Assignment 7

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Question : EX 5.39

A random variable x has a geometric distribution if

$$P(x = k) = pqk \ \ k = 0, 1, \ldots \ p + q = 1$$

Find $\Gamma(z)$ and show that $\eta_{\,x} = \frac{q}{p}$, $\sigma_y^2 = \frac{q}{p^2}$

Solution

We already know

$$p + q = 1 \tag{1}$$

$$\Gamma\left(z\right) = \sum_{k=0}^{\infty} pq^k z^k \tag{2}$$

from (1)

$$= \frac{p}{1 - qz} \tag{3}$$

Now ,

$$\Gamma'(z) = \frac{pq}{(1-qz)^2} \tag{4}$$

and

$$\Gamma''(z) = \frac{2pq^2}{(1-qz)^3} \tag{5}$$

And

$$\Gamma'(1) = \frac{pq}{(1-q)^2} = \frac{q}{p} = \eta_x \tag{6}$$

and

$$\Gamma''(1) = \frac{2pq^2}{(1-q)^3} = \frac{2q^2}{p^2} = m_2 - m_1 \tag{7}$$

Then

$$\sigma^2 = m_2 - (m_1)^2 \tag{8}$$

$$= \frac{2q^2}{p^2} + m_1 - (m_1)^2 \tag{9}$$

$$= \frac{q}{p^2} \tag{10}$$

$\therefore \ \sigma_y^2 = \frac{q}{p^2}$ and $\eta_{\ x} = \frac{q}{p}$