MATHEMATICS

Class - VI

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Foreword

The Government of Andhra Pradesh has unleashed a new era in school education by introducing extensive curricular reforms from the academic year 2020-21. The Government has taken up curricular reforms intending to enhance the learning outcomes of the children with focus on building solid foundational learning and to build up an environment conducive for an effective teaching-learning process. To achieve this objective, special care has been taken in designing the textbooks to achieve global standards.

As a part of the curricular reform, in order to support the designing of textbooks, with better pedagogical strategies, handbooks are given to teachers with elaborate lesson plans. For the practice of the students, workbooks are given which will reinforce the learning in the classroom. Parental handbooks are prepared to impart awareness regarding the teaching-learning process to the parent community. The textbooks are also designed in such a way that the initial two months will focus on the school readiness of the children in order to create a learning environment in the school at the start of the academic year.

The textbook has given importance for foundational learning which includes the basic mathematical concepts like computational skills, algorithms and the necessary glossary to acquire the subject knowledge. The mathematical concepts in the textbook are developed based on the themes like Number System, Arithmetic, Algebra, Geometry, Mensuration and Statistics.

The textbook attempts to enhance the conceptual foundation with elaborate activities required for hands on experience in the form of 'Let's do', 'Let's Think' and 'Let's explore' and 'Projects'. Learning outcomes are provided in each chapter to develop teaching strategies and to assess the student's performance time to time. After completion of every sub-topic of the chapter, an opportunity is provided to the child in the form a small exercise under 'check your progress', and also a cumulative exercise is given under 'unit exercise' at the end of each chapter. 'Fun corner' and 'ICT Corner' were provided to make the children free from fear of mathematics, it also helps them learn the new techniques in the learning process.

In this textbook, concepts are introduced through activities related to daily life incidents, situations, contexts and conversations. To strengthen these concepts, whole class activity, group activity and individual activities are designed. QR codes are incorporated in each chapter for additional information on the concepts. Care has been taken to ensure that the new textbook is calibrated with the learning requirement of the 21st century.

We are grateful to Honourable Chief Minister Sri.Y.S. Jagan Mohan Reddy for being our source of inspiration to carry out this extensive reform in the education department. We extend our gratitude to Dr. Adimulapu Suresh, Honourable Minister of Education for striving towards qualitative education. Our special thanks to Sri. Budithi Rajsekhar, IAS, Principal secretary, School Education, Sri. Vadrevu Chinaveerabhadrudu, IAS, Commissioner, School Education, Ms. Vetriselvi.K, IAS, Special Officer for their constant motivation and guidance.

We convey our thanks to the expert team who studied curriculum from Chicago to Singapore and recommended best practices across the globe to reach global standards. Our sincere thanks to SCERT of Kerala, Tamilnadu, Karnataka and Haryana in designing the textbooks. We also thank our textbook writers, editors, artists and layout designers for their contribution in the development of this textbook. We invite constructive feedback from the teachers and parents in the further refinement of the textbook.

Dr. B. Pratap Reddy
Director
SCERT – Andhra Pradesh

Our National Anthem

- Rabindranath Tagore

Jana-gana-mana-adhinayaka jaya he

Bharata-bhagya-vidhata

Panjaba-Sindhu-Gujarata-Maratha

Dravida-Utkala-Banga

Vindhya-Himachala-Yamuna-Ganga

uchchala-jaladhi-taranga

Tava Subha name jage, tave subha asisa mage,

gahe tava jaya-gatha.

Jana-gana-mangala-dayaka jaya he

Bharata-bhagya-vidhata.

Jaya he, Jaya he, Jaya he,

jaya jaya jaya jaya he.

Pledge

- Pydimarri Venkata Subba Rao

India is my country. All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall give my parents, teachers and all elders respect,
and treat everyone with courtesy. I shall be kind to animals.
To my country and my people, I pledge my devotion.
In their well-being and prosperity alone lies my happiness.

Contents

V9S8U6
Teacher Corner

J2U6V9
Student Corner

CHAPTER 1

Numbers all around us

T4M7J9

Learning Outcomes:-

The Students are able to

- read, write, compare and use large numbers in daily life situations.
- estimate sum, difference, product and quotient of numbers.
- form different numbers with given digits and recognise greatest and smallest numbers among them.
- write expanded form and simplest form in representation of numbers.
- convert numbers from Hindu-Arabic (Indian) system to international numbers and vice versa.
- estimate and verify the results in four fundamental operations.

Content Items

Grandfather of Tabitha deposited some money in a bank and returned home.

Tabitha : Hi Grandpa

Grandfather : Hello Tabitha.

Tabitha : What is that paper in your hand?

Grandfather : I deposited some amount for you. Here is the fixed deposit receipt. Take it and observe.

Tabitha : Deposited amount is Rs.85,000 (Eighty five thousand rupees)

Grandfather : Nice. Read out maturity amount in that paper.

She couldn't read the maturity amount ₹1,75,423

Grandfather : Okay, Try to learn numbers larger than lakhs.

Tabitha : Definitely. I will learn it Grandpa.

1.1 Introduction

Number system is backbone to mathematics. Numbers are used in many different contexts and in many ways. Let us think various situations where we use numbers.

> 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits.
> All numbers are written by using these 10 digits.

Observe the following table and fill with the appropriate numbers in the table and write in words

Situation	Number	Number in words
Number of planets	U	
Number of students in your class	T U	
Length of coast line of Andhra Pradesh (Kms)	H T U	
Ramanujan Number	Th H T U	
Cost of 10 grams gold in your city	Tth Th H T U	
Cost of a new tractor	L Tth Th H T U	
Successor of 999999	TL L Tth Th H T U	
Present population of A.P. (approx)	Cr TL L Tth Th H T U	

We have learnt these numbers in previous classes. Let us revise the number concepts.

Observe

Lakhs	Thousands		Ones			
Lakhs	Ten thousands	Thousands	Hundreds	Tens	Units	
					2	← Place value of 2 is 2
				2	3	← Place value of 2 is 20
			2	3	4	← Place value of 2 is 200
		2	3	4	5	← Place value of 2 is 2,000
	2	3	4	5	6	← Place value of 2 is 20,000
2	3	4	5	6	7	← Place value of 2 is 2,00,000

The value of each digit in a number depends on its place in the number. Proceeding from right to left, the place value increases by 10 times.

HISTORICAL NOTE

Who invented zero ?
INDIANS

The ancient Indian Bakhshali manuscript, containing the world's oldest recorded origin of the zero symbol we use today, represented as dots.

1.2 Comparing and ordering of Numbers

To compare and order of a number

- align the digits by place value.
- compare the digits in each place, starting with the greatest place.

Example-1: Compare and write the following numbers in ascending and descending orders.

29,845 | 29,923 | 38,962 | 1,26,845 | 8,496 | 36,897

Solution:

Ascending Order (Increasing order)

Ascending order means arrangement from the smallest to the greatest.

8,496, 29,845, 29,923, 36,897, 38,962, 1,26,845

Descending Order (Decreasing order)

Descending order means arrangement from the greatest to the smallest.

1,26,845, 38,962, 36,897, 29,923, 29,845, 8,496

Exercise - 1.1

1) Identify the greatest and smallest among the following numbers.

S.No.	Numbers	Greatest	Smallest
1.	67456, 76547, 15476, 75460		
2.	64567, 66000, 78567, 274347		
3.			

Create Your Own Problem on Block No:3 and fill the above table

2) Write the given numbers in ascending and descending order.

S.No.	Numbers	Ascending Order
1.	75645, 77845, 24625, 85690	
2.	6790, 27895, 16176, 50000	

S.No.	Numbers	Descending Order
1.	75645, 77845, 24625, 85690	
2.	6790, 27895, 16176, 50000	

3) Write the numbers in word form.

S.No.	Number	Word Form
1.	73,062	
2.	1,80,565	
3.	25,45,505	
4.		

Create Your Own Problem on Block No:4 and fill the above table

4) Write the numbers in figures.

S.No.	Word Form	Number
1.	Sixty thousand sixty six.	60,066
2.	Seventy eight thousand four hundred and fourteen.	
3.	Nine lakhs nintey six thousand and nintey.	
4.		

Create Your Own Problem on Block No:4 and fill the above table

5) Write 4-digit numbers as many as possible with 6, 0, 5, 7 digits.

6) Form the greatest and smallest numbers with given digits and find the difference (without repetition)

S.No.	Given digits	Greatest Number	Smallest Number	Difference
1.	4, 5, 6, 3	6543	3456	6543 – 3456 = 3087
2.	5, 8, 7, 2			
3.	6, 0, 8, 9, 4			
4.	3, 4, 8, 7, 9			

7) Observe the table and fill the empty boxes.

S.No.	Digit	Smallest	Greatest	Smallest - Greatest	Total Numbers
1.	1 digits	0	9	0 – 9	10
2.	2 digits	10	99	10 – 99	90
3.	3 digits	100	999	100 – 999	900
4.	4 digits	1,000	9999		
5.					90,000
6.			9,99,999		
7.		10,00,000	99,99,999		
8.	8 digits	1,00,00,000	9,99,99,999	1,00,00,000 – 9,99,99,999	9,00,00,000

Observe that at each stage

Total number of same digit numbers = Greatest number – smallest number + 1

Total number of eight digit numbers = 9,99,99,999 – 1,00,00,000 + 1

= 9,00,00,000

The number of 8-digit numbers is 9 crores.

1.3 Method of Numeration

There are two commonly used methods of numeration.

(i) Indian System of Numeration (Hindu-Arabic)

India, Pakistan, Bangladesh, Nepal, Srilanka and some other countries follow this system.

(ii) International System of Numeration

Most of the countries follow this system.

The Place value chart of Numbers in Indian system of Numerations is presented in the given table

Place Value Chart

Crores			Lakhs		Thousands		Units (ones)		
Hundred Crores	Ten Crores 10,00,00,000	Crores 1,00,00,000	Ten Lakhs 10,00,000	Lakhs 1,00,000	Ten Thousands 10,000	Thousands 1,000	Hundreds 100	Tens 10	Units 1
							2	3	5
					4	0	5	7	2
			5	7	4	8	7	6	8
	9	6	0	8	5	4	0	3	9
8	5	7	9	0	0	0	7	5	6

Let us write above numbers in expanded form.

(i) *Number form* : 235

Expanded form : 2 × 100 + 3 × 10 + 1 × 5

Word form : Two hundred and thirty five.

(ii) 40,572 = 4 × 10,000 + 0 × 1,000 + 5 × 100 + 7 × 10 + 2 × 1

Forty thousand five hundred and seventy two.

(iii) 57,48,768 = 5×10,00,000+7×1,00,000+4×10,000+8×1,000+7×100 + 6×10 + 8×1

Fifty seven lakhs forty eight thousand seven hundred and sixty eight.

(iv) 96,08,54,039 =

(v) 857,90,00,756 =

Introducing Ten Crores and Hundred Crores :

The greatest eight digit number is 9,99,99,999. If we add 1 to 9,99,99,999 then we will get 10,00,00,000 which is the smallest 9 digit number, 10 crores.

No. of digits	*Value*	*From - To*
9 digits	Ten Crores	10,00,00,000 - 99,99,99,999
10 digits	Hundred Crores	100,00,00,000 - 999,99,99,999

1 Crore	=	10 Ten Lakhs
	=	100 Lakhs
	=	1000 Ten Thousands
	=	10,000 Thousands
	=	1,00,000 Hundreds
	=	10,00,000 Tens
	=	1,00,00,000 Units

CHECK YOUR PROGRESS Write 10 Crores and 100 Crores as in the above table.

Example-2 : Insert commas to separate periods and write the names of the following numerals.

(i) 356485 (ii) 4075675 (iii) 7056702725

Solution :

(i) By separating periods using commas, 356485 can be written as 3,56,485
This can be read as "Three lakh fifty six thousand four hundred and eighty five".

(ii) By separating periods using commas, 4075675 can be written as 40,75,675
This can be read as "Forty lakhs seventyfive thousand six hundred and seventy five".

Example-3 : Write the following numerals by inserting commas correctly

(i) Four crores four lakhs four thousand four hundred and four.

(ii) Ninty five crores sixty lakhs seventy two thousand four hundred and twenty five.

Solution :

(i) Four crores four lakhs four thousand four hundred and four.
4,04,04,404

(ii) Ninty five crores sixty lakhs seventy two thousand four hundred and twenty five.
95,60,72,425

Example-4 : Find the difference between the place values of two '7's in the number 857065723.

Solution : Given number is 857065723. By inserting commas to separate periods the given number can be written as 85,70,65,723.

Place value of '7' in hundreds place	=	7×100	= 700
Place value of '7' in ten lakhs place	=	$7 \times 10,00,000$	= 70,00,000
Hence, required difference	=	70,00,000 – 700	= 69,99,300

Exercise - 1.2

1. Write each of the following in numeral form.
   (i) Sixty crores seventy five lakhs nintey two thousand five hundred and two.
   (ii) Nine hundred forty four crores six lakhs fifty five thousand four hundred and eighty six.
   (iii) Ten crores ten thousand and ten.
2. Insert commas in the correct positions to separate periods and write the following numbers in words.
   (i) 57657560   (ii) 70560762   (iii) 97256775613
3. Write the following in expanded form.
   (i) 756723   (ii) 60567234   (iii) 8500756762
4. Determine the difference between the place value and the face value of 6 in 86456792

1.4 International System of Numeration

In this system also numbers are written by using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 with each symbol getting a value depending on the place it occupies in the place value chart.

To read numbers in the International system of numeration, we first separate groups (periods) of ones, thousands, millions, billions and trillions by commas. Each period has 3 places.

International system place value chart

Billions			Millions			Thousands			Ones			
100,000,000,000 Hundred Billions	10,000,000,000 Ten Billion	1,000,000,000 Billion	100,000,000 Hundred Million	10,000,000 Ten Million	1,000,000 Million	100,000 Hundred Thousands	10,000 Ten thousands	1,000 Thousands	100 Hundreds	10 Tens	1 Units	
						7	8	1	2	3		
					9	3	4	5	6	7		
			8	9	6	8	0	0	7	0	5	
		9	9	2	4	0	6	7	2	5	6	
2	3	9	1	7	6	5	0	7	8	5	7	
												← Your problem
												← Your problem
												← Your problem

Let us try to read the given numbers in International system.

Example-5: 78123

Solution: *Step-1 :* Put comma for each period (3 places) 78, 123

Step-2 : In *expanded form*

78123 = 7 × 10,000 + 8 × 1000 + 1 × 100 + 2 × 10 + 3

Step-3 : In *word form:* Seventy eight thousand one hundred twenty three.

Example-6: 934567

Solution: *Step-1 :* Put comma for each period (3 places) 934,567

Step-2 : In *expanded form*

934567 = 9 × 100,000 + 3 × 10,000 + 4 × 1000 + 5 × 100 + 6 × 10 + 7

Step-3 : In *word form*

Nine hundred thirty four thousand and five hundred sixty seven.

Example-7: 9924067256

Step-1 : Put comma for each period (3 places) 9,924,067,256

Step-2 : In *expanded form*

9 × 1,000,000,000 + 9 × 100,000,000 + 2 × 10,000,000 + 4 ×1,000,000 + 0 × 100,000 + 6 × 10,000 + 7 × 1000 + 2 × 100 + 5 × 10 + 6

Step-3 : In *word form*

Nine billion, nine hundred twenty four millions sixty seven thousand and two hundred fifty six.

CHECK YOUR PROGRESS

1. Write remaining numbers of the above table in the International System.

2. Fill the boxes in the table with your own numbers and write in words in the international system.

Observe

Indian		***International***
1 lakh	=	100 Thousands
10 lakhs	=	1 million
1 crore	=	10 million
10 crore	=	100 million
100 crores	=	1 billion

Exercise - 1.3

1. Write each of the following numbers in digits by using International place value chart. Also write them in expanded form.
   (i) Nine million seven hundred thousand and six hundred five.
   (ii) Seven hundred million eight hundred seventy two thousand and four hundred seven.

