Assignment 3

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Question : Trains X and Y arrive at a station at random between 8 A.M. and 8.20 A.M. Train X stops for four minutes and train Y stops for five minutes. Assuming that the trains arrive independently of each other, determine various probabilities related to the times x and y of their respective arrivals.

Solution :

The event

A = X arrives in the interval (t_1, t_2)

$$\implies P(A) = \frac{t_2 - t_1}{20} \tag{1}$$

The event

B = Y arrives in the interval (t_3, t_4)

$$\implies P(B) = \frac{t_4 - t_3}{20} \tag{2}$$

Interpreting the independence of the rival times as independence of the events A and B, we obtain ,

$$\implies P(AB) = P(A)P(B) = \frac{(t_4 - t_3)(t_2 - t_1)}{400} \tag{3}$$

(i) We shall determine the probability that train X arrives before train Y.
This is the probability of the event

$$C = \{x \leq y\}$$

This event is a triangle with area 200. Hence

$$\implies P(C) = \frac{200}{400} \tag{4}$$

(ii) We shall determine the probability that the trains meet at the station. For the trains to meet, x must be less than y + 5 and y must be, less than x + 4. This is the event

$$D = \{-4 \leq x - y \leq 5\}$$

The region D consists of two trapezoids with common base, and its area equals 159.5. Hence

$$\implies P(D) = \frac{159.5}{400} \tag{5}$$

(iii) Assuming that the trains met, we shall determine the probability that train X arrived before train Y. We wish to find the conditional probability P(C/D). The event CD is a trapezoid and its area equals 72. Hence

$$P(C/D) = \frac{P(CD)}{P(D)} = \frac{72}{159.5} \tag{6}$$