Assignment 7

KARTHIK RAVULA
AI21BTECH11024

Question : A random variable x has a geometric distribution if

P$(x = k)$ = pqk $k = 0, 1, ...$ $p + q = 1$

Find $\Gamma(z)$ and show that $\eta_{\ x}$ = $\frac{q}{p}$, σ_y^2 = $\frac{q}{p^2}$

Solution :

We already know

$$p + q = 1 \tag{1}$$

$$\Gamma\left(z\right) = \sum_{k=0}^{\infty} pq^k z^k \tag{2}$$

from (1)

$$= \frac{p}{1 - qz} \tag{3}$$

Now ,

$$\Gamma'\left(z\right) = \frac{pq}{(1 - qz)^2} \tag{4}$$

and

$$\Gamma''\left(z\right) = \frac{2pq^2}{(1 - qz)^3} \tag{5}$$

And

$$\Gamma'\left(1\right) = \frac{pq}{(1 - q)^2} = \frac{q}{p} = \eta_x \tag{6}$$

and

$$\Gamma''\left(1\right) = \frac{2pq^2}{(1 - q)^3} = \frac{2q^2}{p^2} = m_2 - m_1 \tag{7}$$

Then

$$\sigma^2 = m_2 - (m_1)^2 \tag{8}$$

$$= \frac{2q^2}{p^2} + m_1 - (m_1)^2 \tag{9}$$

$$= \frac{q}{p^2} \tag{10}$$

$\therefore$ σ_y^2 = $\frac{q}{p^2}$ and $\eta_{\ x}$ = $\frac{q}{p}$