2. Rewrite each of the following numerals with proper commas in the International system of numeration and write the numbers in word form.
(i) 717858 (ii) 3250672 (iii) 75623562 (iv) 956237676

3. Write the following number names in both Indian and International systems.
(i) 6756327 (ii) 45607087 (iii) 8560707236

4. Express the following numbers in other system.

Indian	International
42,56,876	
	800,000,000
956,76,72,345	
	6303448433

5. Write the following numbers in International system (Word Form).
i) Twenty Nine crore thirty five lakh forty six thousand seven hundred and fifty three.
ii) Thousand crore ninty nine lakh and forty three.

6. Write following numbers in Indian system (Word Form).
i) Nine billion twenty four million fifty thousand and seventy two.
ii) Seven hundred billions six millions four thousand seven hundred and five.

1.5 Large numbers used in daily life situations

Let us observe the area of top 7 countries in the world in sq.km (km^2). (Approx) Take teacher's help to understand km^2.

S.No.	Name of the Country	Area in Sq.km International system	Indian System
1.	Russia	17,100,000	1,71,00,000
2.	Canada	9,984,000	99,84,000
3.	United States of America	9,630,000	96,30,000
4.	China	9,600,000	96,00,000
5.	Brazil	8,510,000	85,10,000
6.	Australia	7,690,000	76,90,000
7.	India	3,290,000	32,90,000

Read above numbers loudly and write India's Area in words in both Indian and International system.

Use of large numbers

Capacity:

To measure water flow we use Cusecs and TMC.

Cusecs = Cubic feet per second

= Flow of 28.316 litre per second

One thousand million cubic feet = 1 T.M.C.

= 28316000000 litre

= 28.316 billion litre

= 2831.6 crores litre

To read big numbers for convenience we use '.' (point) symbol and make the number simplify to read.

For example 28.316 is read as twenty eight point three one six

1 millilitre = 1 cubic centimetre

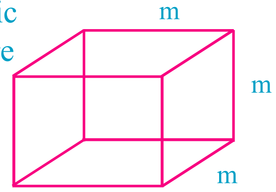
1 litre = 1000 ml

1 m^3 = 1000 litres

1 mega litre = 1000 m^3 = 1000 × 1000 litres

= 1,000,000 litres

$1m^3$= Cubic metre



Cubic kilometre (km^3) = 1,000,000,000 m^3 = 1 billion litres

= 1,000,000,000,000 litres = 1 trillion litres

Weight:

To measure the weights of products like food grains and others we use the following

1 gram = 1 g

1 kilogram = 1000 g

1 quintal = 100 kg = 100000 grams

1 tonne (metric ton) = 1000 kg = 1000000 grams

1 mega tonne (mt) = 1000000000 kg = 1000000000000 grams

1 giga tonne (gt) = 1000000000000 kg = 1000000000000000 grams

Length:

To measure distances we use *mm, cm, m, km* etc.

10 millimetres = 1 centimeter

100 centimeters= 1 meter

1000 meters = 1 kilometer

Collect the population of India from Census 1991-2001, 2011 and write in words using Indian and International system.

Do you know

Pacific ocean contains 700,000,000 km^3 water

Do you know

Earth weight is 5,972 sextillion (1000 trilion metric tons)

Exercise - 1.4

1. Write some daily life situation where we can use large numbers.
2. A box of medicine contains 3,00,000 tablets each weighing 15mg. What is the total weight of all the tablets in the box in grams and in kilograms ?
3. Damodhar wants to buy onions in Kurnool market. Each onion bag weighs 45 kg. He loaded 326 onion bags with 45kg in a lorry. Find the total weight of onions in kilograms and quintals ?
4. The population of 4 South Indian States according to 2011 Census : Andhra Pradesh 8,46,65,533: Karnataka : 6,11,30,704, Tamil Nadu : 7,21,38,958 and Kerala:3,33,87,677. What is the total population of South Indian States?
5. A famous cricket player has so far scored 28,754 runs in International matches. He wishes to complete 50,000 runs in his career. How many more runs does he need ?
6. In an election, the successful candidate registered 1,32,356 votes and his nearest rival secured 42,246 votes. Find the majority of successful candidate.
7. Write the greatest and smallest six digit numbers formed by all the digits 6, 4, 0, 8, 7, 9 and find the sum and difference of those numbers.
8. Haritha has ₹ 1,00,000 with her. She placed an order for purchasing 124 ceiling fans at ₹ 726 each. How much money will remain with her after the purchase ?

1.6 Estimation and Rounding off Numbers

The population of Vizianagaram district in Census - 2011 is 23,44,474. To express the population approximately, a rounding of number 23,44,000.

Rules to round off a number to a given place :

- Find the *place* you are rounding to
- Look at the digit to its right

In order to estimate or round off a number to the nearest

(a) Tens :

(i) If the digit at ones place is less than 5, replace the one's digit by zero. Other digits remain same.

(ii) If the digit at ones place is greater than or equal to 5 increase tens digit by '1' and replace units digit by 0. Other digits remain same.

(b) Hundreds :

(i) Observe the digit at tens place. If it is less than 5, then hundreds place remains same. Put zeroes in tens and units places.

(ii) If the digit at ten's place is greater than 5, then add '1' to hundred's place. Put zeroes in tens and hundreds places.

If the digit is less than 5 round Down

If the digit is 5 or greater round Up

Example-8:

Round off 536724 to nearest ten, hundreds and thousands.

Solution :

i) Nearest 10 :

Given number is 5,36,724

The digit to the right to tens place is 4, 4 < 5 Rounded Down 5,36,720.

ii) Rounding to nearest 100 :

Given number is 5,36,724

The digit to the right to hundreds place is 2, 2 < 5 Rounded Down 5,36,700.

iii) Rounding to nearest 1000 :

Given number is 5,36,724

The digit to the right to thousands place 7, 7 > 5 Rounded Up 5,37,000.

CHECK YOUR PROGRESS

Round off each to the nearest ten, hundred and thousands.

(1) 56,789 (2) 86,289 (3) 4,56,726 (4) 5,62,724

LET'S EXPLORE

Discuss with your friends about rounding off numbers. Consider the population of A.P., Telangana and India in 2011. Round off the numbers to the nearest lakhs.

Four Fundamental Operations

Let us estimate the results in addition, subtraction, multiplication and division of numbers

Addition

Add : 8162 and 5789

First estimate by rounding = 8000 + 6000 = 14000.

Now add

$$\begin{array}{r} {}^{1}\ {}^{1}\quad \\ 8\ 1\ 6\ 2 \\ 5\ 7\ 8\ 9 \\ \hline 1\ 3,\ 9\ 5\ 1 \\ \hline \end{array}$$

The sum is 13,951

Think

13,951 is close to the estimate of 14,000.

- Estimate the sum by rounding and verify the result

1. 8756 + 723
2. 56723 + 4567 + 72 + 5
3. 656724 + 8567
4. 60756 + 2562 + 72

Subtraction

Example : 5723 – 2867 = ____??____

First estimate by rounding 6000 – 3000 = 3000

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{16}{\cancel{7}}\,\overset{11}{\cancel{2}}\,\overset{13}{\cancel{3}} \\ -\;2\;8\;6\;7 \\ \hline 2\;8\;5\;6 \\ \hline \end{array}$$

Think

2,856 is close to the estimate of 3,000.

The difference is 2,856.

- Estimate the difference by rounding and verify the result

1. 7023 – 856
2. 9563 – 2847
3. 52007 – 6756
4. 95625 – 4235

Multiplication

Multiply 58 × 67

First estimate by rounding 60 × 70 = 4200, Rounding the result to hundreds = 4000

Then multiply

$$\begin{array}{r} 5\;8 \\ \times\;\;6\;7 \\ \hline 4\;0\;6 \\ 3\;4\;8\;0 \\ \hline 3\;8\;8\;6 \\ \hline \end{array}$$

Think

3886 is close to the estimate of 4000.

- Estimate the product by rounding and verify the result

1. 63 × 85
2. 636 × 78
3. 506 × 85
4. 709 × 98

Division

Divide 976 ÷ 18

First estimate by rounding 1000 ÷ 20 = 50

Steps :

$$\begin{array}{r|l} & \;\;5\;4 \\ \hline 18 & 9\;7\;6 \\ - & 9\;0\downarrow \\ \hline & \;\;7\;6 \\ - & \;\;7\;2 \\ \hline & \;\;\;\;4\;\;R \end{array}$$

Think

54 is close to the estimate of 50

- Estimate the quotient by rounding and verify the result.

1. 936 ÷ 7
2. 956 ÷ 17
3. 859 ÷ 23
4. 708 ÷ 32

E7K8F4

Unit Exercise

1. Write each of the following in numeral form.
   (i) Hundred crores hundred thousands and hundred.
   (ii) Twenty billion four hundred ninty seven million ninty six thousands four hundred seventy two.
2. Write each of the following in words in both Hindu-Arabic and International system.
   (i) 8275678960 (ii) 5724500327 (iii) 1234567890
3. Find the difference between the place values of the two eight's in 98978056.
4. How many 6 digit numbers are there in all ?
5. How many thousands make one million ?
6. Collect '5' mobile numbers and arrange them in ascending and descending order ?
7. Pravali has one sister and one brother. Pravali's father earned one million rupees and wanted to distribute the amount equally. Estimate approximate amount each will get in lakhs and verify with actual division.
8. Government wants to distribute ₹13,500 to each farmer. There are 2,27,856 farmers in a district. Calculate the total amount needed for that district? (First estimate, then calculate)
9. Explain terms Cusec, T.M.C, Metric tonne, Kilometer.

Points to Remember

1. 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called digits and a group of digits denoting a number is called numeral or number.
2. The method of representing a number in digits or figures is called Notation. The method of expressing a number in words is called numeration. There are two methods of numeration.
   (i) Indian system (Hindu-Arabic) (ii) International system.
3. Place value of a digit in a number = Its face value × Position value.
4. Small and large numbers are used to in measuring, capacity, weight and distances.
5. Try to estimate sum, difference, product or quotient before you go to actual calculation.

Srinivasa Ramanujan (22.12.1887 - 26.04.1920)

An Indian genius in number theory.
First Indian elected to the Fellow of Royal Society (England).
1729 is the Ramanujan's Number.
National Mathematics Day is celebrated every year on the eve of his birthday (22^{nd} December)

CHAPTER 2

Whole Numbers

U3V2E9

Learning Outcomes:-

The students are able to

- solve problems related to addition, subtraction and multiplication on number line.
- verify properties of whole numbers.
- represent whole numbers on number line.
- understand the importance of patterns in mathematics and try to create new patterns.

Content Items

2.0 Introduction

2.1 Whole numbers

2.2 Representation of whole numbers on number line

2.3 Properties of whole numbers

2.4 Patterns on whole numbers

2.0 Introduction

We use numbers 1, 2, 3, ...in counting. These numbers are called natural numbers. We express the set of natural numbers in the form

N = {1, 2, 3, 4, ...}

The next number of any natural number is called its *successor* and the number just before a number is called the *predecessor.*

For example,

the successor of 9 is 10 and

the predecessor of 9 is 8.

Fill the following table with the successor and predecessor of the numbers provided.

S.No.	Natural number	Predecessor	Successor
1.	135		
2.	237		
3.	999		

Discuss

1. Which number has no successor?
2. Which number has no predecessor?

2.1 Whole Numbers

You know that '1' has no predecessor in natural numbers. The natural numbers along with the zero form the collection of Whole numbers.

Set of Whole Numbers W = {0, 1, 2, 3,...}

CHECK YOUR PROGRESS

Which is the smallest whole number?

Let's Think

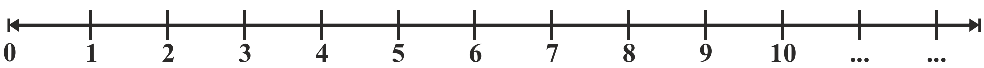
1. Are all natural numbers whole numbers?

2. Are all whole numbers natural numbers?

2.2 Representation of Whole Numbers on Number Line

Draw a line. Mark a point on it. Label it as '0'. Mark points at equal distance to the right of 0. Label the points as 1, 2, 3, 4, ... respectively. The distance between any two consecutive points is unit distance.

The number line for whole numbers is:



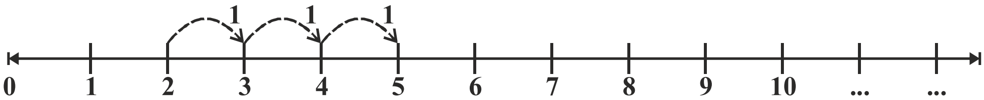
Observe that the successor of any number lies on the right of that number. For example, the successor of 3 is 4. 4 is greater than 3 and lies on the right side of number 3.

Now can we say that all the numbers that lie on the right of a number are greater than that number? Yes. Consider the numbers 12 and 8.

Numbers	Position on number line	Relation between numbers
12, 8	12 lies on the right of 8	12 > 8

Addition on Number Line

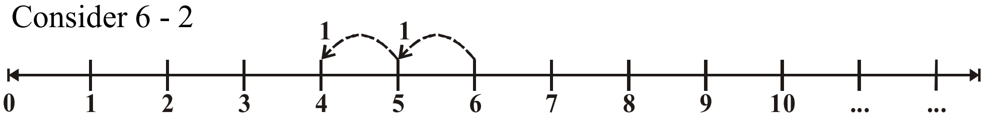
Addition of whole numbers can be represented on number line. Addition of 2 and 3 on number line is shown below.



Start from 2. We make 3 jumps to the right of 2 on the number line. We reach 5.
So, 2 + 3 = 5. Whenever we add two numbers, we move on the number line towards right starting from the first number.

Subtraction on Number Line

Consider 6 - 2



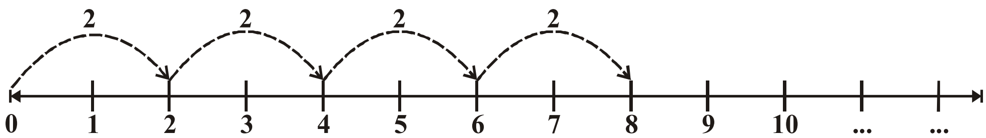
Start from 6. We take 2 steps to the left from 6 on the number line. We reach 4. So, 6 - 2 = 4. Thus, moving towards left means subtraction.

Show these on number line:

i) 5 + 3 ii) 5 - 3 iii) 3 + 5 iv) 10 + 1 v) 8-5

Multiplication on Number Line

Consider the multiplication of whole numbers on the number line. For example, 4x2. It is 2+2+2+2, which means 4 times 2. So, we take a jump of 2 steps at a time for 4 times towards right.



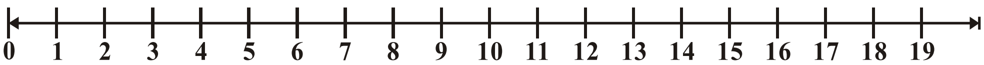
Start from 0, move 2 units to the right each time, making 4 such moves, we reach 8.

So, $4 \times 2 = 8$

- Find the following by using number line:

 1. Which number should be deducted from 8 to get 5?
 2. Which number should be deducted from 6 to get 1?
 3 Which number should be added to 6 to get 8?
 4. How many 6s are needed to get 30?

- Raju and Gayatri together made a number line and played a game on it.



Raju asked "Gayatri, where will you reach if you jump thrice, taking leaps of 3, 8 and 5"? Gayatri said 'the first leap will take me to 3 and then from there I will reach 11 in the second step and another five steps from there to 16'.

