

Assignment 12

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Question : EX 6.41

The processes $x(t)$ and $y(t)$ are Jointly normal with zero mean.
Show that:

(a) If $w(t) = X(t + \lambda)y(t)$, then

$$C_{ww} = C_{xy}(\lambda + \tau)C_{xy}(\lambda - \tau) + C_{xx}(\tau)C_{yy}(\tau)$$

(b) If the functions $C_{xy}(\tau)$, $C_{yy}(\tau)$ and $C_{xx}(\tau)$ tend to 0 as $\tau \rightarrow \infty$ then the processes $x(t)$ and $y(t)$ are cross-variance ergodic.

Solution

(a) With $E \{w(t)\} = C_x y(\lambda)$

$$\begin{aligned} R_{ww}(\tau) &= C_{xy}(\lambda + \tau)C_{xy}(\lambda - \tau) + C_{xx}(\tau)C_{yy}(\tau) + C_{xx}^2(\lambda) \\ &= C_{ww}(\tau) + C_{xx}^2(\lambda) \end{aligned}$$

(b) It follows from (a) that if

$$C_{xx}(\tau) \rightarrow 0$$

$$C_{yy}(\tau) \rightarrow 0$$

$$C_{xy}(\tau) \rightarrow 0$$

then $C_{ww}(\tau) \rightarrow 0$ as $|\tau| \rightarrow \infty$; hence the process $x(t)$ and $y(t)$ are covariance ergodic .