Assignment 11

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June 2022

Question : EX 10.21

Given a WSS process $x(t)$ and a set of Poisson points t independent of $x(t)$ and with average density λ. we form the sum
$X_c\left(w\right) = \sum_{|t_i| \leq c} x\left(t_i\right) e^{-jwt_i}$
Show that if $E\left\{x(t)\right\} = 0$ and $\int_{-\infty}^{\infty} |X_c(w)| d\tau < \infty$, then for large c
$E\left\{|X_c(w)^2|\right\} = 2cS_x(w) + \frac{2c}{\lambda} R_x(0)$.

Solution

We shall show that if

$$\underline{X}_c(w) = \frac{1}{\lambda} \sum_{|t_i| \le c} \underline{x}\left(t_i\right) e^{-jwt_i}$$
$$= \frac{1}{\lambda} \int_{-a}^{a} \underline{x}\left(t\right) \underline{z}\left(t\right) e^{-jwt}$$

where $z(t) = \sum \delta(t - \underline{t_i})$ is a Poisson impulse train, then $E\left\{|X_c(w)^2|\right\} = 2cS_x(w) + \frac{2c}{\lambda} R_x(0)$

proof

Since $R_z(\tau) = \lambda^2 + \lambda\delta(r)$, it follows that

$$E\left\{|\underline{X}_c(w)^2|\right\} = \frac{1}{\lambda^2}\int_{-c}^{c}\int_{-c}^{-c} R_x(t_1 - t_2)e^{-jw(t_1-t_2)}dt_1 dt_2 \tag{1}$$

$$= \int_{-c}^{c} e^{jwt_2}\int_{-c}^{-c} R_x(t_1 - t_2)e^{-jwt_1}dt_1 dt_2 + \frac{1}{\lambda}\int_{-c}^{c} R_x(0)dt_2 \tag{2}$$

If $\int_{-\infty}^{\infty}|R_x(\tau)| < \infty$ then for sufficient large c , the inner integral on the right is nearly equal to $S_x(w)e^{-}jwt_2$ and (i) follows.