Draw Gayatri's steps and verify her answers.

Play this game using addition and subtraction on this number line with your friend.

Exercise - 2.1

1. How many whole numbers are there in between 27 and 46?
2. Find the following using number line.

   i) $6 + 7 + 7$ ii) $18 - 9$ iii) 5×3
3. In each pair, state which whole number on the number line is on the right of the other number.

   i) 895, 239 ii) 1001, 10001 iii) 15678, 4013
4. Mark the smallest whole number on the number line.

2.3 Properties of Whole Numbers

Take any two whole numbers and add them.

Is the result a whole number?

$2 + 3 = 5$, a whole number

$0 + 7 = 7$, a whole number

$20 + 51 = 71$, a whole number

$0 + 0 = 0$, a whole number

Think of some more examples and check.

Observe that the ***sum of any two whole numbers is always a whole number.***

Can you find any pair of whole numbers, which when added will not give a whole number? We see that no such pair exists and the collection of whole numbers is closed under addition. This property is known as the ***closure property of addition for whole numbers.***

Let us check whether the collection of whole numbers is also closed under multiplication.

$5 \times 6 = 30$, a whole number

$11 \times 0 = 0$, a whole number

$16 \times 5 = 80$, a whole number

The product of any two whole numbers is found to be a whole number too. Hence, we say that the collection of ***whole numbers is closed under multiplication.***

Whole numbers are closed under addition and multiplication.

Let's Think

1. Are the whole numbers closed under subtraction?

$7 - 5 = 2$, a whole number

$5 - 7 = ?$, not a whole number

Take as many examples as possible and check.

2. Are the whole numbers closed under division?

$6 \div 3 = 2$, a whole number

$5 \div 2 = \frac{5}{2}$, not a whole number

Confirm by taking a few more examples.

Division by Zero

Let us find $6 \div 2$

6 divided by 2 means, we subtract 2 from 6 repeatedly i.e. we subtract 2 from 6 again and again till we get zero.

$6 - 2 = 4$ once

$4 - 2 = 2$ twice

$2 - 2 = 0$ thrice

So, $6 \div 2 = 3$

Let us consider $3 \div 0$,

Here, we have to subtract zero again and again from 3.

$3 - 0 = 3$ once

$3 - 0 = 3$ twice

$3 - 0 = 3$ thrice and so on.....

Will this ever stop? No. So, $3 \div 0$ is not a number that we can reach.

So, division of a whole number by 0 does not give a known number as answer.

Division by zero is not defined.

CHECK YOUR PROGRESS

1. Find out $12 \div 3$ and $42 \div 7$.
2. What would $6 \div 0$ and $9 \div 0$ be equal to?

Commutativity of Whole numbers

Observe the following additions.

$2 + 3 = 5 \qquad 3 + 2 = 5$

In both cases, we get the same answer 5.

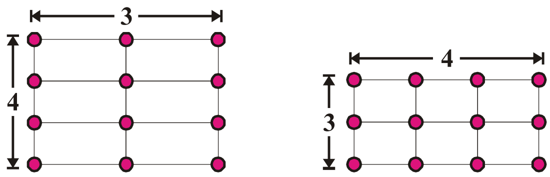
$7 + 8 = 15 \qquad 8 + 7 = 15$

Observe that 7 + 8 and 8 + 7 are equal.

Here, the sum is same, though the order of addition of a pair of whole numbers is altered. Check it for few more examples.

Thus, it is clear that we can add two whole numbers in any order. We say that ***addition is commutative over whole numbers.***

Observe the following figure:



$4 \times 3 = 12 \qquad 3 \times 4 = 12$

We observe that, the product is same, though the order of multiplication of two whole numbers is changed. Check it for few more whole numbers.

Multiplication is commutative over whole numbers.

Addition and multiplication are commutative over whole numbers

LET'S EXPLORE

Take few examples and check whether

- subtraction is commutative over whole numbers or not?
- division is commutative over whole numbers or not?

Associativity of addition and multiplication

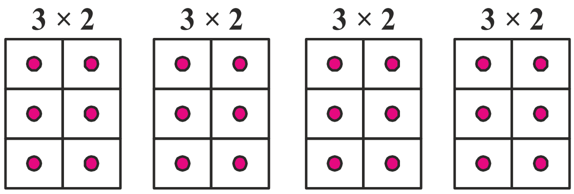
Observe the following:

i. $(3 + 4) + 5 = 7 + 5 = 12$ ii. $3 + (4 + 5) = 3 + 9 = 12$ So, $(3 + 4) + 5 = 3 + (4 + 5)$.

In (i) we add 3 and 4 first and then add 5 to the sum and in (ii) we add 4 and 5 first, and then add the sum to 3. The result is the same. This is called ***associative property of addition for whole numbers.***

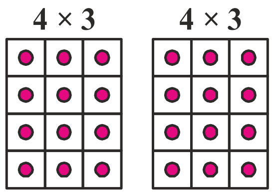
Create 10 more examples and check them for it. Could you find any example where the sums are not equal?

Observe the following:



$4 \times (3\times2)$ = four times (3×2)

Fig. (a)



$2 \times (4\times3)$ = twice of (4×3)

Fig. (b)

In fig (a), each 3×2 block has 6 dots. There are four blocks. So, the total number of dots in blocks is $4 \times (3\times2) = 24$. In fig. (b) each block has 4×3 dots. There are two blocks. So, the total number of dots in blocks is $2 \times (4 \times 3) = 24$. Thus, $4 \times (3\times2) = 2 \times (4 \times 3)$.

This is ***associative property for multiplication of whole numbers.***

Addition and multiplication are associative over whole numbers

CHECK YOUR PROGRESS

Verify the following.

i. $(5 \times 6) \times 2 = 5 \times (6 \times 2)$ ii. $(3 \times 7) \times 5 = 3 \times (7 \times 5)$

Example-1. Find $196 + 57 + 4$.

***Solution*:** $196 + (57 + 4)$

$= 196 + (4 + 57)$ [Commutative property]

$= (196 + 4) + 57$ [Associative property]

$= 200 + 57 = 257$

Here we used a combination of commutative and associative properties for addition.

Do you think using the commutative and associative properties make the calculations easier?

Example-2. Find $5 \times 9 \times 2 \times 2 \times 3 \times 5$

***Solution*:** $5 \times 9 \times 2 \times 2 \times 3 \times 5$

$= 5 \times 2 \times 9 \times 2 \times 5 \times 3$ [Commutative property]

$= (5 \times 2) \times 9 \times (2 \times 5) \times 3$ [Associative property]

$= 10 \times 9 \times 10 \times 3$

$= 90 \times 30 = 2700$

Here, we used a combination of commutative and associative properties for multiplication.

Do you think using the commutative and associative properties make the calculations easier?

CHECK YOUR PROGRESS

Use the commutative and associative properties to simplify the following:

i. $319 + 69 + 81$ ii. $431 + 37 + 69 + 63$

iii. $2 \times (71 \times 5)$ iv. $50 \times 17 \times 2$

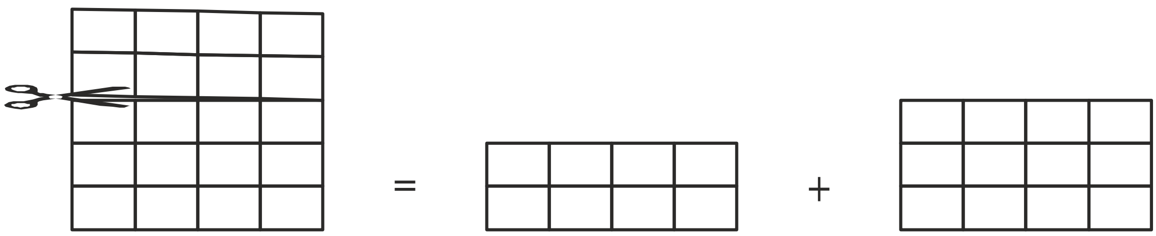
Let's Think

Is $(8 \div 2) \div 4 = 8 \div (2 \div 4)$?

Is there any associative property for division?

Check if this property holds for subtraction of whole numbers too.

Observe the following



5 × 4 = **2 × 4** + **3 × 4**

The grid paper 5×4 has been divided into two pieces 2×4 and 3×4.

Thus, $5 \times 4 = (2 \times 4) + (3 \times 4)$

$= 8 + 12 = 20$

Also since $5 = 2 + 3$, we have $5 \times 4 = (2 + 3) \times 4$

Thus, we can say $(2 + 3) \times 4 = (2 \times 4) + (3 \times 4)$

In the same way, $(5 + 6) \times 7 = 11 \times 7 = 77$ and $(5 \times 7) + (6 \times 7) = 35 + 42 = 77$. We see that both are equal. This is known as ***distributive property of multiplication over addition.***

• Find using distributive property : i) $2 \times (5 + 6)$ ii) $5 \times (7 + 8)$ iii) $19 \times 7 + 19 \times 3$

Example-3. Find 12×75 using distributive property.

Solution: 12×75

$= 12 \times (70 + 5)$ or $= 12 \times (80 - 5)$

$= (12 \times 70) + (12 \times 5)$ or $= (12 \times 80) - (12 \times 5)$

$= 840 + 60 = 900$ $= 960 - 60 = 900$

• Find: i) 25×78 ii) 17×26 iii) $49 \times 68 + 32 \times 49$ using distributive property.

Identity (for addition and multiplication)

When you add 7 and 5, you get another whole number 12. Addition of two whole numbers gives another whole number. Is this always so, for all whole numbers? Observe the table.

When we add zero to a whole number, we get the same whole number again.

2	+	0	=	2
9	+	0	=	9
0	+	11	=	11
0	+	25	=	25

We call ***0 as the additive identity in whole numbers.***

Observe:

$1 \times 9 = 9$
$6 \times 5 = 30$
$6 \times 4 = 24$
$5 \times 1 = 5$
$11 \times 1 = 11$

In multiplication of 2 whole numbers, if one number is 1, then the product is equal to the other number.

So, ***the multiplicative identity for whole numbers is 1.***

Exercise - 2.2

1. Find the sum by suitable rearrangement.

   i) $238 + 695 + 162$ ii) $154 + 197 + 46 + 203$

2. Find the product by suitable rearrangement.

   i) $25 \times 1963 \times 4$ ii) $20 \times 255 \times 50 \times 6$

3. Find the product using suitable properties.

   i) 205×1989 ii) 1991×1005

4. A milk vendor supplies 56 litres of milk in the morning and 44 litres of milk in the evening to a hostel. If the milk costs ₹50 per litre, how much money he gets per day?

2.4 Patterns in Whole Numbers

Whole numbers can be shown in elementary shapes made up of dots. Observe the following.

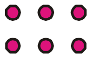
- Number 1 can be represented as a ●
- Numbers can be arranged as a line.

  The number 2 is shown as ● ●

  The number 3 is shown as ● ● ● and so on.

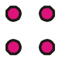
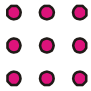
- Some numbers can also be shown as rectangle.

  For example,

  The number 6 can be shown as 

  In this rectangle, observe that there are 2 rows and 3 columns.

- Some numbers like 4 or 9 can also be arranged as squares.

  4  9 

  What are the other numbers that form squares like this? We can see a pattern here.

  $4 = 2 \times 2$. This is a perfect square.

  $9 = 3 \times 3$. This is also a perfect square.

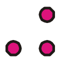
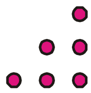
  What will be the next number which can be arranged like a square?

  Easily, we say that $4 \times 4 = 16$ and 16 is the next number which is also a perfect square.

  Find the next 3 numbers that can be arranged as squares.

  Give 5 numbers that can be arranged as rectangles that are not squares.

- Some numbers can also be arranged as triangles.

  3  6 

Note that the arrangement as a triangle would have its two sides equal. The number of dots from the bottom row can be like 4, 3, 2, 1. The top row always contains only one dot, so as to make one vertex.

What are the next possible triangles?

Do you observe any pattern here? Observe the number of dots in each row and think about it. Now, complete the following table.

Number	Line	Rectangle	Square	Triangle
2	Yes	No	No	No
3	Yes	No	No	Yes
4	Yes	No	Yes	No
5				
.....				
25				

LET'S EXPLORE

1. Which numbers can be shown as a line only?
2. Which numbers can be shown as rectangles?
3. Which numbers can be shown as squares?
4. Which numbers can be shown as triangles?

Patterns of numbers

We can use patterns in simplifying processes. Study the following.

1. $296 + 9 = 296 + 10 - 1 = 306 - 1 = 305$
2. $296 - 9 = 296 - 10 + 1 = 286 + 1 = 287$
3. $296 + 99 = 296 + 100 - 1 = 396 - 1 = 395$
4. $296 - 99 = 296 - 100 + 1 = 196 + 1 = 197$

Let us see one more pattern.

1. $65 \times 99 = 65\ (100 - 1) = 6500 - 65 = 6435$
2. $65 \times 999 = 65\ (1000 - 1) = 65000 - 65 = 64935$
3. $65 \times 9999 = 65\ (10000 - 1) = 650000 - 65 = 649935$
4. $65 \times 99999 = 65\ (100000 - 1) = 6500000 - 65 = 6499935$ and so on.

Here, we can see a shortcut to multiply a number by numbers of the form 9, 99, 999, ... This type of shortcuts enable us to do sums mentally.

Observe the following pattern: It suggests a way of multiplying a number by 5, 15, 25, ...

(You can think of extending it further).

a. $46 \times 5 = 46 \times \frac{10}{2} = \frac{460}{2} = 230 = 230 \times 1$

b. $46 \times 15 = 46 \times (10 + 5)$

$= 46 \times 10 + 46 \times 5 = 460 + 230 = 690 = 230 \times 3$

c. $46 \times 25 = 46 \times (20 + 5)$

$= 46 \times 20 + 46 \times 5 = 920 + 230 = 1150 = 230 \times 5$ .....

Think some more examples.

Exercise - 2.3

1. Study the pattern.

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$

Write the next four steps. Can you find out how the pattern works?

2. How would we multiply the numbers 13680347, 35702369 and 25692359 with 9 mentally? What is the pattern that emerges?

Unit Exercise

1. Choose the appropriate symbol from $<$ or $>$ and place it in the blanks.

i.	8 7	ii.	5 2
iii.	0 1	iv.	10 5

2. Present the successor of 11 and predecessor of 5 on the number line.

3. Which of the statements are true (T) and which are false (F). Correct the false statements.

   i. There is a natural number that has no predecessor.

   ii. Zero is the smallest whole number.

   iii. A whole number on the left of another number on the number line, is greater than that number.

4. Give the results without actually performing the operations, using the given information.

i)	28	×	19	=	532	then	19	×	28	=
ii)	a	×	b	=	c	then	b	×	a	=
iii)	85	+	0	=	85	then	0	+	85	=

5. Find the value of the following:

   i. $368 \times 12 + 18 \times 368$ ii. $79 \times 4319 + 4319 \times 11$

6. Chandana and Venu purchased 12 note books and10 note books respectively. The cost of each note book is ₹15. Then how much amount should they pay to the shop keeper?

7. Match the following.

i. $3+1991+7 = 3+7+1991$	[ ]	A. Additive identity
ii. $2\times68\times50 = 2\times50\times68$	[ ]	B. Multiplicative identity
iii. 1	[ ]	C. Commutative under addition
iv. 0	[ ]	D. Distributive property of multiplication over addition
v. $879\times(100+30) = 879\times100+879\times30$	[ ]	E. Commutative under multiplication

8. Study the pattern:

$$91 \times 11 \times 1 = 1001$$

$$91 \times 11 \times 2 = 2002$$

$$91 \times 11 \times 3 = 3003$$

Write next seven steps. Check, whether the result is correct.

