

Assignment 6

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Question : Show that if x has a Rayleigh density with parameter α and $y = b+cx^2$, then $\sigma_y^2 = 4c^2\alpha^4$

Solution :

If x has a Rayleigh density

$$f(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} U(x)$$

then

$$E \{x^n\} = \frac{1}{\sigma^2} \int_0^\infty x^{n+1} e^{\frac{-x^2}{2\sigma^2}} dx \quad (1)$$

$$= \frac{1}{2\sigma^2} \int_{-\infty}^\infty |x|^{n+1} e^{\frac{-x^2}{2\sigma^2}} dx \quad (2)$$

from this we get ;

$$E \{x^n\} = \begin{cases} 1.3\dots n\sigma^n \sqrt{\frac{\pi}{2}}, & n = 2k + 1 \\ 2^k k! \sigma^{2k}, & n = 2k \end{cases} \quad (3)$$

for the given question ;

from (3)

$$E \{x^2\} = 2\alpha^2 \quad (4)$$

$$E \{x^4\} = 8\alpha^4 \quad (5)$$

If $y = b+cx^2$, then

$$E \{y\} = b + 2\alpha^2 c \quad (6)$$

$$E \{y^2\} = b^2 + 4\alpha^2 bc + 8\alpha^4 c^2 \quad (7)$$

$$\sigma_y^2 = E \{y^2\} - E \{y\}^2 = 4c^2\alpha^4. \quad (8)$$