Intermodulation:

- Intermodulation products are generated whenever more than one signal is carried by a nonlinear device.
- The characteristics of a transponder can be modeled by a cubic curve to illustrate the generation of third – order intermodulation.
- Third – order intermodulation is important because third-order IM products often have frequencies close to the signals that generate the intermodulation.
- To illustrate the generation of third- order IM products, we will model nonlinear characteristics of the transponder HPA with cubic voltage relationship and apply two unmodulated carriers at frequencies f_1 & f_2 at the input of the amplifier.

- $$V_{out} = Av_{in} + b(V_{in})^3$$
  $$\text{where } A >> b$$
- The amplifier input signal is
  $$V_1 \cos w_1 t + V_2 \cos w_2 t$$

The amplifier output signal is

$$V_{out} = AV_{in} + b(V_{in})^3$$
$$= \underbrace{AV_1 \cos\omega_1 t + AV_2 \cos\omega_2 t}_{\text{linear term}} + \underbrace{b(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^3}_{\text{cubic term}} \quad (6.3)$$

The linear term simply amplifies the input signal by a voltage gain A. The cubic term, which will be denoted as V_{3out}, can be expanded as

$$V_{3out} = (V_1 \cos\omega_1 t + V_2 \cos\omega_2 t)^3$$
$$= b[V_1^3 \cos^3\omega_1 t + V_2^3 \cos^3\omega_2 t +$$
$$2V_2^2 \cos^2\omega_2 t \times V_2 \cos\omega_2 t + 2V_2^2 \cos^2\omega_2 t \times V_1 \cos\omega_1 t] \quad (6.4)$$

The first two terms contain frequencies f_1, f_2, $3f_1$, and $3f_2$. The triple frequency components can be removed from the amplifier output with band-pass filters. The second two terms generate the third-order IM frequency components.

We can expand the cosine squared terms using the trig identity $\cos^2 x = \frac{1}{2}[\cos 2x + 1]$. Hence the IM terms of interest become

$$V_{IM} = bV_1^2 \times V_2[\cos\omega_2 t \times (\cos 2\omega_1 t + 1)] +$$
$$bV_2^2 \times V_1[\cos\omega_1 t \times (\cos 2\omega_2 t + 1)]$$
$$= bV_1^2 \times V_2[\cos\omega_2 t \cos 2\omega_1 t + \cos\omega_2 t] +$$
$$bV_2^2 \times V_1[\cos\omega_1 t \cos 2\omega_2 t + \cos\omega_1 t] \quad (6.5)$$

The terms at frequencies f_1 and f_2 add to the wanted output of the amplifier, so the third-order intermodulation products are generated by the $f_1 \times 2f_2$ and $f_2 \times 2f_1$ terms.

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Using another trig identity

$$\cos x \cos y = \cos(x + y) + \cos(x - y)$$

The output of the amplifier contains IM frequency components given by

$$V'_{IM} = bV_1^2 \times V_2[\cos(2\omega_1 t + \omega_2 t) + \cos(2\omega_1 t - \omega_2 t)]$$
$$+ bV_2^2 \times V_1[\cos(2\omega_2 t + \omega_1 t) + \cos(2\omega_2 t - \omega_1 t)] \quad (6.6)$$

We can filter out the sum terms in Eq. (6.6), but the difference terms, with frequencies $2f_1 - f_2$ and $2f_2 - f_1$ may fall within the transponder bandwidth. These two terms are known as the third-order intermodulation products of the high-power amplifier, because they are the only ones likely to be present at the output of a transponder which incorporates a narrow bandpass filter at its output. Thus the third-order intermodulation products that are of concern are given by V_{3IM} where

$$V_{3IM} = bV_1^2 V_2 \cos(2\omega_1 t - \omega_2 t) + bV_2^2 V_1 \cos(2\omega_2 t - \omega_1 t) \quad (6.7)$$

The magnitude of the IM products depends on the parameter b, which describes the nonlinearity of the transponder, and the magnitude of the signals. The wanted signals at the transponder output, at frequencies f_1 and f_2, have magnitudes AV_1 and AV_2. The wanted output from the amplifier is

$$V_{out} = AV_1 \cos\omega_1 t + AV_2 \cos\omega_2 t$$

The total power of the wanted output from the HPA, referenced to a 1 ohm load, is therefore

$$P_{out} = \tfrac{1}{2}A^2V_1^2 + \tfrac{1}{2}A^2V_2^2 = A^2(P_1 + P_2)\ \text{W} \quad (6.8)$$

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- The power of the IM products at the output of HPA is $P_{IM} = b^2 (P_1^3 + P_2^3)$