Y7D5Q9

Points to Remember

1. The numbers N = {1, 2, 3, ...} which we use for counting are known as natural numbers.
2. Every natural number has a successor. Every natural number except 1 has a predecessor.
3. Collection of whole numbers W = {0, 1, 2, ...}
4. Every whole number has a successor. Every whole number except zero has a predecessor.
5. All natural numbers are whole numbers, and all whole numbers except zero are natural numbers.
6. Whole numbers can be represented on a number line.Operations of addition, subtraction and multiplication can easily be performed on a number line.
7. Addition corresponds to moving to the right on the number line, where as subtraction corresponds to moving to the left. Multiplication corresponds to making jumps of equal distance from zero.
8. Whole numbers are closed under addition and multiplication. But, whole numbers are not closed under subtraction and division.
9. Division by zero is not defined.
10. Zero is the additive identity and 1 is the multiplicative identity of whole numbers.
11. Addition and multiplication are commutative over whole numbers.
12. Addition and multiplication are associative over whole numbers.
13. Multiplication is distributive over addition for whole numbers.
14. Commutativity, associativity and distributivity of whole numbers are useful in simplifying calculations and we often use them without being aware of them.
15. Pattern with numbers are not only interesting, also especially for mental calculations. They help us to understand properties of numbers better.

CHAPTER 3

H.C.F and L.C.M

Highest Common Factor — Least Common Multiple

F6T3A4

Learning Outcomes:-

The students are able to

- state and apply divisibility rules whereever necessary.
- recognize and appreciate the broad classification of numbers as prime, composite, co-primes, perfect numbers etc.,
- understand factors and multiples of a number.
- find out the HCF and LCM of any two or more numbers using different methods.
- apply H.C.F or L.C.M in daily life situations.

Content Items

3.0 Introduction

To find whether a given number is divisible by another number, we perform division and check whether the remainder is zero or not.

In some cases, it is very tedious. For example is 489347561 divisible by 3? We can answer this type of questions by using divisibility rules, without performing actual division.

3.1 Divisibility Rules

The process of checking whether a number is divisible by a given number or not without actual division is called divisibility rule for that number. Let us start.

Number Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3.1.1. Divisibility by 2

Look at the number chart.

Cross all the multiples of 2. Do you see any pattern in the ones place of these numbers?

These numbers have the digits 0. 2, 4, 6 & 8 only in the ones place. We observe that ***a number is divisible by 2 if it has any of the digits 0, 2, 4, 6, or 8 in its ones place.***

CHECK YOUR PROGRESS

1) Are the numbers 900, 452, 9534, 788 divisible by 2? Why?
2) Are the numbers 953, 457, 781, 325, 269 divisible by 2? Why?
3) Are the numbers 452, 673, 259, 356 divisible by 2? Verify.

3.1.2. Divisibility by 3

Encircle all the multiples of 3 in the chart. Observe the encircled numbers.

Consider the numbers of 24, 39, 57, 72 & 87

Ex: 2 + 4 = 6   3 + 9 = ________   5 + 7 = ________

7 + 2 = ________   8 + 7 = ________

All these sums are divisible by 3.

Thus, we can say that ***if the sum of the digits of a number is a multiple of 3, then that number is divisible by 3.*** Check this rule for other circled numbers.

Note: ***The digital root*** (repeated ***digital sum) of a natural number is the single digit value obtained by repeated process of summing digits.***

Ex: Digital root of 129 is 1+2+9 = 12. Now 1+2 = 3. Thus, Digital root of 129 is 3.

CHECK YOUR PROGRESS

Check whether the following numbers are divisible by 3 (using rule)? Verify by actual division.

(i)12345   (ii) 61392   (iii) 8747

3.1.3. Divisibility by 6

Put a △ on the numbers which are multiples of 6 in the number chart.

Do you notice anything special about them? Yes. They are divisible by both 2 and 3.

If a number is divisible by both 2 and 3, then it is also divisible by 6.

LET'S EXPLORE

1) Is 8430 divisible by 6? Why?
2) Take any three 4 digit numbers and check whether they are divisible by 6.
3) Can you give an example of a number which is divisible by 6 but not by 2 and 3? Why?

3.1.4. Divisibility by 9

Put a □ (box) on the numbers which are multiples of 9 in the number chart.

Now try to find a pattern or rule for checking the divisibility of 9. (Hint: sum of digits).

Sum of digits in these numbers is also divisible by 9.

For example : If we take 279 ; 8739

Sum of digits of 279 = 2 + 7 + 9 = 18 is divisible by 9.

Sum of digits of 8739 = 8 + 7 + 3 + 9 = 27 is divisible by 9.

A number is divisible by 9, if the sum of the digits of the number is divisible by 9.

CHECK YOUR PROGRESS

1) Test whether 6669 is divisible by 9.
2) Without actual division, find whether 8989794 is divisible by 9.

3.1.5. Divisibility by 5

Are all the numbers 10, 15, 20, 25, 30 divisible by 5? Yes, divisible by5.

Is 93 divisible by 5? No. Why?

The numbers with zero or five at ones place are divisible by 5.

CHECK YOUR PROGRESS

Are the numbers 28570, 90875 divisible by 5? Verify by actual division also.

3.1.6. Divisibility by 10

Mark all the numbers divisible by 10 in the number chart.

What do you observe?

1) All of them have ‘0’ at their ones place.
2) All of them are divisible by both 5 and 2 which are the factors of 10.

3.1.7. Divisibility by 4

Can you quickly give five 3-digit numbers divisible by 4?

One such number is 312. Think of such 4-digit numbers. One example is 1316.

Observe the number formed by ones and tens places of 312. It is 12; which is divisible by 4. For 1316 it is 16, which is divisible by 4.

Try this exercise with other such numbers, for example with 5620; 4524; 7628; 3532.

Is 386 divisible by 4? No.

Is 86 divisible by 4? No.

So, we see that ***a number with 3 or more digits is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4, and also zeros on both places.***

CHECK YOUR PROGRESS

Check whether the number 598, 864, 4782 and 8976 are divisible by 4.
Use divisibility rule and verify by actual division.

3.1.8. Divisibility by 8

Are the numbers 1000, 2104, 1418, 1352 divisible by 8?

You can verify that they are divisible by 8 except 1418 by actual division.

Let us try to see the pattern. Look at the number formed by the digits at ones, tens, hundreds place of these numbers. These are 000, 104, 418, 352 respectively. These too are divisible by 8 but 418 is not divisible by 8.

Find some more numbers in which the number formed by the digits at ones, tens, hundreds place (i.e. last 3 digits) is divisible by 8. For example 9216, 8216, 7216 etc,. You will find that 216 is divisible by 8. Hence these numbers are divisible by 8.

We find that ***a number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8, are also zeros on three places.***

Is 76512 divisible by 8?

The divisibility for numbers with 1, 2 or 3 digits by 8 has to be checked by actual division.

Exercise 3.1

1. Which of the following numbers are divisible by 2, by 3 and by 6?

(i)237192	(ii) 193272	(iii) 972312	(iv) 1790184
(v) 312792	(vi) 800552	(vii) 4335	(viii) 726352

2. Determine which of the following numbers are divisible by 5 and by 10.
   25, 125, 250, 1250, 10205, 70985, 45880.
   Check whether the numbers that are divisible by 10 are divisible by 2 and 5.

3. Make 3 different 3 digit numbers using 2, 3, 4 where each digit can be used only once.
   Check which of these numbers is divisible by 9.

4. Write different 2 digit numbers using digits 5, 6, 7. Check whether these numbers are divisible by 2, 3, 5, 6 and 9?
5. Find the smallest number that must be added to 128, so that it becomes exactly divisible by 5.
6. Find the smallest number that has to be subtracted from 276 so that it becomes exactly divisible by 10.
7. Write all the numbers between 100 and 200 which are divisible by 6.
8. Write the greatest four digit number which is divisible by 9. Is it divisible by 3? What do you notice?
9. Which of the following are divisible by 8?

   (i) 1238 (ii) 13576 (iii) 93624 (iv) 67104
10. Write the nearest number to 12345 which is divisible by 4.

3.1.9. Divisibility by 11

Fill the blanks and complete the table.

Number	Sum of the digits at odd places (from the right)	Sum of the digits at even places (from the right)	Difference	Is the given number divisible by 11
29843	3 + 8 + 2 = 13	4 + 9 = 13	13 – 13 = 0	Yes
90002				
80927				
19091908	8 + 9 + 9 + 9 = 35	0 + 1 + 0 + 1 = 2	35 – 2 = 33	Yes

What do you observe from the table? Observed that in each case the difference is either 0 or divisible by 11. All these numbers are divisible by 11.

A given number is divisible by 11, if the difference between the sum of the digits at odd places and the sum of the digits at even places (from the right) is either '0' or multiple of 11.

Example-1: **Is 6535 divisible by 11?**

Solution :

Sum of the digits at odd places = 5 + 5 = 10

Sum of the digits at even places = 3 + 6 = 9

Their difference = 10 – 9 = 1

Is 1 divisible by 11? No.

So, 6535 is not divisible by 11.

Example-2: **Is 1221 divisible by 11?**

Solution:

Sum of the digits at odd places = 1 + 2 = 3

Sum of the digits at even places = 2 + 1 = 3

Their difference = 3 – 3 = 0

So, 1221 is divisible by 11.

1221 is a polindrome number, which on reversing its digits gives the same number. Thus, every polindrome number with even number of digits is always divisible by 11. Write polindrome number of 6 – digits.

Exercise – 3.2

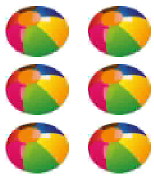
1) Using divisibility rules, determine which of the following numbers are divisible by 11.
   (i) 6446 (ii) 10934 (iii) 7138965 (iv) 726352
2) Write all the possible numbers between 2000 and 2100, that are divisible by 11.
3) Write the nearest number to 1234 which is divisible by 11.

3.2 Factors

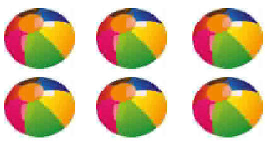
Arun has 6 balls with him. He wants to arrange them in rows in such a way that each row has the same number of balls. He arranges them in the following ways and matches the total number of balls.

(i) 1 ball in each row.
Number of rows = 6
Total number of balls = 6 x 1 = 6

(ii) 2 balls in each row.
Number of rows = 3
Total number of balls = 3 x 2 = 6



(iii) 3 balls in each row.
Number of rows = 2.
Total number of balls = 2 x 3 = 6



(iv) He could not think of any arrangement in which each row had 4 balls or 5 balls. So, the only possible arrangement left was with all the 6 balls in a row.
Number of rows = 1
Total number of balls = 1 x 6 = 6

From these arrangements, Arun observes that 6 can be written as a product of two numbers in the following ways.

6 = 1 x 6 | 6 = 2 x 3 | 6 = 3 x 2 | 6 = 6 x 1

From 6 = 2 x 3, it can be said that 2 and 3 exactly divide 6.

So, 2 and 3 are exact divisors of 6.

From the other product 6 = 1 x 6, the exact divisors of 6 are found to be 1 and 6.

Thus, 1, 2, 3 and 6 are exact divisors of 6. They are called the factors of 6.

Try to arrange 12 balls in rows and find the factors of 12. Observe the following table.

Number	Factors
12	1, 2, 3, 4, 6, 12
18	1, 2, 3, 6, 9, 18
20	1, 2, 4, 5, 10, 20
24	1, 2, 3, 4, 6, 8, 12, 24

From the above table, we notice that

1) 1 is a factor of every number and is the smallest factor of all.
2) Every number is a factor of itself and is the greatest of its factors.
3) Every factor is less than or equal to the given number.
4) Number of factors of a given number is countable.

CHECK YOUR PROGRESS

1) Find the factors of 60.
2) Do all the factors of a given number divide the number exactly? Find the factors of 30 and verify by division.
3) 3 is a factor of 15 and 24. Is 3 a factor of their difference also?

3.3 Perfect Number

The factors of 6 are 1, 2, 3 and 6.

Also 1 + 2 + 3 + 6 = 12 = 2 x 6

= 2 x the number

We find that the sum of the factors of 6 is twice the number 6.

All the factors of 28 are 1, 2, 4, 7, 14 and 28.

Adding these we have, 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 x 28

= 2 x the number

The sum of the factors of 28 is equal to twice the number 28. ***A number for which sum of all its factors is equal to twice the number is called a perfect number.*** The numbers 6 and 28 are perfect numbers. Is 15 a perfect number? No, Why?

Try to find one perfect number other than 6 and 28.

3.4 Prime and Composite Numbers.

Number	1	2*	3*	4	5*	6	7*	8	9	10
Factors	1	1,2	1,3	1,2,4	1,5	1,2,3,6	1,7	1,2,4,8	1,3,9	1,2,5,10
No. of Factors	1	2	2	3	2	4	2	4	3	4

From the table which numbers have only two factors? The numbers are 2, 3, 5, 7

The numbers whose only factors are 1 and the number itself are called prime numbers.

Which numbers have more than two factors?

Numbers having more than two factors like 4, 6, 8, 10 etc ***are called composite numbers.***

Which number has only one factor?

The number 1 has only one factor (i.e., itself). ***1 is neither prime nor composite.***

LET'S EXPLORE

1) What is the smallest prime number?
2) What is the smallest composite number?
3) What is the smallest odd prime number?
4) What is the smallest odd composite number?
5) Write 10 odd and 10 even composite numbers.

Without actually checking the factors of a number, we can find prime numbers from 1 to 100 with an easy method. ***This method was given by the Greek mathematician Eratosthenes in the third century BC***. Let us see the method. List all the numbers from 1 to 100 as shown below:

Number Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

LET'S DO ACTIVITY

Observe the chart and follow the steps.

Step 1: Cross out 1 because it is neither prime nor composite.

Step 2: Encircle 2. Cross out all the other multiples of 2 i.e., 4, 6, 8 and so on.

Step 3: You will find that the next uncrossed number is 3. Encircle 3 and cross out all the other multiples of 3.

Step 4: The next uncrossed number is 5. Encircle 5 and cross out all the other multiples of 5.

Step 5: Continue this process with the uncrossed numbers 7, 11, 13, 17, 19, 23 and so on and their crossed out multiples, till all the numbers in the list are either encircled or crossed out.

Step 6: Write all the encircled numbers like 2, 3, 5, 7

All the encircled numbers are *Prime Numbers.* All the crossed out numbers other than 1 are *Composite Numbers.*

LET'S EXPLORE

1) Can you guess a prime number which when on reversing its digits, gives another prime number? (Hint a 2 digit prime number)
2) 311 is a prime number. Can you find the other two prime numbers just by rearranging the digits?

3.4.1 Co-Prime or Relatively prime numbers

Observe the numbers 2 and 9.

The factors of 2 are 1 and 2. The factors of 9 are 1, 3, 9.

The common factor for both 2 and 9 is 1 only.

Thus, the numbers which have only 1 as the common factor are called co-prime or relatively prime numbers. Observe

- 7 and 11 are prime.

7 and 11 are Co-prime.

- 4 is not a prime and 5 is a prime.

4 and 5 are Co-prime.

Can we say any two primes are co-prime but each co-prime need not be primes?

Twin Primes:

Two prime numbers are said to be twin primes, if they differ each other by 2.

For example (3,5); (5,7); (11, 13); (17, 19); … are all twin primes.

Are all twin primes relatively prime numbers? Discuss.

CHECK YOUR PROGRESS

1) From the following numbers identify different pairs of co-primes. 2, 3, 4, 5, 6, 7, 8, 9 and 10.
2) Write the pairs of twin primes less than 50.

