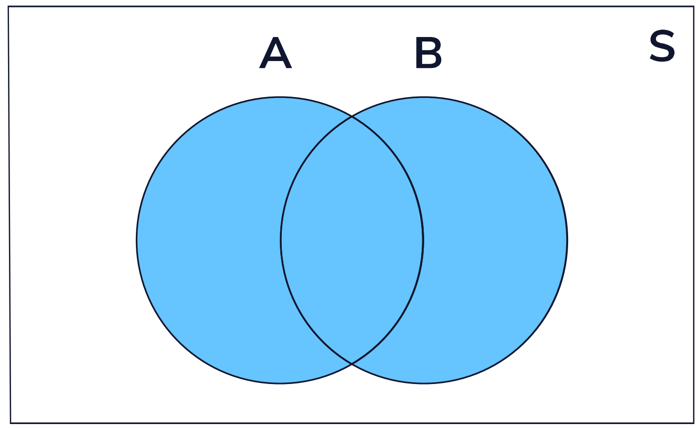
Probability for ML/AI Engineers

Union

The *union* of two sets encompasses any element that exists in either one or both of them. We can represent this visually as a *venn diagram* as shown. Union is often represented as:

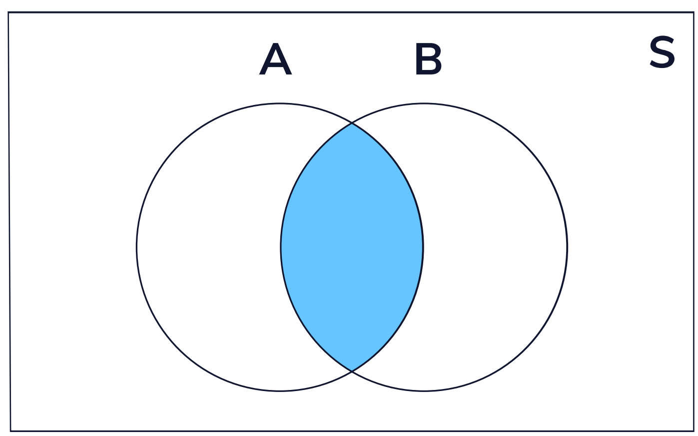
`(A\ or\ B)`



Intersection

The intersection between two sets encompasses any element that exists in BOTH sets and is often written out as:

`(A\ and\ B)`



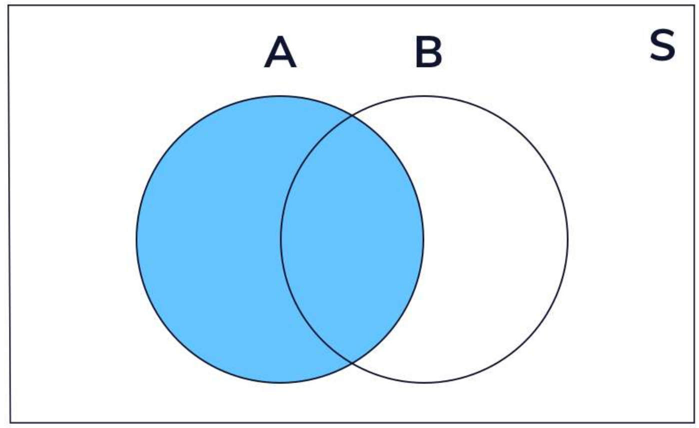
Addition Rule

If there are two events, A and B, the addition rule states that the probability of event A or B occurring is the sum of the probability of each event minus the probability of the intersection:

`P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)`

If the events are mutually exclusive, this formula simplifies to:

`P(A\ or\ B) = P(A) + P(B)`



P(A or B) = P(A)

Multiplication Rule

The multiplication rule is used to find the probability of two events, *A* and *B*, happening simultaneously. The general formula is:

`P(A \text{ and } B) = P(A) \cdot P(B \mid A)`

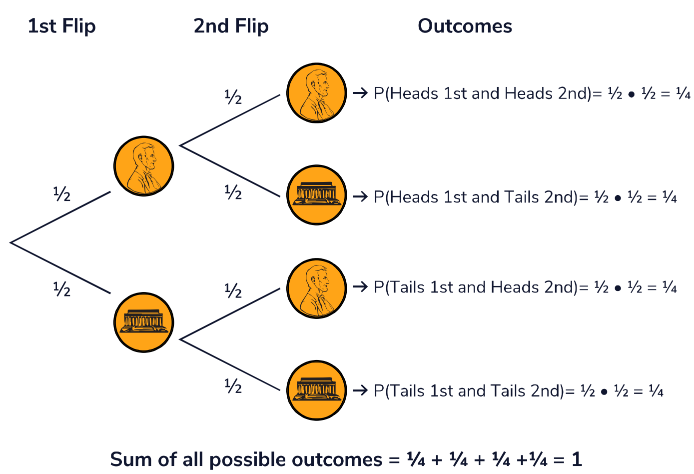
For independent events, this formula simplifies to:

`P(A \text{ and } B) = P(A) \cdot P(B)`

This is because the following is true for independent events:

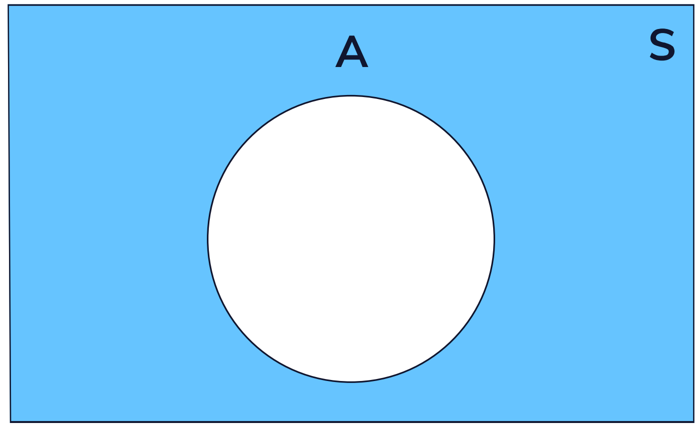
`P(B \mid A) = P(B)`

The tree diagram shown displays an example of the multiplication rule for independent events.



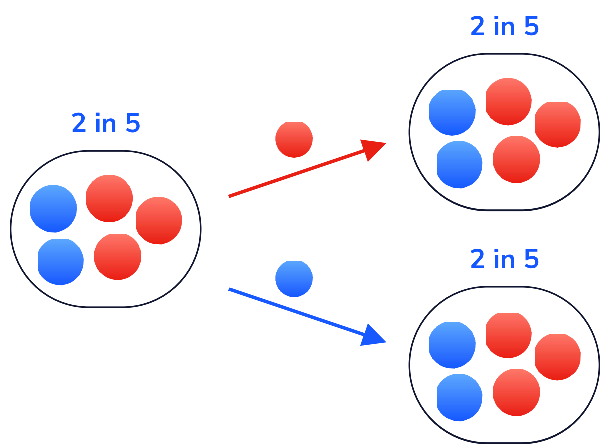
Complement

The complement of a set consists of all possible outcomes outside of the set.
Let's say set *A* is rolling an odd number with a 6-sided die: *{1, 3, 5}*. The complement of this set would be rolling an even number: *{2, 4, 6}*.
We can write the complement of set *A* as A^C. One key feature of complements is that a set and its complement cover the entire sample space. In this die roll example, the set of even numbers and odd numbers would cover all possible rolls: *{1, 2, 3, 4, 5, 6}*.



