The measurement of the sensitivity of a material to strain is called the gauge factor (GF). It is the ratio of the change in resistance $\Delta R/R$ to the change in the length $\Delta l/l$

i.e.
$$\text{GF}(K) = \frac{\Delta R/R}{\Delta l/l} \tag{13.1}$$

where K = gauge factor
ΔR = the change in the initial resistance in Ω's
R = the initial resistance in Ω (without strain)
Δl = the change in the length in m
l = the initial length in m (without strain)

Since strain is defined as the change in length divided by the original length, i.e. $$\sigma = \frac{\Delta l}{l}$$

Eq. (13.1) can be written as

$$K = \frac{\Delta R / R}{\sigma} \tag{13.2}$$

where σ is the strain in the lateral direction.

The resistance of a conductor of uniform cross-section is

$$R = \rho \frac{\text{length}}{\text{area}}$$

$$R = \rho \frac{l}{\pi r^2}$$

Since $$r = \frac{d}{2} \quad \therefore \quad r^2 = \frac{d^2}{4}$$

$$\therefore \quad R = \rho \frac{l}{\pi d^2/4} = \rho \frac{l}{\pi/4\, d^2} \tag{13.3}$$

where ρ = specific resistance of the conductor
l = length of conductor
d = diameter of conductor

When the conductor is stressed, due to the strain, the length of the conductor increases by Δl and the simultaneously decreases by Δd in its diameter. Hence the resistance of the conductor can now be written as

$$R_s = \rho \frac{(l + \Delta l)}{\pi/4 (d - \Delta d)^2} = \frac{\rho (l + \Delta l)}{\pi/4 (d^2 - 2d\, \Delta d + \Delta d^2)}$$

Since Δd is small, Δd^2 can be neglected

$$\therefore \quad R_s = \frac{\rho (l + \Delta l)}{\pi/4 (d^2 - 2d\, \Delta d)}$$

$$= \frac{\rho (l + \Delta l)}{\pi/4 d^2 \left(1 - \frac{2\Delta d}{d}\right)} = \frac{\rho l (1 + \Delta l / l)}{\pi/4 d^2 \left(1 - \frac{2\Delta d}{d}\right)} \tag{13.4}$$

Now, Poisson's ratio μ is defined as the ratio of strain in the lateral direction to strain in the axial direction, that is,

$$\mu = \frac{\Delta d / d}{\Delta l / l} \tag{13.5}$$

$$\therefore \quad \frac{\Delta d}{d} = \mu \frac{\Delta l}{l} \tag{13.6}$$

Substituting for $\Delta d/d$ from Eq. (13.6) in Eq. (13.4), we have

$$R_s = \frac{\rho l\,(1+\Delta l/l)}{(\pi/4)\,d^2\,(1-2\mu\,\Delta l/l)}$$

Rationalising, we get

$$R_s = \frac{\rho l\,(1+\Delta l/l)}{(\pi/4)\,d^2\,(1-2\mu\,\Delta l/l)}\,\frac{(1+2\mu\,\Delta l/l)}{(1+2\mu\,\Delta l/l)}$$

$$\therefore \quad R_s = \frac{\rho l}{(\pi/4)\,d^2}\left[\frac{(1+\Delta l/l)}{(1-2\mu\,\Delta l/l)}\,\frac{(1+2\mu\,\Delta l/l)}{(1+2\mu\,\Delta l/l)}\right]$$

$$\therefore \quad R_s = \frac{\rho l}{(\pi/4)\,d^2}\left[\frac{1+2\mu\,\Delta l/l + 2\,\Delta l/l + 2\mu\,\Delta l/e\,\Delta l/l}{1-4\mu^2\,(\Delta l/l)^2}\right]$$

$$\therefore \quad R_s = \frac{\rho l}{(\pi/4)\,d^2}\left[\frac{1+2\mu\,\Delta l/l + \Delta l/l + 2\mu\,\Delta l^2/l^2}{1-4\mu^2\,\Delta l^2/l^2}\right]$$

Since Δl is small, we can neglect higher powers of Δl.

$$\therefore \quad R_s = \frac{\rho l}{(\pi/4)\,d^2}\,[1+2\,\mu\,\Delta l/l + \Delta l/l]$$

$$R_s = \frac{\rho l}{(\pi/4)\,d^2}\,[1+(2\,\mu+1)\,\Delta l/l]$$

$$R_s = \frac{\rho l}{(\pi/4)\,d^2}\,[1+(1+2\,\mu)\,\Delta l/l]$$

$$\therefore \quad R_s = \frac{\rho l}{(\pi/4)\,d^2} + \frac{\rho l}{(\pi/4)\,d^2}\,(\Delta l/l)\,(1+2\,\mu)$$

Since from Eq. (13.3), $R = \dfrac{\rho l}{(\pi/4)\,d^2}$ R + DR

$$\therefore \quad R_s = R + \Delta R \tag{13.7}$$

where

$$\Delta R = \frac{\rho l}{(\pi/4)\,d^2}\,(\Delta l/l)\,(1+2\,\mu)$$

$\therefore$ The gauge factor will now be

$$K = \frac{\Delta R/R}{\Delta l/l} = \frac{(\Delta l/l)\,(1+2\mu)}{\Delta l/l}$$

$$= 1+2\,\mu$$

$$\therefore \quad K = 1+2\,\mu \tag{13.8}$$

$$\Delta l / l \qquad \Delta l / l$$

$$= 1 + 2\mu$$

$$\therefore \quad K = 1 + 2\mu \tag{13.8}$$