Exercise – 3.3

1) Write all the factors for the following numbers.
   (i) 24 (ii) 56 (iii) 80 (iv) 98
2) What is the greatest prime number between 50 and 100?
3) The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find 2 more such pairs of prime numbers below 100.
4) Express the following numbers as the sum of two odd primes.
   (i) 18 (ii) 24 (iii) 36 (iv) 44
5) Write seven consecutive composite numbers less than 100.
6) Write two prime numbers whose difference is 10.
7) Write three pairs of prime numbers less than 20, whose sum is divisible by 5.

3.5 Prime factorisation

The process to express the given number as the product of prime numbers is called prime-factorisation.

For Example : Prime factorization of 30.

$$30 = 2 \times 15$$
$$= 2 \times 3 \times 5$$

Therefore, $30 = 2 \times 3 \times 5$

Example-3: Express 100 as the product of prime numbers.

Solution :

$$100 = 2 \times 50$$
$$= 2 \times 2 \times 25$$

Therefore, $100 = 2 \times 2 \times 5 \times 5$

3.5.1 Methods of prime factorisation

1) Division Method:

For prime factorisation of 42, using division method, we proceed as follows:

Start dividing by the least prime factor.

Continue division till the resulting number to be divided is 1.

$$\begin{array}{c|c} 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore$ Prime factorisation of 42 is $2 \times 3 \times 7$

2) Factor Tree Method:

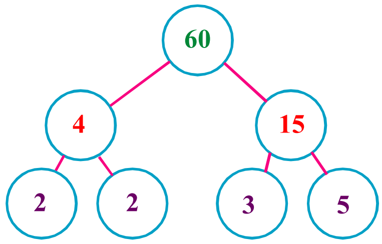
We can find the prime factorisation of 60 by drawing a factor tree, for that, we proceed as follow:

Step-1: Express 60 as product of two numbers i.e., 4 and 15.

Step-2: Factorise 4 and 15 further, since they are composite numbers.

Step-3: Continue the process till all the factors are prime numbers.

$\therefore$ Prime factorisation of 60 is $60 = 2 \times 2 \times 3 \times 5$



Exercise – 3.4

1) Prepare a factor tree for 90.
2) Factorise 84 by division method.
3) Write the greatest 4 digit number and express it in the form of its prime factors.
4) Write the prime factorisation of 96 by factor tree method.
5) I am the smallest number, having four different prime factors. Can you find me?
6) Write the prime factorisation of 28 and 36 through division method. Write the prime factorisation of 42 by factor tree method.

3.6 Common Factors

Observe the following table:

Number	12	18
Factors of the number	1, 2, 3, 4, 6, 12	1, 2, 3, 6, 9, 18

Common factors of 12 and 18 are 1, 2, 3 and 6.

Common factors are those numbers which are factors of all the given numbers.

Now, find common factors of 20 and 24.

3.6.1. Highest Common Factor (H.C.F)

From the above table, we found that common factors of 12 and 18 are 1, 2, 3 and 6.

What is the highest of these common factors?

It is 6. So, we say that the Highest Common Factor (HCF) of 12 and 18 is 6.

The highest common factor (HCF) of two or more given numbers is the highest or greatest of their common factors. It is also called as Greatest Common Divisor (GCD).

3.6.2. Methods of Finding HCF

1) Finding HCF by prime factorisation method

The HCF of 12, 30 and 36 can also be found by prime factorisation, which is as follows:

2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

3	36
3	12
2	4
2	2
	1

Thus $12 = \mathbf{2} \times \mathbf{3} \times 2$

$30 = \mathbf{2} \times \mathbf{3} \times 5$

$36 = \mathbf{2} \times \mathbf{3} \times 2 \times 3$

The common factor of 12, 30 and 36 is $2 \times 3 = 6$.

Hence, HCF of 12, 30 and 36 is 6.

- Find the HCF of 12, 16 and 28.

HISTORICAL NOTE

The prime factorisation method of finding HCF was invented by the famous Greek mathematician Euclid.

2) HCF by continued division method

Divide the larger number by the smaller and then divide the previous divisor by the remainder until the remainder is 0. The last divisor is the HCF of the numbers.

Example-4 : Find the HCF of 32 and 40.

Solution :

32) 40 (1
- 32

8) 32 (4
- 32

0

Last divisor is 8 as remainder became 0.

$\therefore$ HCF of 32 and 40 is 8

Example-5 : Find the HCF of 40, 56 and 60.

Solution :

Step - 1: First let us find the HCF of 40 and 56.

```
40) 56 (1
   -40
    16) 40 (2
       -32
         8) 16 (2
           -16
             0
```

Last divisor is 8 when remainder becomes zero.

$\therefore$ *HCF of 40 and 56 = 8*

Step – 2: Then, find the HCF of the third number and the HCF of first two numbers i.e., let us find HCF of 60 and 8.

```
8) 60 (7
  -56
   4) 8 (2
     -8
      0
```

Last divisor is 4 when remainder becomes zero.

$\therefore$ *HCF of 60 and 8 = 4*

Step – 3: HCF of the given three numbers 40, 56 and 60 is 4.

What is the HCF of any two

(i) Consecutive numbers ?

(ii) Consecutive even numbers ?

(iii) Consecutive odd numbers? What do you observe? Discuss with your friends.

Example-6: Two tankers contain 850 litres and 680 litres of Kerosene oil, respectively. Find the maximum capacity of a container which can measure the Kerosene oil of both the tankers when used an exact number of times.

Solution :

The required container has to measure both the tankers in a way that the count is an exact number of times. So, its capacity must be an exact divisor of the capacities of both the tankers. More over, this capacity should be maximum. Thus, the maximum capacity of such a container will be the HCF of 850 and 680. The HCF of 850 and 680 is 170.

```
680) 850 (1
     680
     ---
     170) 680 (4
          680
          ---
            0
          ---
```

Therefore, maximum capacity of the required container is 170 litres. It will fill the first container in 5 and the second container in 4 refills.

Exercise – 3.5

1) Find the HCF of the following by prime – factorisation and by continued division method.

(i) 48, 64 (ii) 126, 216 (iii) 40, 60, 56 (iv) 10, 35, 40

2) Two milk cans have 60 and 165 litres of milk. Find a can of maximum capacity which can exactly measure the milk in two cans.

3) Three different containers contain different quantities of milk whose measures are 403 lit, 465 lit, 527 litres. What biggest measure must be there to measure all different quantities in exact number of times?

3.7 Common Multiples

Observe the following table.

Number	4	6
Multiples of Number	4, 8, 12, 16, 20, 24,…	6, 12, 18, 24, 30, ….

Common multiples of both 4 and 6 are 12, 24,…

3.7.1. Least Common Multiple (LCM)

Common multiples of both 4 and 6 are 12, 24, 36, ….

Least of them is 12.

That means, 12 is the lowest among the common multiples of both 4 and 6.

$\therefore$ Least common multiple (LCM) of 4 and 6 is 12.

LCM: ***Least common multiple of two or more numbers is the smallest natural number, which is the multiple of each of the numbers.***

Instead of writing all the common multiples of the given numbers every time to identify the least one of them, we can just find the LCM of those numbers directly.

3.7.2. Methods of Finding LCM

1) Prime Factorisation Method:

For example, the LCM of 36 and 60 can be found by prime factorisation method.

Step – 1: Express each number as a product of prime factors.

Factors of 36 = 2 x 2 x 3 x 3

Factors of 60 = 2 x 2 x 3 x 5

Step – 2: Take the common factors of both 2 x 2 x 3

Step – 3: Take the extra factors of both 36 and 60 i.e., 3 and 5.

Step – 4: LCM is found by the product of all common prime factors of two numbers and extra prime factors of both.

Hence, the LCM of 36 and 60 = (2 x 2 x 3) x 3 x 5 = 180

- Find LCM of (i) 3, 4 (ii) 10, 11 (iii) 10,30 (iv) 12, 24 (v) 3, 12

2) Division method:

Find the LCM of 24 and 90.

Step – 1: Arrange the given numbers in a row.

Step – 2: Then, divide by a least prime number which divides at least two of the given numbers and carry forward the numbers which are not divisible by the number if any.

Step – 3: Repeat the process till the two numbers have no common factor other than 1.

Step – 4: LCM is the product of the divisors and the remaining numbers.

2	24, 90
3	12, 45
	4, 15

Thus, the LCM of 24 and 90 is 2 x 3 x 4 x 15 = 360

Example-7: Find the L.C.M of 21, 35 and 42.

Solution :

7	21, 35, 42
3	3, 5, 6
	1, 5, 2

Thus, the L.C.M of 21, 35 and 42 is 7 x 3 x 5 x 2 = 210.

Exercise – 3.6

1) Find the LCM of the following numbers by prime factorisation method.

   (i) 12 and 15 (ii) 15 and 25 (iii) 14 and 21

2) Find the LCM of the following numbers by division method.

   (i) 84, 112, 196 (ii) 102, 119, 153 (iii) 45, 99, 132, 165

3) Find the smallest number which when added to 5 is exactly divisible by 12, 14 and 18.

4) Find the greatest 3-digit number which when divided by 75, 45 and 60 leaves

   (i) No remainder (ii) The remainder 4 in each case.

5) Two bells ring together. If the bells ring at every 3 minutes and 4 minutes respectively, at what interval of time, will they ring together again?

3.8 Relationship between HCF and LCM

Consider the numbers 18 and 27.

Factorisation of $18 = 2 \times 3 \times 3$

$27 = 3 \times 3 \times 3$

LCM of 18 and 27 is $3 \times 3 \times 2 \times 3 = 54$

HCF of 18 and 27 is $3 \times 3 = 9$

LCM x HCF $= 54 \times 9 = 486$

Product of 18 and 27 $= 18 \times 27 = 486$

What do you notice? We see that

Product of LCM and HCF of two numbers = Product of the two numbers

Example-8: Find the LCM of 8 and 12 and find their HCF using their relation.

Solution: LCM of 8 and 12 $= 2 \times 2 \times 2 \times 3$

$= 24$

$$\begin{array}{r|l} 2 & 8,\ 12 \\ \hline 2 & 4,\ 6 \\ \hline & 2,\ 3 \end{array}$$

We know that LCM x HCF = Product of the two numbers

$$\text{HCF} = \frac{\text{Product of the two numbers}}{\text{LCM}}$$

$$\text{HCF} = \frac{8 \times 12}{24} = 4$$ Hence, HCF of 8 and 12 = 4

• What is the LCM and HCF of twin prime numbers?

Exercise – 3.7

1) Find the LCM and HCF of the following numbers and check their relationship.

   (i) 15, 24   (ii) 8, 25   (iii) 12, 48   (iv) 30, 48

2) If the LCM of two numbers is 290 and their product is 7250, what will be its HCF?
3) The product of two numbers is 3276. If their HCF is 6, find their LCM.
4) The HCF of two numbers is 6 and their LCM is 36. If one of the numbers is 12, find the other.
5) Can two numbers have 16 as their HCF and 384 as their LCM? Give reason.
6) Can two numbers have 14 as their HCF and 204 as their LCM?

   Give reasons in support of your answer.

Unit Exercise

1) Classify the given numbers according to their divisibility.

Numbers	Numbers Divisible by				
	10	9	8	6	2
972, 5500, 14560, 45873, 1790184					

2) Write the divisibility rule by 11 with one example.
3) Fill the table with correct answer.

	Any two consecutive numbers	Any two consecutive even numbers	Any two consecutive odd numbers
H.C.F of			

4) Find HCF of 70, 105, and 175 by prime factorisation method.
5) Find HCF of 18, 54, 81 by continued division method.
6) Find LCM of 4, 12, 24 by two methods.
7) What is the capacity of the largest vessel which can empty the oil from three vessels containing 32 litres, 24 litres and 48 litres respectively, an exact number of times?
8) HCF, LCM of two number are 9 and 54 respectively. If one of those two numbers is 18, find the other number.

Points to Remember

1) (i) A factor of a number is an exact divisor of that number.
   (ii) Every number is a factor of itself. 1 is a factor of every number.
   (iii) Every factor of a number is less than or equal to the given number.
   (iv) Every number is a multiple of each of its factors.
   (v) Every multiple of a given number is greater than or equal to that number.
   (vi) Every number is a multiple of itself.
   (vii) If the sum of all factors of a number is two times the number, then that number is called a perfect number.

2) (i) The number other than 1, with only two factors, namely 1 and the number, is a prime number. Numbers that have more than two factors are called composite numbers. Number 1 is neither prime nor composite.
   (ii) The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
   (iii) Two numbers with only 1 as a common factor are called co-prime numbers or relatively prime numbers.

3) (i) Divisibility by 2, 5 and 10 can be seen by just the last digit.
   (ii) Divisibility by 3 and 9 is checked by finding the sum of all digits.
   (iii) Divisibility by 4 and 8 is checked by the last 2 and 3 digits respectively.
   (iv) Divisibility of 11 is checked by comparing the sum of digits at odd and even places.

4) If two numbers are divisible by a number, then their sum and difference are also divisible by the number.

5) (i) The highest common factor (HCF) of two or more given numbers is the highest of their common factors.
   (ii) The least common multiple (LCM) of two or more given numbers is the lowest of their common multiples.

6) If one of the two given numbers is a multiple of the other, then the greater number will be their LCM.

7) Relationship between LCM and HCF of two numbers
   LCM x HCF = Product of the two numbers.

P8L3Y4

CHAPTER 4

Integers

D1N1Z7

Learning Outcomes:-

The students are able to

- know the necessity and importance of Integers beyond whole numbers.
- understand the need of negative numbers in real life situations.
- represent the integers on the number line.
- compare and arrange integers in ascending and descending order.
- understand the concept and operations in addition and subtraction of integers.
- verify the properties of integers under addition and subtraction.

Content Items

4.0 Introduction

We have learnt about natural numbers, whole numbers and their properties in the previous classes.

There are several situations in our daily life,where we use negative numbers to represent loss in business, low temperature, below a surface…etc. These numbers are lesser than zero. We call them as ***negative numbers.*** The numbers, which are greater than zero are called ***positive numbers*** which represent profit in business, high temperature, above the surface...etc.

4.1 Integers

To distinguish between the numbers mentioned above, we use the symbol ***(+) for positive numbers*** and ***(-) for negative numbers.*** You know that zero and positive numbers are called whole numbers. When we include negative numbers in the group,we call the new set of numbers as ***"Integers".***

Note that ***Zero(0) is neither a positive nor a negative number.***

Thus, ***the positive numbers, zero and negative numbers together form Integers. The set of Integers is denoted by the symbol "Z".***

Z = { …, -4, -3, -2, -1, 0, 1, 2, 3, 4,…}

Note: A number without any sign is considered as positive number only.

i.e., 12 is considered as +12.

Ex: Profit of Rs.200/- is represented as +200 and 4°C below 0° is represented as -4°C.

HISTORICAL NOTE

Brahmagupta (598-670 AD) first used a special sign (–) for negative numbers and stated the rules for dealing with positive and negative quantities.

The letter “Z” was first used by the Germans because the word for Integers in the German language is “Zehlen”, which means “Number”.

Brahmagupta

CHECK YOUR PROGRESS

1. Write any five positive integers.
2. Write any five negative integers.
3. Which number is neither positive nor negative?
4. Represent the following situations with integers.

   i) A gain of ₹ 500 (   )

   ii) Temperature is below 5°C (   )

5. Represent the following using either positive or negative numbers.

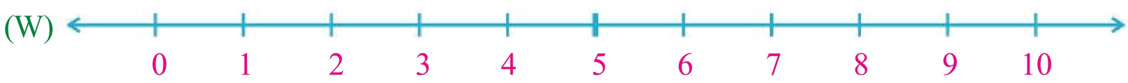
a) A bird is flying at a height of 25 metres above the sea level and a fish at a depth of 2 metres.

b) A helicopter is flying at a height of 60m above the sea level and a submarine is at 400m below sea level.

