AI1110 Assignment 2

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Question 2(a): Using properties of determinants prove that

$$\begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix} = (a+b+c)(a-c)^2 \tag{1}$$

Solution: : Let the given determinant be

$$M = \begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix} \tag{2}$$

The above determinant can be simplified as

$$\xrightarrow{R_1 \to R_1 + R_3} \begin{vmatrix} a+b & b+c & a+2b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix}, \tag{3}$$

$$\xrightarrow{R_1 \to R_1 + R_2} \begin{vmatrix} a+b+c & a+b+c & 2(a+b+c) \\ c & a & c+a \\ b & c & a+b \end{vmatrix}, \tag{4}$$

$$\xrightarrow{C_1 \to C_1 + C_2} \begin{vmatrix} 2(a+b+c) & a+b+c & 2(a+b+c) \\ c+a & a & c+a \\ b+c & c & a+b \end{vmatrix}, \tag{5}$$

$$\xrightarrow{C_3 \to C_3 - C_1} \begin{vmatrix} 2(a+b+c) & a+b+c & 0 \\ c+a & a & 0 \\ b+c & c & a-c \end{vmatrix} \tag{6}$$

$$= (a+b+c)(a-c) \begin{vmatrix} 2 & 1 & 0 \\ c+a & a & 0 \\ b+c & c & 1 \end{vmatrix} \tag{7}$$

$$= (a+b+c)(a-c)^2 \tag{8}$$

$$\therefore M = \begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix} = (a+b+c)(a-c)^2 \tag{9}$$