Assignment 8

Ravula Karthik (AI21BTECH11024)

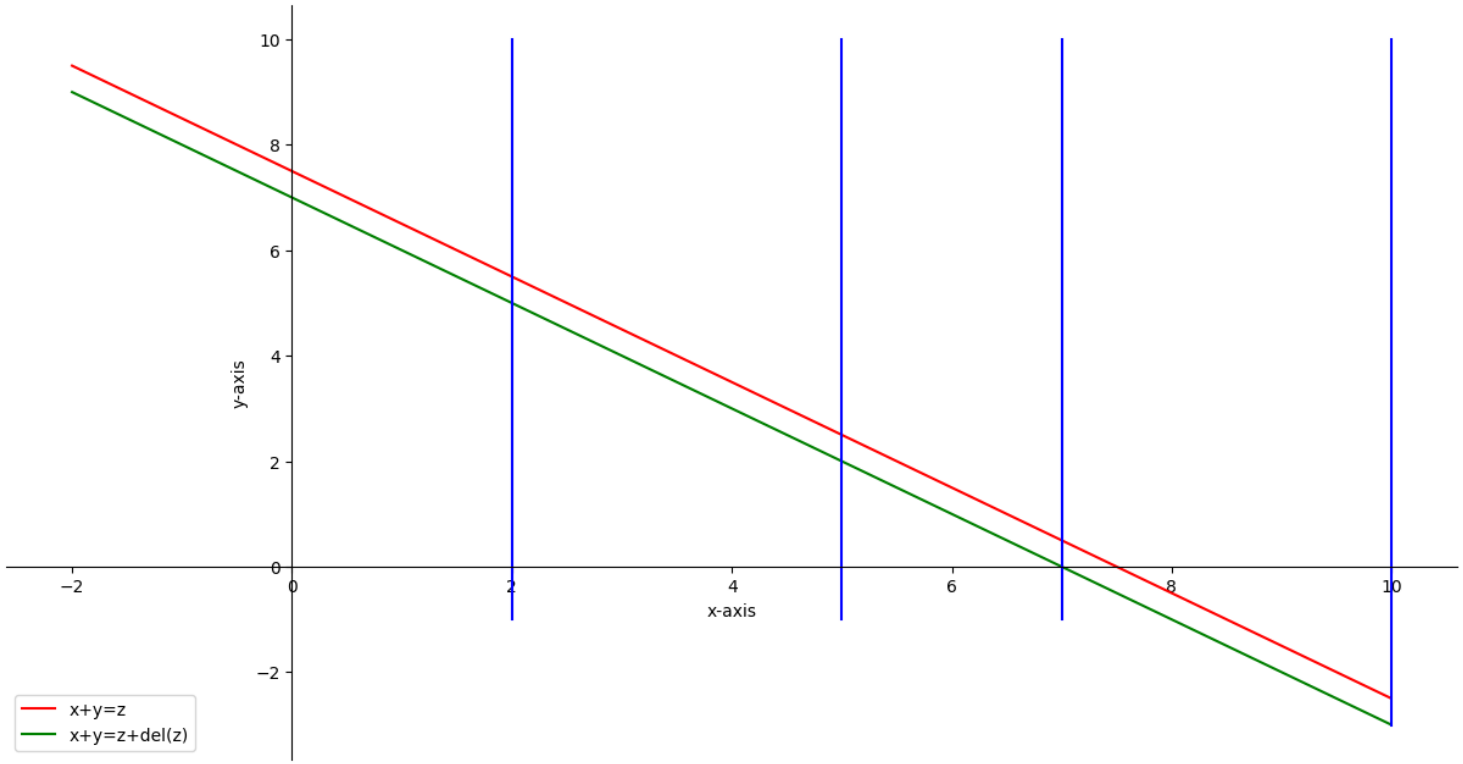
June 2022

Question : EX 6.41

The random variable x is of discrete type taking the values x_n with $P\{x = x_1\} = p_n$ and the random variable y is of continuous type and independent of x. Show that if $z = x + y$ and $w = xy$, then

Solution

(a)