4.2 Representation of Integers on a number line:

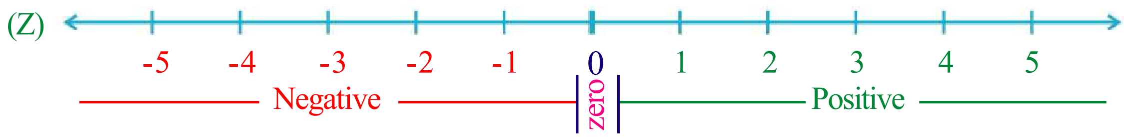
We know that when zero is included to the set of Natural numbers, then the set of numbers is called as Whole numbers.

Now, let us recall the number line which represent the set of Whole numbers.

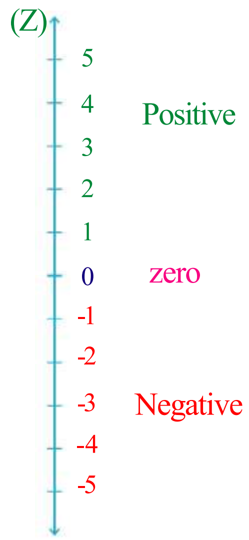


We have seen the need to extend the number line beyond Zero(0) to it’s left. We call the numbers -1 ,-2 ,-3 ,-4 ,……..(to the left of zero) as negative integers which are quite opposite (i.e., to the right of zero) to positive integers.

Hence, the integers(Z) are shown on the number line as in next page.

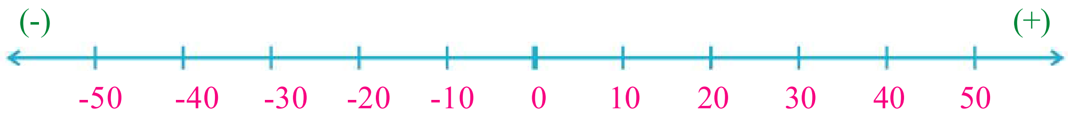


The Integer number line can also be represented vertically as given below.



Natural numbers,{1,2,3,4,…} are also called ***as positive integers*** and ***Whole numbers***,{0,1,2,3,4,…} are also called ***as non-negative integers.***

Example-1: Read the following number line and answer the following questions (1cm=10°C)



i) Write the temperatures which lie between 0°C and -30°C

ii) Write the temperatures which lie between 10°C and 40°C

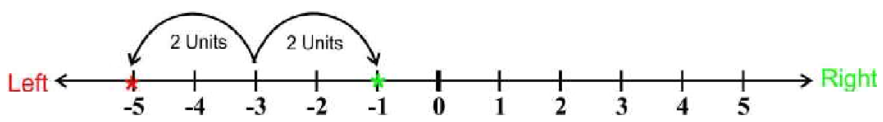
Solution:

i) The temparatures lie between 0°C and -30°C are -10°C and -20°C

ii) The temperatures lie between 10°C and 40°C are 20°C and 30°C

Example-2: Find the numbers that are at a distance of 2 units in the opposite direction to -3 on the number line.

Solution: From -3 , we can move 2 units to the left and then to the right as shown in the figure. The required numbers 2 units from -3 on either side are



-5 is on the left side of -3 and -1 is on the right side of -3

CHECK YOUR PROGRESS

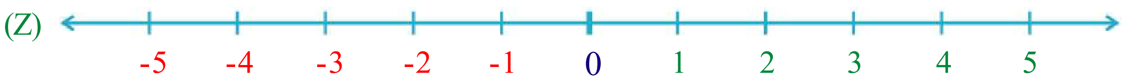
1) Draw a vertical number line and represent -5,4,0,-6,2, and 1 on it.

2) Represent opposite integers of -200 and +400 on integer number line.

Exercise - 4.1

1. Write True or False against each of the following statements.
   i) -7 is on the right side of -6 on the number line.
   ii) Zero is a positive number
   iii) 29 is on the right side of zero on the number line
   iv) -1 lies between the integers -2 and 1
   v) There are nine integers between -5 and +5.

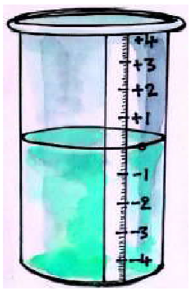
2. Observe the following number line and answer the following questions.



   i) Which is the nearest positive integer to -1?
   ii) How many negative numbers you will find on the left side of Zero?
   iii) How many integers are there in between -3 and 7?
   iv) Write 3 integers lesser than -2.
   v) Write 3 integers more than -2.

3. Represent the integers on a number line as given below.
   i) Integers lie between -7 and -2.
   ii) Integers lie between -2 and 5.

4.3 Ordering of Integers

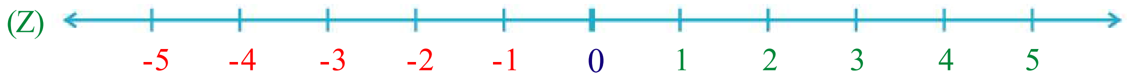


Pavan and Harish are friends and they noticed that the water level in the well of their village (where steps are observed) reduces during day time and rises during night time. The level was shown by the steps of the well. They used the idea of the number of steps of the well and prepared a model of the well using a glass jar. They pasted a strip showing integers with steps below zero as -1, -2, -3 and steps above zero as 1, 2, 3, 4 and so on. They took zero as the level of water on the steps, on the day they observed.

They use this jar to represent the water level, taking out water when water level falls in the well and adding water when it rises. They were able to record the water level in the well above the base step level as positive and below it as negative.

Now say 1. What happens when water is poured into the jar?

2. What happens when water is removed from the jar from the zero level?



Let us observe the integers which are represented on the number line.

We know that 4 > 2 and that 4 is to the right of 2 on the number line. Similarly, 2 > 0 and is to the right of 0. Now, since 0 is to the right of -3, we say 0 > -3.

Thus, we see that on a number line, the number increases as we move to right and decreases as we move to the left.Therefore, -3 < -2, -2 < -1, -1 < 0 and 0 < 1, 1 < 2, 2 < 3 so on.

Let's Think *For any two integers, say 3 and 4, we know that 3 <4. Is it true to say -3 < -4? Give reason.*

Example-3 : Arrange the following integers in ascending order. -8, 0, -1, 3, -5, -20 and 12

Solution:

Step-1: First, separate the positive and negative integers from the given data.

Positive integers are 3,12

Negative integers are -8 ,-1 ,-5 ,-20

Step-2: Arrange the negative integers in ascending order as -20,- 8,-5,-1 also positive integers in ascending order 3 ,12.

Step-3: As zero(0) is neither positive nor negative, it stays in the middle of the arrangement.

Step-4: The ascending order of the given integers is -20, -8, -5, -1, 0, 3, 12.

Example-4: Write the integers on either side of the integers (lesser, greater)

i) -5 ii) 0 iii) 3

Solution:

i) The integers on either side of -5 are -6 and -4.

ii) The integers on either side of 0 are -1 and +1

iii) The integers on either side of 3 are 2 and 4.

Exercise - 4.2

1. Put appropriate symbol > or < in the boxes given.

(i) -1 $\boxed{}$ 0 (ii) -3 $\boxed{}$ -7 (iii) -10 $\boxed{}$ +10

2. Write the following integers in increasing and decreasing order:

(i) -7, 5, -3 (ii) -1, 3, 0 (iii) 1, 3, -6 (iv) -5, -3, -1

3. Write True or False.

   (i) Zero is on the right of -3 ( )

   (ii) -12 and +12 represent on the number line the same integer ( )

   (iii) Every positive integer is greater than zero ( )

   (iv) (-100) > (+100) ( )

4. Find all integers which lie between the given two integers. Represent them on number line:

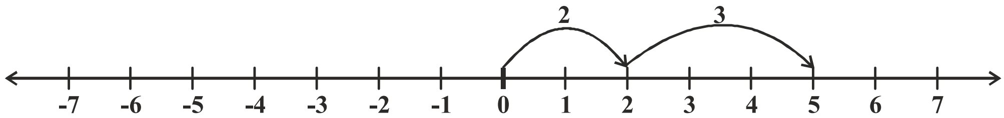
   (i) -1 and 1 (ii) -5 and 0 (iii) -6 and -8 (iv) 0 and -3

5. The temperature recorded in Shimla is -4° c and in Kufri is -6°c on the same day. Which place is colder on that day? Why?

4.4 Addition of integers on the number line

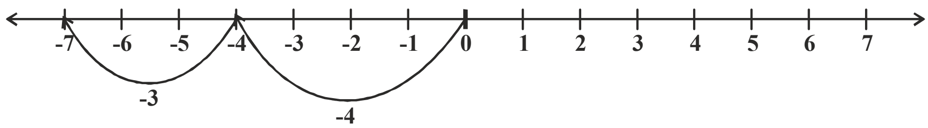
Let us see how we can add any two integers using a number line.

1. Let us add 2 and 3 on a number line.



On the number line, we first move 2 steps to the right from 0 to reach 2. Then we move 3 steps to the right of 3. Now we are at 5. Thus, we get 2 + 3 = 5.

2. Let us add (-4) and (-3).

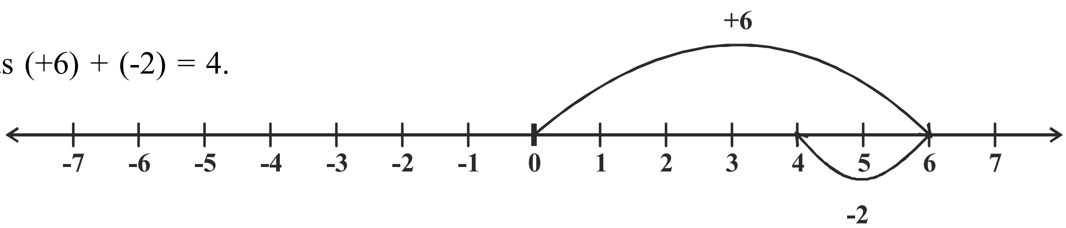


On the number line, we first move 4 steps to the left of 0 to reach -4. Then we move 3 steps to the left of -3 and reach -7.

Thus, (-4) + (-3) = -7.

3. Suppose we wish to find the sum of (+6) and (-2) on the number line. First we move to the right of 0 by 6 steps to reach 6. Then we move 2 steps to the left of 6 to reach 4.

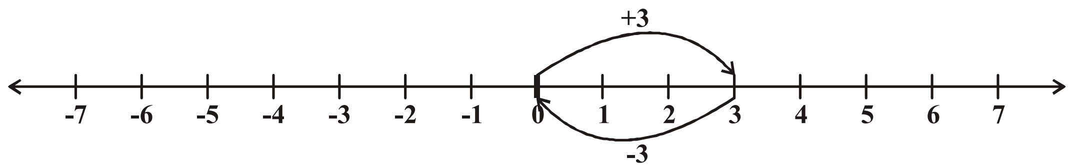
Thus (+6) + (-2) = 4.



4. Suneetha adds 3 and -3. She first moves from 0 to +3 and then from +3 she move 3 points

to the left. Where does she reach ultimately?

From the figure 3 + (-3) = 0



Similarly, if we add 1 and -1, 2 and -2, 3 and -3 ..... so on we obtain the sum as zero. They are called additive inverses of each other i.e. any two distinct numbers that give zero when added to each other are additive inverse of each other.

- What is additive inverse of 7?
- What is additive inverse of -8?

LET'S EXPLORE

1. Find the value of the following using a number line.

(i) (-3) + 5 (ii) (-5) + 3

Make two questions on your own and solve them using the number line.

2. Find the solution of the following:

(i) (+5) + (-5) (ii) (+6) + (-7) (iii) (-8) + (+2)

Ask your friend five such questions and solve them.

- Observe the following:

(i) 3 + 2 = 5 (ii) 20 + 6 = 26 (iii) 30 + 22 = 52

We see that ***"the sum of two positive integers is also a positive integer".***

- Look at the following now:

(i) -4 + (-6) = -10 (ii) -8 + (-12) = -20 (iii) -3 + (-9) = -12

What do you learn from this? ***"The sum of two negative integers is always a negative integer".***

- What happens if one integer is positive and the other negative? Let us see

(i) 15 + (-17) = -2 (ii) -23 + 4 = -19 (iii) -11 + 16 = 5 (iv) -12 + 12 = 0

From the above, we can conclude that when ***"the sum of two integers one of which is positive and the other negative, then the sum may be either positive, negative or zero".***

Take some more examples to verify the above results on your own.

Example-5 : Find the sum of (-20), (-82), (-28) and (-14).

Solution: We have (-20) + (-82) + (-28) + (-14) = -144

Example-6: Find the sum of 25 + (-21) + (-20) + (+17) + (-1)

Solution: We have 25 + (-21) + (-20) + (+17) + (-1) = 0

Exercise - 4.3

1. Add the following integers using number line.
   (i) 7 + (-6)   (ii) (-8) + (-2)   (iii) (-6) + (-5) + (+2)
2. Add without using number line.
   (i) 10 + (-3)   (ii) -10 + (+16)   (iii) (-8) + (+8)
3. Find the sum of:
   (i) 120 and -274   (ii) -68 and 28
4. Simplify:
   (i) (-6) + (-10) + 5 + 17   (ii) 30 + (-30) + (-60) + (-18)

4.5 Subtraction of Integers:

We saw that to add 5 and (-2) on a number line, we can start from 5 and then move 2 steps to the left of 5.

We reach at 3. So, we have 5 + (-2) = 3

Thus, we find that to add a positive integer we move towards the right on a number line and for adding a negative integer we move towards left. We have also seen that while subtracting whole numbers on a number line, we would move towards left.

For example, take 5 - 2 = ?

We start from 5 and take two steps to the left and end up at 3.

What does subtraction of a negative integer mean?

Let us observe the following example.

Consider 5 and -5. We know that additive inverse of 5 is -5.

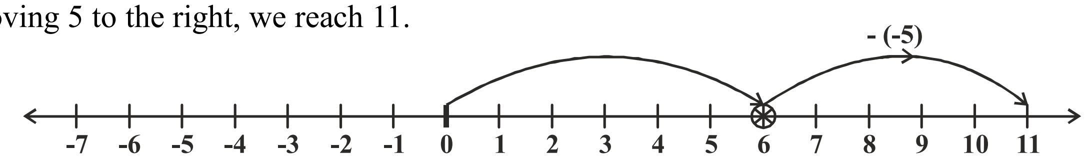
So, additive inverse of -5 is 5. We can write additive inverse of (-5) as -(-5) also.

So, -(-5) = 5. Thus, ***for any integer a, - (-a) = a.***

Example-7: Subtract -5 from 6.

Solution: To subtract -5 from 6, let us start at 6. For -5, we would have moved left but for -(-5) = 5 we would move in the opposite direction. i.e., towards right.

Moving 5 to the right, we reach 11.



We have 6 - (-5) = 11

i.e. to subtract -5 from 6 add 5 (the additive inverse of -5) to 6.

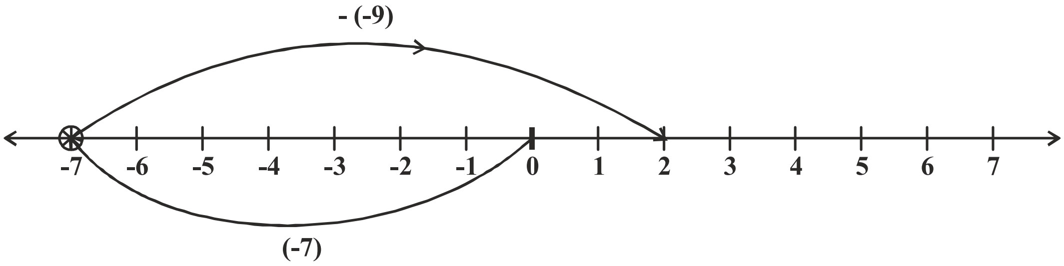
6 - (-5) = 6 + 5 = 11

Example-8: Find the value of (-7) - (-9) using number line.