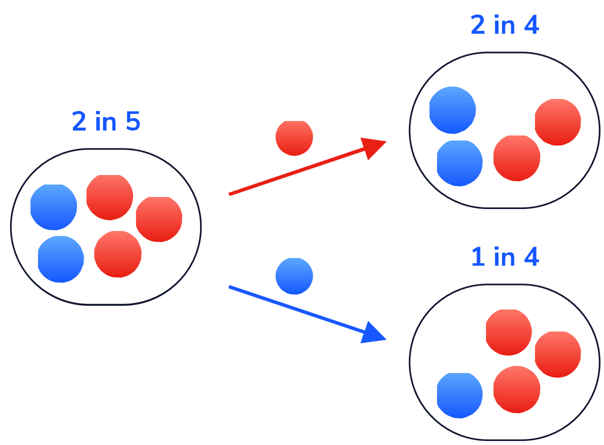
Independent Events

Two events are *independent* if the occurrence of one event does not affect the probability of the other one occurring.
Let's say we have a bag of five marbles: three are red and two are blue. If we select two marbles out of the bag WITH replacement, the probability of selecting a blue marble second is independent of the outcome of the first event.
The diagram below outlines the independent nature of these events. Whether a red marble or a blue marble is chosen randomly first, the chance of selecting a blue marble second is always 2 in 5.



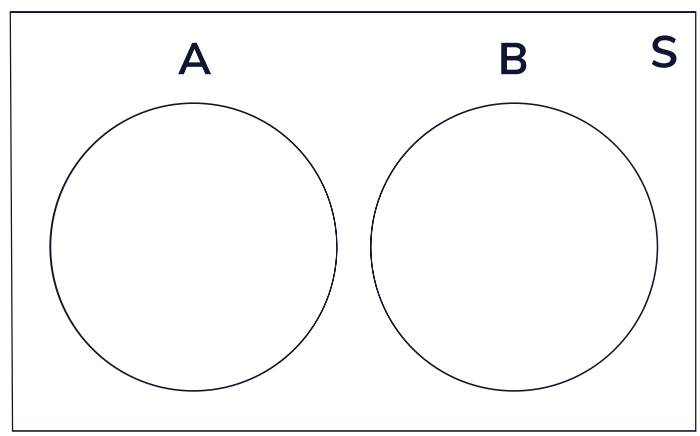
Dependent Events

Two events are *dependent* if the occurrence of one event does affect the probability of the other one occurring. Let's say we have a bag of five marbles: three are red and two are blue. If we select two marbles out of the bag WITHOUT replacement, the probability of selecting a blue marble second depends on the outcome of the first event.
The diagram below outlines this dependency. If a red marble is randomly selected first, the chance of selecting a blue marble second is 2 in 4. Meanwhile, if a blue marble is randomly selected first, the chance of selecting a blue marble second is 1 in 4.



Mutually Exclusive Events

Two events are considered *mutually exclusive* if they cannot occur at the same time. For example, consider a single coin flip: the events "tails" and "heads" are mutually exclusive because we cannot get both tails and heads on a single flip.
We can visualize two mutually exclusive events as a pair of non-overlapping circles. They do not overlap because there is no outcome for one event that is also in the sample space for the other.



Conditional Probability

Conditional probability is the probability of one event occurring, given that another one has already occurred. We can represent this with the following notation:

```
\begin{aligned}
\text{Probability of event A occurring given event B has
occurred} \\
P(A \mid B) \\
\end{aligned}
```

For independent events, the following is true for events *A* and *B*:

```
\begin{aligned}

P(A \mid B) = P(A) \\
\text{and} \\
P(B \mid A) = P(B) \\

\end{aligned}
```

Bayes’ Theorem

Bayes’ theorem is a useful tool to find the probability of an event based on prior knowledge. The formula for Bayes’ theorem is:

```
P(B \mid A) = \frac{P(A \mid B) \cdot P(B)}{P(A)}
```

Random Variables

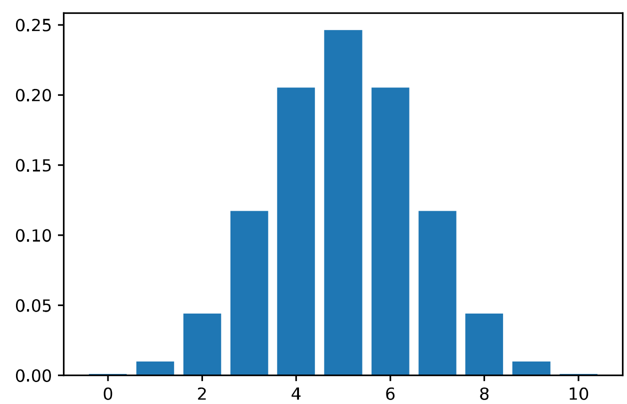
Random variables are functions with numerical outcomes that occur with some level of uncertainty. For example, rolling a 6-sided die could be considered a random variable with possible outcomes {1,2,3,4,5,6}.