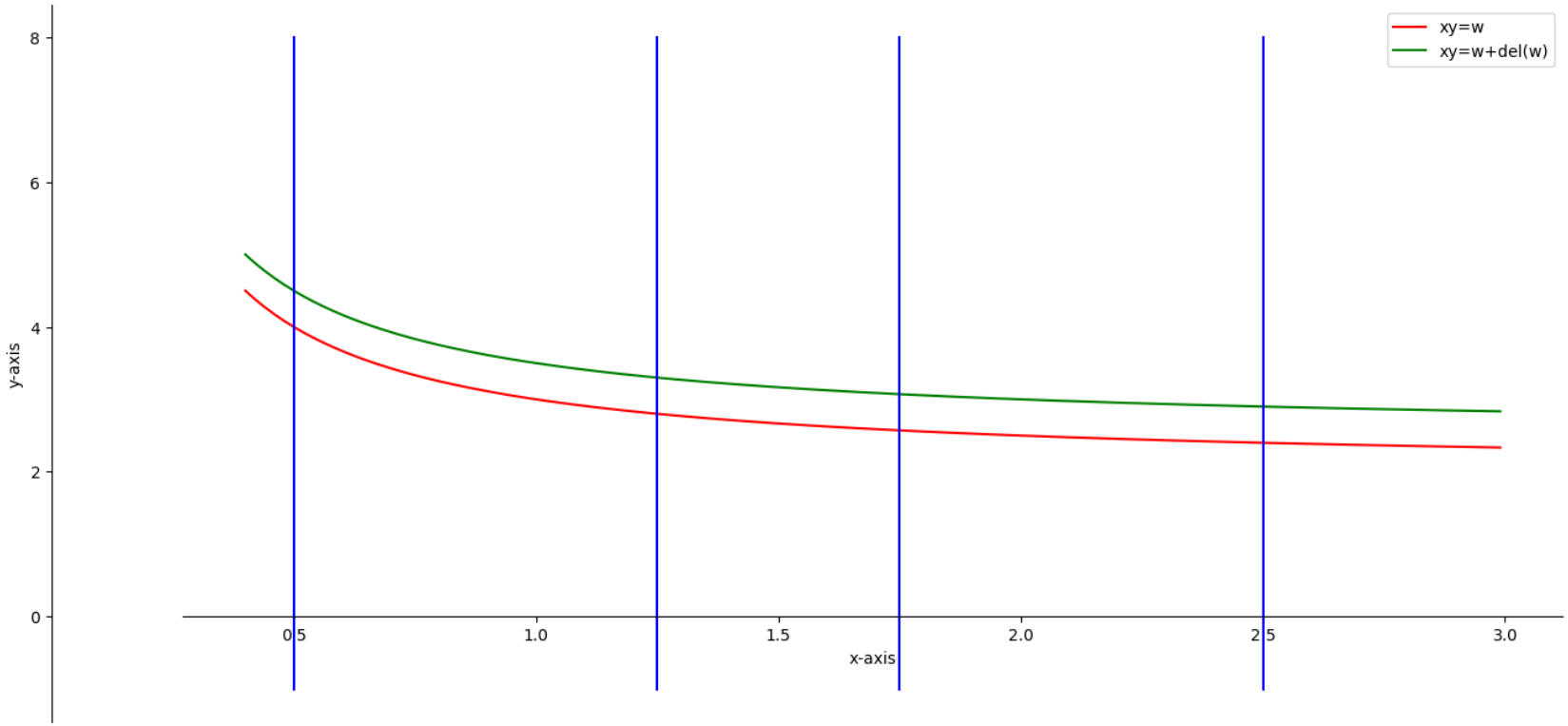


Line masses

$$P\left\{x = x_n, z - x_n \leq \underline{y} \leq z - x_n + \Delta z\right\} = p_n f_n\left(z - x_n\right)\Delta z$$

$$\implies \left\{z \leq \underline{z} \leq z + \Delta z\right\} = \sum_n \left\{x = x_n, z - x_n \leq \underline{y} \leq z - x_n + \Delta z\right\} \tag{1}$$

$$\implies f_z\left(z\right)\Delta z = \sum_n p_n f_y\left(z - x_n\right)\Delta z \tag{2}$$


xy=w
xy=w+del(w)
y-axis
x-axis
8
6
4
2
0
0 5
1.0
1.5
2.0
2 5
3.0

$$P\left\{x = x_n, \frac{w}{x_n} \leq \underline{y} \leq \frac{w + \Delta w}{x_n}\right\} = p_n f_y\left(\frac{w}{x_n}\right)\Delta w$$

$$\implies \{w \leq \underline{w} \leq w + \Delta w\} = \sum_n \left\{x = x_n, \frac{w}{x_n} \leq \underline{y} \leq \frac{w + \Delta w}{x_n}\right\} \tag{3}$$

$$\implies f_w(w)\,\Delta w = \sum_n p_n f_y\left(\frac{w}{x_n}\right)\Delta z \tag{4}$$