Solution:

(-7) - (-9) is equal to -7 + 9 (Since -9 is additive inverse of 9).

On the number line, start from -7 and move 9 units to right, we will reach 2.



So (-7) - (-9) = -7 + 9 = 2.

Example-9: Subtract (+8) from (-8)

Solution: (-8) - (+8) = (-8) + (additive inverse of +8)

= -8 + (-8)

= -16

Example-10: Simplify: (-6) - (+7) - (-24)

Solution: (-6) - (+7) - (-24) = (-6) + (additive inverse of +7) + (additive inverse of -24)

= -6 + (-7) + (+24)

= -13 + 24

= 11.

Example-11: Write any real life situation that represent the integer -3.

Solution: Nagamani answered 20 answers correctly and 23 answers in wrongly in a talent test. If 1 mark is given for each correct answer and -1 mark is given for wrong answer then the total score obtained in the test will be -3

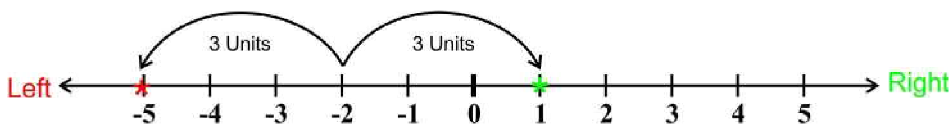
i.e., 20(+1) + 23(-1) =20 -23 = -3

Example-12: Identify the integers and mark on the number line that are at a distance of 3 units from -2.

Solution: The integers which are at a distance of 3 units from -2 are -5 and 1.

-5 is 3 units from -2 on the left side

1 is 3 units from -2 on the right side of the number line.



LET'S EXPLORE

Take any two integers **a** and **b.** Check whether **a+b** is also an integer?

Observe that ***sum of any two integers is also an integer.***

Integers are closed under addition ***(Closure Property).***

• Check the following properties on integers. **a, b, c** are any integers.

i) Closure Property under subtraction

ii) Commutative Property under addition and subtraction (a+b=b+a ?, a-b=b-a ?)

iii) Associative property under addition and subtraction.

(a+b)+c=a+(b+c)?   (a-b)-c=a-(b-c)?

Exercise - 4.4

1. Find :

(i) 40 - (22)   (ii) 84 - (98)   (iii) (-16) + (-17)

(iv) (-20) - (13)   (v) (38) - (-6)   (vi) (-17) - (-36)

2. Fill in the boxes with >, < or = sign:

(i) (-4) + (-5) $\square$ (-5) - (-4)

(ii) (-16) - (-23) $\square$ (-6) + (-12)

(iii) 44 - (-10) $\square$ 47 + (-3)

3. Fill in the blanks:

(i) (-13) + ____________ = 0   (ii) (-16) + 16 = ____________

(iii) (-5) + ____________ = -14   (iv) ____________ + (2-16) = -22

Unit Exercise

1. Write the integers for the following situations.

i) A kite is flying at a height of 225 m in the sky. (   )

ii) A whale is at a depth of 1250 m in the ocean. (   )

iii) The temperature in Sahara desert is 12°C below freezing temperature. (   )

iv) Ravi withdrawn ₹ 3800 from ATM using his debit card. (   )

2. Justify the following statements with an example.

i) A positive integer is always greater than a negative integer.

ii) All positive integers are natural numbers.

iii) Zero is greater than a negative integer.

iv) There exist infinite integers in the number system.

v) All whole numbers are integers.

3. Represent i) 3+4 ii) 8+(-3) iii) -7-2 iv) 6-(5) v) -5-(-4) on number line.

4. Write all the integers lying between the given numbers.

   i) 7 and 12   ii) -5 and -1   iii) -3 and 3   iv) -6 and 0

5. Arrange the following integers in ascending and descending order.

   -1000, 10 ,-1 ,-100 ,0 ,1000, 1 ,-10

6. Write a real life situation for each of the following integers.

   i) -200 m   ii) +42°C   iii) ₹ 4800(cr)   iv) -3.0 kg

G8A8W1

7. Find i) (-603) + (603) ii) (-5281) + (1825) iii) (-32)+ (-2) + (-20) +(-6)
8. Find i) (-2) - (+1) ii) (-270) - (-270) iii) (1000) - (-1000).
9. In a quiz competition,where negative score for wrong answer is taken,Team A scored +10, -10, 0, -10, 10,-10 and Team B scored 10,10, -10,0,0,10 in 6 rounds successively.Which team wins the competition? How?

10. An apartment has 10 floors and two cellars for car parking under the basement.A lift is now,at the ground floor.Ravi goes 5 floors up and then 3 floors up,2 floors down and then 6 floors down and come to lower cellar for taking his car. Count how many floors does Ravi travel all together? Represent the result on a vertical number line.

Points to Remember

1. Integers are a collection of natural numbers, Zero and negative numbers.
2. The set of numbers, -3 ,-2 ,-1, 0 ,1, 2, 3, ….is called Integers.It is denoted by the letter Z.
3. The number zero(0) is neither positive nor negative.
4. Negative numbers are important to represent just opposite numbers for positive numbers.
5. Every integer can be represented on a number line horizontally or vertically.
6. Addition and subtraction of integers can be represented on number line.
7. Every negative integer is always lesser than zero.
8. The sum of two positive integers is positive and sum of two negative numbers is negative.
9. The sum of a positive and a negative number is either a positive or negative.
10. We add the opposite of an integer(additive inverse) to an integer in subtraction.
11. Integers are widely used in our daily life i.e., business, engineering, games, temperatures, medicine etc.,
12. Integers are closed under addition and subtraction.
13. Integers are commutative and associative under addition.

CHAPTER 5 Fractions and Decimals

Learning Outcomes:-

The students are able to

- recall the different types of fractions.
- perform the four fundamental operations involving fractions.
- understand Addition and Subtraction of decimals.
- understand the use of fractions and decimals in daily life situations.
- solve various types of problems involving fractions and decimals.
- convert the fractions into decimals and Vice - Versa.

N1W9D9

5.0 Introduction

You have learnt the fundamental concepts of fractions, types of fractions (Proper, Improper, Mixed), comparision of fractions, equivalant fractions and their representation on the numberline. You have also studied the addition and subtraction of fractions. We shall now learn multiplication and division of fractions and also decimal numbers, after reviewing all the above.

Content Items

5.1 Types of Fractions:

A fraction is a number representing a part of the whole. The whole may be a single object or a group of objects.

The fraction $\frac{3}{5}$ represents three out of five. Here, in the fraction $\frac{3}{5}$, 3 is called the ***numerator*** and 5 is called the ***denominator***.

Proper fraction: ***A fraction that is whose numerator is less than the denominator is called a proper fraction.***

$\frac{3}{4}, \frac{5}{7}, \frac{11}{17}$ ... etc., are proper fractions.

All proper fractions are less than 1.

Improper fraction: ***A fraction whose numerator is equal or more to the denominator is called an improper fraction.***

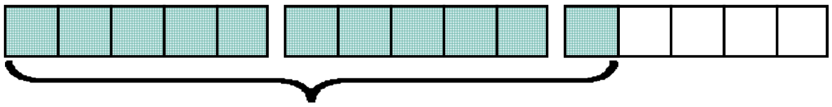
$\frac{7}{3}, \frac{11}{5}, \frac{85}{16}$ ... are improper fractions.

All improper fractions are greater than or equal to 1.

A mixed fraction is a combination of a whole number and a proper fraction.

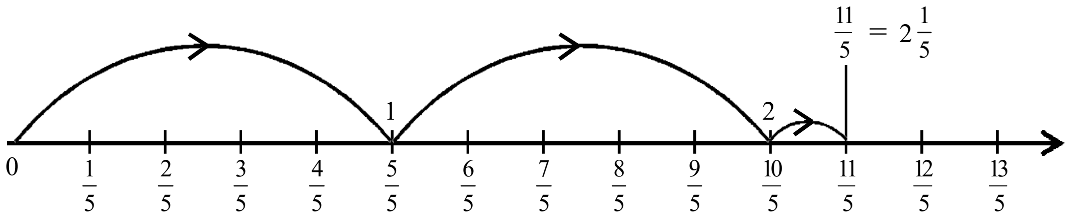
Example-1: Represent $\frac{11}{5}$ as a mixed fraction.

Solution: $\frac{11}{5}$



Mixed fraction is $2\frac{1}{5}$.

It can also be shown on a number line.

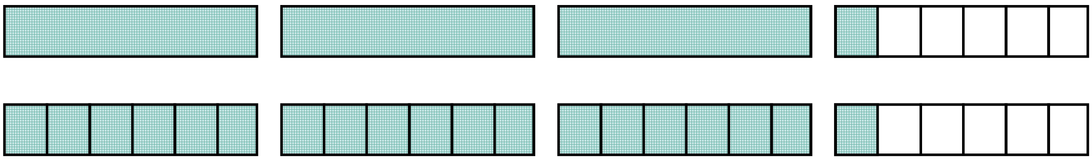


Example-2 : Represent $3\frac{1}{6}$ as an improper fraction.

Solution : Pictorially it can be shown as

$3\frac{1}{6}$

$\frac{19}{6}$

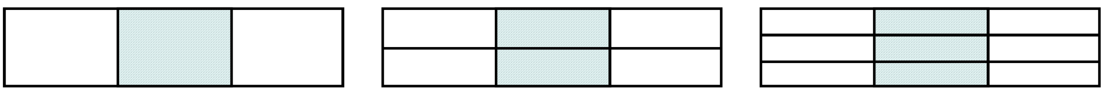


$$\frac{6}{6} + \frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{19}{6}$$

$$3\frac{1}{6} = 3 + \frac{1}{6} = \frac{18}{6} + \frac{1}{6} = \frac{3\times6+1}{6} = \frac{19}{6}$$

5.1.1 Equivalent Fractions:

Observe the fraction $\frac{1}{3}$ in different figures given below.



Shaded part = $\frac{1}{3}$ Shaded part = $\frac{2}{6}$ Shaded part = $\frac{3}{9}$

Observe that in these figures, the same part of the rectangle bars is shaded i.e. $\frac{1}{3}$ of the big rectangle is shaded. Thus, the fractions $\frac{1}{3}$, $\frac{2}{6}$ and $\frac{3}{9}$ represent the same portion (Part)

Equivalent Fractions have the same value even though they may look different

$$\therefore \frac{1}{3} = \frac{2}{6} = \frac{3}{9}$$

Example-3: Write the equivalent fractions of $\frac{2}{7}$.

Solution: Equivalent fractions of $\frac{2}{7}$ are

$$\frac{2\times 2}{7\times 2} = \frac{4}{14}, \quad \frac{2\times 3}{7\times 3} = \frac{6}{21}, \quad \frac{2\times 4}{7\times 4} = \frac{8}{28}$$

$$\frac{2\times 8}{7\times 8} = \frac{16}{56}, \quad \frac{2\times 12}{7\times 12} = \frac{24}{84}$$

5.1.2 Like and unlike fractions :

Fractions with the same denominators are called "Like fractions".

$\frac{5}{12}, \frac{7}{12}, \frac{13}{12}, \frac{17}{12}$.... are like fractions.

Fractions with different denominators are called "Unlike fractions".

$\frac{2}{15}, \frac{7}{17}, \frac{8}{19}, \frac{53}{117}$... are unlike fractions.

5.1.3 Fraction in lowest form

A fraction is said to be in its lowest terms, if the numerator and denominator have no common factor other than 1 (If the HCF of its numerator and denominator is 1). Out of the equivalent fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$,.... etc, $\frac{1}{2}$ is the lowest form or standard form.

5.1.4 Comparision of fractions

i) Out of two fractions with the same denominator, the one having the smallest numerator is the small fraction.

$\frac{2}{11}$ and $\frac{5}{11}$ i.e., $\frac{2}{11} < \frac{5}{11}$.

ii) Out of two fractions with the same numerator, the one having the smaller denominator is greater than other.

$\frac{3}{7}$ and $\frac{3}{11}$ i.e., $\frac{3}{7} > \frac{3}{11}$.

iii) To compare unlike fractions, convert them into equivalent fractions with LCM as denominators, then compare the like fractions.

Compare $\frac{2}{5}$ and $\frac{3}{4}$.

LCM of 4 and 5 is 20.

$$\frac{2}{5} = \frac{8}{20} \text{ i.e., } \frac{3}{4} = \frac{15}{20}$$

$$\therefore \frac{15}{20} > \frac{8}{20} \text{ and hence } \frac{3}{4} > \frac{2}{5}.$$

5.1.5 Addition and Subtractions of Fractions

i) Like Fractions

We add or subtract their numerators and retain the common denominator.

(i) $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$ (ii) $\frac{23}{35} - \frac{9}{35} = \frac{23-9}{35} = \frac{14}{35}$

(iii) $5\frac{2}{7} + \frac{1}{7} - 4\frac{3}{7} = \frac{37}{7} + \frac{1}{7} - \frac{31}{7} = \frac{37+1-31}{7} = \frac{7}{7} = 1$

ii) Unlike fractions

(i) Find the LCM of the denominators.

(ii) Convert each fraction into an equivalent fraction whose denominator is equal to the LCM, to make them like fractions.

(iii) Add or subtract like fractions as given.

$$\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

Example-4 : Simplify

(i) $1\frac{4}{5} + 2\frac{5}{6}$ (ii) $5\frac{1}{4} - 2\frac{5}{6}$

Solutions :

(i) $1\frac{4}{5} + 2\frac{5}{6} = \frac{9}{5} + \frac{17}{6}$

The LCM of denominators 5 and 6 is 30.

$$\frac{9}{5} = \frac{9\times 6}{5\times 6} = \frac{54}{30}; \quad \frac{17}{6} = \frac{17\times 5}{6\times 5} = \frac{85}{30}$$

$$\frac{9}{5} + \frac{17}{6} = \frac{54}{30} + \frac{85}{30} = \frac{139}{30} = 4\frac{19}{30}$$

(ii) $5\frac{1}{4} - 2\frac{5}{6} = \frac{21}{4} - \frac{17}{6}$

LCM of 4 and 6 is = 12

$\frac{21}{4} = \frac{21\times3}{4\times3} = \frac{63}{12}$; $\frac{17}{6} = \frac{17\times2}{6\times2} = \frac{34}{12}$

$\therefore \frac{21}{4} - \frac{17}{6} = \frac{63}{12} - \frac{34}{12} = \frac{63-34}{12} = \frac{29}{12} = 2\frac{5}{12}$

$\therefore 5\frac{1}{4} - 2\frac{5}{6} = 2\frac{5}{12}$

Example 5 : Satish bought $1\frac{2}{5}$ m. of ribbon and Padma $2\frac{3}{4}$ m. of ribbon. What was the total length of the ribbon they bought?

Solution : Total length of the ribbon Satish and Padma bought = $1\frac{2}{5} + 2\frac{3}{4}$ m

Now $1\frac{2}{5} + 2\frac{3}{4} = \frac{7}{5} + \frac{11}{4}$ LCM of 5, 4 is 20.

$= \frac{7\times4}{5\times4} = \frac{28}{20}$; $\frac{11\times5}{4\times5} = \frac{55}{20}$

$= \frac{28}{20} + \frac{55}{20}$

$= \frac{28+55}{20} = \frac{83}{20} = 4\frac{3}{20}$

Example 6 : What should be added to $2\frac{4}{5}$ to get $5\frac{2}{3}$.

Solution : The number added to $2\frac{4}{5}$ to get $5\frac{2}{3}$, subtract $2\frac{4}{5}$ from $5\frac{2}{3}$

i.e $5\frac{2}{3} - 2\frac{4}{5}$

$= \frac{17}{3} - \frac{14}{5}$ LCM of 3, 5 is 15.