Discrete and Continuous Random Variables

Discrete random variables have countable values, such as the outcome of a 6-sided die roll.
Continuous random variables have an uncountable amount of possible values and are typically measurements, such as the height of a randomly chosen person or the temperature on a randomly chosen day.

Probability Mass Functions

A probability mass function (PMF) defines the probability that a discrete random variable is equal to an exact value. In the provided graph, the height of each bar represents the probability of observing a particular number of heads (the numbers on the x-axis) in 10 fair coin flips.



Probability Mass Functions in Python

The `binom.pmf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value in a random experiment.
For example, the provided code calculates the probability of observing exactly 4 heads from 10 fair coin flips.

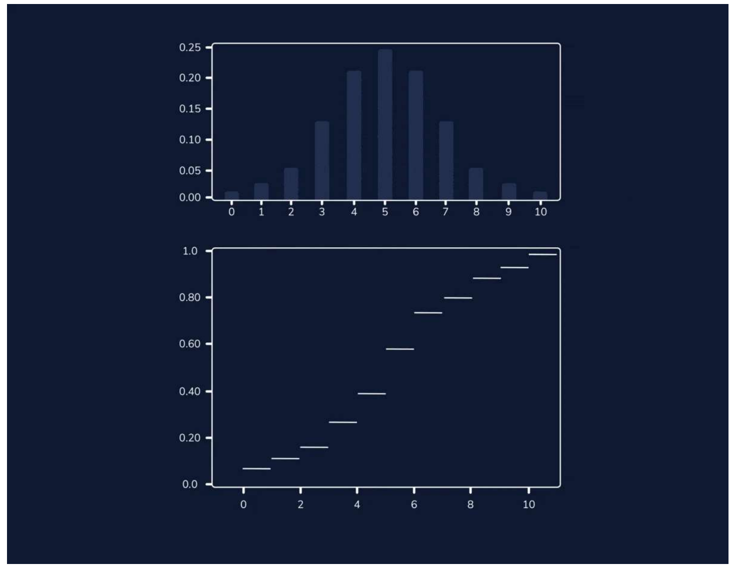
```
import scipy.stats as stats

print(stats.binom.pmf(4, 10, 0.5))

# Output:
# 0.20507812500000022
```

Cumulative Distribution Function

A cumulative distribution function (CDF) for a random variable is defined as the probability that the random variable is less than or equal to a specific value.
In the provided GIF, we can see that as x increases, the height of the CDF is equal to the total height of equal or smaller values from the PMF.



Calculating Probability Using the CDF

The `binom.cdf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value or less using the cumulative density function.
The given code calculates the probability of observing 4 or fewer heads from 10 fair coin flips.

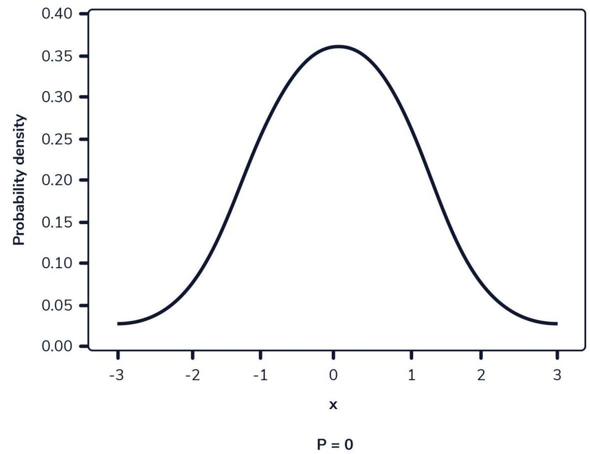
```
import scipy.stats as stats

print(stats.binom.cdf(4, 10, 0.5))

# Output:
# 0.3769531250000001
```

