Assignment 1

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Question 3.8: The system function of a causal linear time-invariant system is

$H(z) = \frac{1-z^{-1}}{1+\frac{3}{4}z^{-1}}$

The input to this system is

$x[n] = (\frac{1}{3})^n u[n] + u[-n-1]$

Find the output $y[n]$.

SOLUTION:

$h[n] = (\frac{-3}{4})^n u[n] - (\frac{-3}{4})^{n-1} u[n-1]$

$$Y(z) = X(z)H(z) \tag{1}$$

$$= \frac{\frac{-2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1+\frac{3}{4}z^{-1})} \implies |z| > \frac{3}{4} \tag{2}$$

$$= \frac{\frac{-8}{13}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{-2}{3}}{1+\frac{3}{4}z^{-1}} \tag{3}$$

ROC:

We get 3 cases ;

poles : $|z| = \frac{1}{3}$; $|z| = \frac{3}{4}$

$$\text{case 1 :} |z| < \frac{1}{3}, |z| > \frac{3}{4} (\text{no ROC}) \tag{4}$$

$$\text{case 2 :} \frac{1}{3} < |z| < \frac{3}{4}; \tag{5}$$

$$\text{case 3 :} |z| > \frac{1}{3}, |z| > \frac{3}{4} \implies |z| > \frac{3}{4} \tag{6}$$

But causality condition(given in question) is satisfied only in case 3 .

$\implies$ ROC : $|z| \geq \frac{3}{4}$

By using Z-transform ;

. $\implies y[n] = \frac{-8}{13}(\frac{1}{2})^n u[n] + \frac{8}{13}(\frac{-3}{4})^n u[n]$