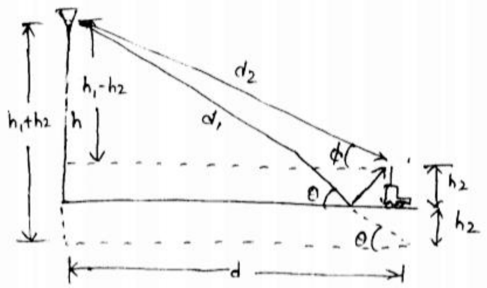
* The phase difference between a Direct path and Ground Reflected path:

For Direct wave signal : $e_0$

For Reflected wave signal : $e_0 \cdot a_v \, e^{j\Delta\phi}$




$$\Rightarrow \quad e_0 + e_0 a_v e^{j\Delta\phi}$$

$$e_0 \left(1 + a_v e^{j\Delta\phi}\right)$$

In terms of power, the received power is

$$P_r = \text{Direct wave signal power} \times \left[1 + a_v e^{j\Delta\phi}\right]^2$$

$$P_r = \frac{P_0 \lambda^2}{(4\pi d)^2} \left[1 + a_v e^{j\Delta\phi}\right]^2 \quad \text{—} \; (1)$$

where $a_v$ is reflection co-efficient

$\Delta\phi$ is the phase difference b/w direct & reflected paths

$P_0$ is the transmitted power

$d$ is the distance

$\lambda$ is wavelength

In a mobile radio Environment $a_v = -1$ because of the small incident angle of ground wave caused by a relatively low cell site antenna height

thus

$$P_r = \frac{P_0 \lambda^2}{(4\pi d)^2} \left[1 + (-1)(\cos\Delta\phi + j\sin\Delta\phi)\right]^2$$

$$= \;\; '' \;\; \left[\underbrace{1 - \cos\Delta\phi}_{a} - j\underbrace{\sin\Delta\phi}_{b}\right]^2$$

$$\because \boxed{|a - jb|^2 = \left(\sqrt{(a^2+b^2)}\right)^2 = a^2 + b^2}$$

$$= \;\; '' \;\; \left[(1-\cos\Delta\phi)^2 + \sin^2\Delta\phi\right]$$

$$= \;\; '' \;\; \left[1 + \cos^2\Delta\phi - 2\cos\Delta\phi + \sin^2\Delta\phi\right]$$

$$= \;\; '' \;\; \left[2 - 2\cos\Delta\phi\right] \Rightarrow \frac{P_0 \lambda^2}{(4\pi d)^2} \, 2(1 - \cos\Delta\phi)$$

$$\boxed{\because (1 - \cos\theta) = 2\sin^2\theta/2}$$

$$= \frac{P_0 \lambda^2}{(4\pi d)^2} \cdot 2\left(2\sin^2\frac{\Delta\phi}{2}\right)$$

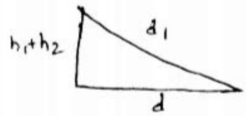
$$P_r = \frac{P_0 \lambda^2 \, 4 \sin^2\frac{\Delta\phi}{2}}{(4\pi d)^2}$$

where $\Delta\phi = \beta \Delta d$

and $\Delta d$ is the difference i.e $\Delta d = d_1 - d_2$

From fig :

$$d_1 = \sqrt{(h_1+h_2)^2 + d^2}$$




$$d_2 = \sqrt{(h_1-h_2)^2 + d^2}$$

$$\therefore \Delta d = \sqrt{(h_1+h_2)^2 + d^2} - \sqrt{(h_1-h_2)^2 + d^2}$$

$$= d\left[\sqrt{\frac{(h_1+h_2)^2}{d^2}+1} - \sqrt{\frac{(h_1-h_2)^2}{d^2}+1}\right]$$

From Binominal Theory Expansion

$$= d\left[\left(1+\frac{1}{2}\frac{(h_1+h_2)^2}{d^2}+\dots\right) - \left[1+\frac{1}{2}\frac{(h_1-h_2)^2}{d^2}+\dots\right]\right]$$

$$= d\left[1+\frac{1}{2d^2}\left(h_1^2+h_2^2+2h_1h_2+\dots\right) - \left[1+\frac{1}{2d^2}\left[h_1^2+h_2^2-2h_1h_2 + \dots\right]\right]\right]$$

$$= \cancel{d}\left[\frac{\cancel{4}^{2} h_1 h_2}{\cancel{2}d^{\cancel{2}}}\right]$$

$$= \frac{2h_1h_2}{d}$$

Here $\Delta\phi = \beta \times \Delta d$

$$= \frac{2\pi}{\lambda} \times \frac{2h_1h_2}{d} \Rightarrow \frac{4\pi h_1 h_2}{\lambda d}$$

If $\Delta\phi < 0.6$ rad then $\sin\frac{\Delta\phi}{2} \approx \frac{\Delta\phi}{2}$ then the Equation is

$$\cancel{P_r = \frac{P_0\lambda^2 4}{(4\pi d)^2}\left(\frac{4\pi h_1h_2}{\lambda d}\right)^2}$$

$$P_r = \frac{P_0\lambda^2 4}{(4\pi d)^2}\left(\frac{\cancel{4}^{2}\pi h_1 h_2}{\cancel{\lambda} d \,/\, \cancel{2}}\right)^2$$

$$= \frac{P_0 \cancel{\lambda^2}\cancel{4}}{\cancel{16}\cancel{\pi^2}d^2} \times \frac{4\cancel{\pi^2}h_1^2h_2^2}{\cancel{\lambda^2}d^2}$$

$$\boxed{P_r = P_0\left(\frac{h_1h_2}{d^2}\right)^2}$$