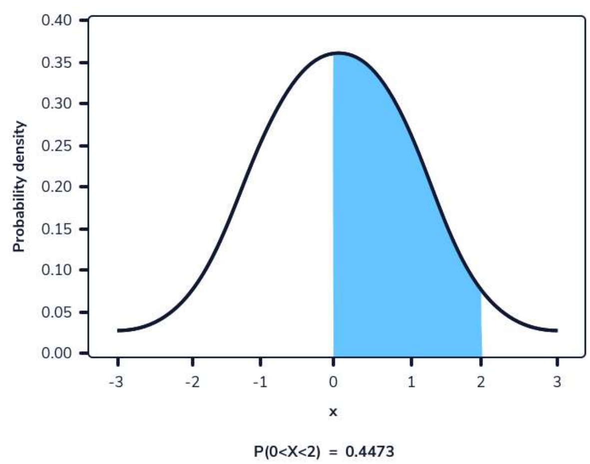
Probability Density Functions

For a continuous random variable, the probability density function (PDF) is defined such that the area underneath the PDF curve in a given range is equal to the probability of the random variable equalling a value in that range. The provided gif shows how we can visualize the area under the curve between two values.



Probability Density Function at a Single Point

The probability that a continuous random variable equals any exact value is zero. This is because the area underneath the PDF for a single point is zero.
In the provided gif, as the endpoints on the x-axis get closer together, the area under the curve decreases. When we try to take the area of a single point, we get 0.



Parameters of Probability Distributions

Probability distributions have parameters that control the exact shape of the distribution.
For example, the binomial probability distribution describes a random variable that represents the number of sucesses in a number of trials (n) with some fixed probability of success in each trial (p). The parameters of the binomial distribution are therefore *n* and *p*. For example, the number of heads observed in 10 flips of a fair coin follows a binomial distribution with n=10 and p=0.5.

The Poisson Distribution

The Poisson distribution is a probability distribution that represents the number of times an event occurs in a fixed time and/or space interval and is defined by parameter λ (lambda).
Examples of events that can be described by the Poisson distribution include the number of bikes crossing an intersection in a specific hour and the number of meteors seen in a minute of a meteor shower.

Expected Value

The *expected value* of a probability distribution is the weighted (by probability) average of all possible outcomes. For different random variables, we can generally derive a formula for the expected value based on the parameters.
For example, the expected value of the binomial distribution is n*p.
The expected value of the Poisson distribution is the parameter λ (lambda).
Mathematically:

```
X \sim Binomial(n, p), \; E(X) = n \times p

Y \sim Poisson(\lambda), \; E(Y) = \lambda
```

Variance of a Probability Distribution

The *variance* of a probability distribution measures the spread of possible values. Similarly to expected value, we can generally write an equation for the variance of a particular distribution as a function of the parameters.
For example:

```
X \sim Binomial(n, p), \; Var(X) = n \times p \times (1-p)

Y \sim Poisson(\lambda), \; Var(Y) = \lambda
```

Sum of Expected Values

For two random variables, *X* and *Y*, the expected value of the sum of *X* and *Y* is equal to the sum of the expected values.
Mathematically:

```
E(X + Y) = E(X) + E(Y)
```

Adding a Constant to an Expected Value

If we add a constant *c* to a random variable *X*, the expected value of *X + c* is equal to the original expected value of *X* plus *c*.
Mathematically:

```
E(X + c) = E(X) + c
```

Multiplying an Expectation by a Constant

If we multiply a random variable *X* by a constant *c*, the expected value of *c*X* equals the original expected value of *X* times *c*.
Mathematically:

```
E(c \times X) = c \times E(X)
```

Adding a Constant to Variance

If we add a constant *c* to a random variable *X*, the variance of the random variable will not change.
Mathematically:

```
Var(X + c) = Var(X)
```

Multiplying Variance by a Constant

If we multiply a random variable *X* by a constant *c*, the variance of *c*X* equals the original expected value of *X* times *c* squared.
Mathematically:

```
Var(c\times X) = c^2 \times Var(X)
```

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