Assignment 6

Ravula Karthik (AI21BTECH11024)

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Question : EX 5.23

Show that if x has a Rayleigh density with parameter α and y $= b+cx^2$, then $\sigma_y^2 = 4c^2\alpha^4$.

Solution

If x has a Rayleigh density

f(x) $= \frac{x}{\sigma^2}\ e^{\frac{-x^2}{2\sigma^2}} U(x)$

then

$$E\left\{x^n\right\} = \frac{1}{\sigma^2}\int_0^\infty x^{n+1} e^{\frac{-x^2}{2\sigma^2}}\, dx \tag{1}$$

$$= \frac{1}{2\sigma^2}\int_{-\infty}^\infty |x|^{n+1} e^{\frac{-x^2}{2\sigma^2}}\, dx \tag{2}$$

from this we get ;

$$E\left\{x^n\right\} = \begin{cases} 1.3...n\sigma^n \sqrt[2]{\frac{\pi}{2}}., & n = 2k+1 \\ 2^k k!\sigma^2 k \ , & n = 2k \end{cases} \tag{3}$$

for the given question ;
from (3)

$$E\left\{x^2\right\} = 2\alpha^2 \tag{4}$$

$$E\left\{x^4\right\} = 8\alpha^4 \tag{5}$$

If y = b+cx^2 , then

$$E\left\{y\right\} = b + 2\alpha^2 c \tag{6}$$

$$E\left\{y^2\right\} = b^2 + 4\alpha^2 bc + 8\alpha^4 c^2 \tag{7}$$

$$\sigma_y^2 = E\left\{y^2\right\} - E\left\{y\right\} = 4c^2\alpha^4. \tag{8}$$