

Assignment 1

KARTHIK RAVULA
AI21BTECH11024

Question 3.8: The system function of a causal linear time-invariant system is

$$H(z) = \frac{1-z^{-1}}{1+\frac{3}{4}z^{-1}}$$

The input to this system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + u[-n-1]$$

Find the output $y[n]$.

SOLUTION:

$$h[n] = \left(\frac{-3}{4}\right)^n u[n] - \left(\frac{-3}{4}\right)^{n-1} u[n-1]$$

$$Y(z) = X(z)H(z) \quad (1)$$

$$= \frac{\frac{-2}{3}z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 + \frac{3}{4}z^{-1}\right)} \implies |z| > \frac{3}{4} \quad (2)$$

$$= \frac{\frac{-8}{13}}{1 - \frac{1}{3}z^{-1}} + \frac{\frac{-2}{3}}{1 + \frac{3}{4}z^{-1}} \quad (3)$$

ROC:

We get 3 cases ;

poles : $|z| = \frac{1}{3}$; $|z| = \frac{3}{4}$

$$\text{case 1 : } |z| < \frac{1}{3}, |z| > \frac{3}{4} \text{ (no ROC)} \quad (4)$$

$$\text{case 2 : } \frac{1}{3} < |z| < \frac{3}{4}; \quad (5)$$

$$\text{case 3 : } |z| > \frac{1}{3}, |z| > \frac{3}{4} \implies |z| > \frac{3}{4} \quad (6)$$

causality : If ROC is the exterior of outermost pole then the system is said to be Causal .

But causality condition(given in question) is satisfied only in case 3 .

$$\implies \text{ROC : } |z| \geq \frac{3}{4}$$

By using Z-transform ;

$$\cdot \implies y[n] = \frac{-8}{13} \left(\frac{1}{2}\right)^n u[n] + \frac{8}{13} \left(\frac{-3}{4}\right)^n u[n]$$