$= \frac{17\times5}{3\times5} - \frac{14\times3}{5\times3} = \frac{85}{15} - \frac{42}{15} = \frac{85-42}{15} = \frac{43}{15} = 2\frac{13}{15}$

Exercise - 5.1

1. Classify the fractions as proper, improper and mixed.

$\frac{3}{4}, \frac{6}{5}, \frac{3}{2}, \frac{4}{1}, \frac{2}{3}, \frac{1}{4}, \frac{18}{13}, 1\frac{5}{7}, \frac{1}{3}, 11\frac{1}{2}$

2. Write the following fractions in an ascending order.

i) $\frac{3}{4}, \frac{3}{2}, \frac{2}{3}, \frac{1}{5}, \frac{18}{7}$ ii) $\frac{2}{7}, \frac{3}{8}, \frac{3}{4}, \frac{5}{7}, \frac{4}{9}$

3. Without doing calculation, find the result $\frac{2}{3} + 1\frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

4. Neha bought a cake. She ate $\frac{7}{15}$th of the cake immediately and in the afternoon she ate the remaining part. How much part she ate in the afternoon ?

5. Simplify.

i) $\frac{2}{5} + \frac{1}{3}$ ii) $\frac{5}{7} + \frac{2}{3}$ iii) $\frac{3}{5} - \frac{7}{20}$ iv) $\frac{17}{20} - \frac{13}{25}$

6. Represent $\frac{16}{5}$ pictorially.

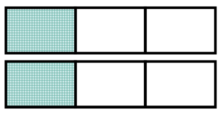
5.2 Multiplication of Fractions

Multiplication of a fraction by a whole number

Observe the pictures. Each shaded part is $\frac{1}{3}$ part of a rectangular strip.

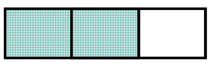
How much will the two shaded parts represent together ?

The shaded part represents $\frac{1}{3} + \frac{1}{3} = \frac{1}{3} \times 2$



Combining the shaded parts of the above the rectangular strip, we get $\frac{1}{3} \times 2$

$2 \times \frac{1}{3} = \frac{2}{3}$



Now, let us find some more results.

$$3 \times \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1+1+1}{5} = \frac{3\times1}{5} = \frac{3}{5}$$

$$\text{So } 3 \times \frac{1}{5} = \frac{3 \times 1}{5} = \frac{3}{5}$$

Thus, we observe that to multiply a fraction by a whole number, first multiply the numerator of the fraction by the whole number and keep the denominator same.

• Is it true to say that $3 \times \frac{1}{5} = \frac{1}{5} \times 3$

To multiply a mixed fraction with a whole number, first convert the mixed fraction to an improper fraction and then multiply with the whole number.

Therefore, $2 \times 5\frac{2}{7} = 2 \times \frac{37}{7} = \frac{74}{7} = 10\frac{4}{7}$

Similarly, $3\frac{2}{5} \times 2 = \frac{17}{5} \times 2 = \frac{17 \times 2}{5} = \frac{34}{5} = 6\frac{4}{5}$

CHECK YOUR PROGRESS

Find (i) $5 \times 3\frac{2}{7}$ (ii) $2\frac{5}{9} \times 3$ (iii) $2\frac{4}{7} \times 3$ (iv) $3 \times 1\frac{3}{4}$

Multiplication of a fraction by a fraction.

Take a rectangular sheet and divide into four equal parts along the length and shade the 3 parts as shown in fig-(i).

The portion shaded represents $\frac{3}{4}$.

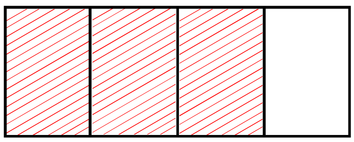


Figure-(i)

Divide this rectangle along the breadth into 6 equal parts and shade 5 parts in blue cross lines as shown in the fig-(ii).

Now the portion shaded in blue cross lines represents $\frac{5}{6}$

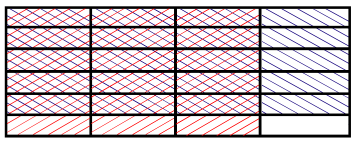


Figure-(ii)

In Figure-(ii), the portion shaded  represents $\frac{15}{24}$

It represents $\frac{5}{6}$ of $\frac{3}{4}$ (or) $\frac{5}{6} \times \frac{3}{4}$.

i.e $\frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{6 \times 4} = \frac{15}{24}$

The value of $\frac{3}{4} \times \frac{5}{6}$ can be found in another way.

Divide the rectangular sheet into six equal parts along the length and shade 5 parts and divide the rectangle along the breadth into 4 equal parts and shade 3 parts. This will represent $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}$.

$$\therefore \frac{3}{4} \times \frac{5}{6} = \frac{3\times5}{4\times6} = \frac{15}{24}. \quad \text{Thus, } \frac{5}{6} \times \frac{3}{4} = \frac{3}{4} \times \frac{5}{6}$$

Example-7 : Multiply (i) $\frac{3}{4} \times \frac{1}{7}$ (ii) $\frac{5}{3}\times\frac{7}{2}$ (iii) $\frac{8}{3} \times \frac{4}{7}$ (iv) $2\frac{1}{5} \times \frac{1}{3}$

Solution : (i) $\frac{3}{4} \times \frac{1}{7} = \frac{3\times1}{4\times7} = \frac{3}{28}$ (ii) $\frac{5}{3}\times\frac{7}{2} = \frac{5\times7}{3\times2} = \frac{35}{6}$

(iii) $\frac{8}{3} \times \frac{4}{7} = \frac{8\times4}{3\times7} = \frac{32}{21}$ (iv) $2\frac{1}{5} \times \frac{1}{3} = \frac{11}{5} \times \frac{1}{3} = \frac{11\times1}{5\times3} = \frac{11}{15}$

So, multiplication of two fractions $= \frac{\text{Product of Numerators}}{\text{Product of Denominators}}$

Example-8 : Multiply

(i) $\frac{2}{9}$ by $\frac{4}{5}$ (ii) $\frac{3}{5}$ by 14 (iii) $3\frac{1}{2}$ by $\frac{1}{7}$ (iv) $4\frac{3}{7}$ by $1\frac{2}{7}$

Solution : We have

(i) $\frac{2}{9} \times \frac{4}{5} = \frac{2\times4}{9\times5} = \frac{8}{45}$ (ii) $\frac{3}{5} \times 14 = \frac{3\times14}{5} = \frac{42}{5}$

(iii) $3\frac{1}{2} \times \frac{1}{7} = \frac{7}{2} \times \frac{1}{7} = \frac{1}{2}$ (iv) $4\frac{3}{7} \times 1\frac{2}{7} = \frac{31}{7} \times \frac{9}{7} = \frac{279}{49}$

Observe the products of fractions.

i) $\frac{1}{5}\times\frac{2}{3} = \frac{2}{15}$ (Product of two proper fractions)

ii) $\frac{3}{2}\times\frac{5}{4} = \frac{15}{8}$ (Product of two improper fractions)

iii) $\frac{2}{3}\times\frac{5}{3} = \frac{10}{9}$ (Product of proper and improper fractions)

Have you observed the products of any two fractions is always lesser or greater than each of its fraction, write conclusion.

Example-9 : In a class of 40 students $\frac{2}{5}$ of the students are boys. How many boys are there in the class?

Solution : Total number of students $= 40$

$\therefore$ Number of boys $= \frac{2}{5}$ of the total number of students.

$$= \frac{2}{5} \text{ of } 40 = \frac{2}{\cancel{5}} \times \cancel{40}^{8} = 16$$

Example-10: A rectangular park is $7\frac{2}{3}$ m. long and $3\frac{1}{5}$ m. wide. What is the area of the park?

Solution : Length of the park $= 7\frac{2}{3}$ m. $= \frac{23}{3}$ m.

Width of the park $= 3\frac{1}{5}$ m. $= \frac{16}{5}$ m.

$\therefore$ Area of the park $=$ Length $\times$ Width

$$= \frac{23}{3} \times \frac{16}{5} m^2$$

$$= \frac{368}{15} m^2 = 24\frac{8}{15} m^2$$

Exercise - 5.2

1. Find the product of the following.

   (i) $3 \times \frac{5}{12}$ (ii) $\frac{15}{8} \times 12$ (iii) $1\frac{3}{4} \times \frac{12}{21}$ (iv) $\frac{4}{5} \times \frac{12}{7}$

2. Which is greater ?

   (i) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$ (ii) $\frac{2}{7}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$

3. Find

   (i) $\frac{7}{11}$ of 330 (ii) $\frac{5}{9}$ of 108 (iii) $\frac{2}{7}$ of 16 (iv) $\frac{1}{7}$ of $\frac{3}{10}$

4. If the cost of a notebook is ₹$10\frac{3}{4}$. Then find the cost of 36 books.

5. A motor bike runs $52\frac{1}{2}$ km using 1 litre of petrol. How much distance will it cover for $2\frac{3}{4}$ litres of petrol?

5.3 Division of Fractions

Division of Whole Number by a Fraction

Let us find how to divide 2 with $\frac{1}{4}$.

Step:1- Take 2 rectangles of same size from the cardboard.

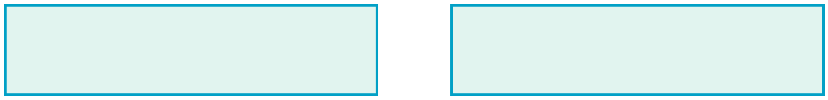


Fig-(1)

Step:2- Divide each rectangle into four equal parts as shown in the given figure.

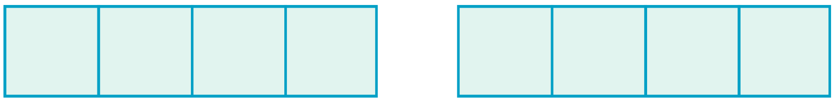


Fig-(2)

Step:3- Record the calculation.

(i) No. of identical rectangles in figure-(1) = 2

(ii) Each rectangle has been divided into four equal parts.

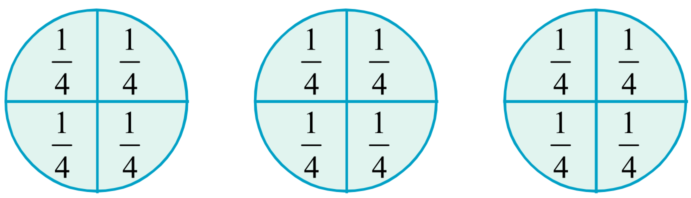
So, each part in a rectangle represents the fraction = $\frac{1}{4}$

(iii) There are in all eight $\frac{1}{4}$ ths in figure-(2).

Therefore, figure-(2) contains eight $\frac{1}{4}$ ths

$\therefore \quad 2 \div \frac{1}{4} = 8.$

Similarly, $3 \div \frac{1}{4}$ = number of $\frac{1}{4}^{th}$ parts obtained when each of the 3 whole, are divided into $\frac{1}{4}$ equal parts.



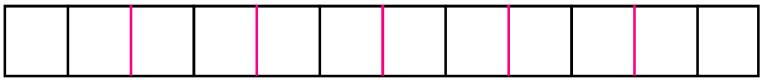
Observe that $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = \frac{3 \times 4}{1} = \frac{12}{1} = 12$

• Find

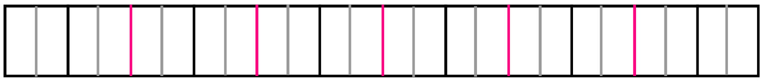
(i) $4 \div \frac{1}{8}$ (ii) $9 \div \frac{3}{4}$ (iii) $7 \div \frac{2}{3}$ (iv) $35 \div \frac{7}{3}$ (v) $4 \div \frac{15}{8}$

Reciprocal of a fraction

Let us take a bar of length 12cm. How many 2cm bars are there in 12cm bar?



$12 \div 2 = 6$ bars



Now, find how many $\frac{1}{2}$ cm bars are there in a 12cm bar? Hence it is 24.

i.e., 24 = 12 x 2

Observe that, dividing a whole number 12 by a fraction $\frac{1}{2}$ is the same as multiplying a whole number 12 by 2, where 2 is the reciprocal of $\frac{1}{2}$.

The number $\frac{2}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{2}$ or inverse of $\frac{1}{2}$. It is also called reciprocal of the fraction.

i.e., reciprocal of $\frac{1}{2} = 2$ or reciprocal of $2 = \frac{1}{2}$.

• Observe these products and fill in the blanks.

$5 \times \frac{1}{5} = 1$	$\frac{5}{8} \times \frac{8}{5} =$ _____
$\frac{1}{8} \times 8 =$ _____	$\frac{2}{7} \times$ ___ $= 1$
$\frac{3}{4} \times \frac{4}{3} = \frac{3 \times 4}{4 \times 3} = \frac{12}{12} = 1$	___ $\times \frac{5}{9} = 1$

Any two non-zero numbers whose product is 1, are called reciprocals to each other.

For example, reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$ and the reciprocal of $\frac{7}{5}$ is $\frac{5}{7}$.

The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$. The reciprocal of $\frac{4}{11}$ is $\frac{11}{4}$.

While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then multiply the whole number with the reciprocal of the improper fraction.

$$\text{Thus } 4 \div 1\frac{3}{5} = 4 \div \frac{8}{5} = 4 \times \frac{5}{8} = \frac{\cancel{4}^{1} \times 5}{\cancel{8}^{2}} = \frac{5}{2}$$

$$\text{Also } 5 \div 3\frac{1}{3} = 5 \div \frac{10}{3} = 5 \times \frac{3}{10} = \frac{\cancel{5}^{1} \times 3}{\cancel{10}^{2}} = \frac{3}{2}$$

CHECK YOUR PROGRESS

Write the reciprocal of fractions in the given table

Fraction	$\frac{2}{9}$		$\frac{4}{7}$	$3\frac{1}{7}$	$\frac{15}{8}$	
Reciprocal		$\frac{5}{5}$			$\frac{1}{5}$	

Division of a fraction by a whole number

What will be $\frac{1}{3} \div 4$?

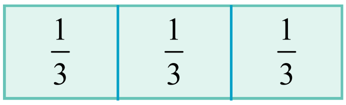


Figure-1

Take a rectangular paper and divide it into 3 equal parts.

Each part is divided into 4 equal parts.

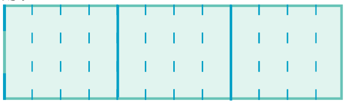


Figure-2

Steps:

(i) Each part in the rectangle represents the fraction $= \frac{1}{3}$

(ii) Each part of it in figure-2 has been again divided into 4 equal parts. So, each part in figure-2 represents $\frac{1}{3} \div 4$

(iii) Hence, the required fraction represents $= \frac{1}{12}$.

Therefore $\frac{1}{3} \div 4 = \frac{1}{12}$

Based on our earlier observation, we have

$$\frac{1}{3} \div 4 = \frac{1}{3} \div \frac{4}{1} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\text{So, } \quad \frac{2}{3} \div 7 = \frac{2}{3} \div \frac{7}{1} = \frac{2}{3} \times \frac{1}{7} = \frac{2 \times 1}{3 \times 7} = \frac{2}{21}$$

$$\frac{5}{7} \div 6 = \frac{5}{7} \div \frac{6}{1} = \frac{5}{7} \times \frac{1}{6} = \frac{5 \times 1}{7 \times 6} = \frac{5}{42}$$

While dividing mixed fractions by whole numbers, convert the mixed fraction into improper fractions and divide.

$$2\frac{1}{3} \div 5 = \frac{7}{3} \div 5 = \frac{7}{3} \div \frac{5}{1} = \frac{7}{3} \times \frac{1}{5} = \frac{7}{15}$$

$$4\frac{1}{2} \div 6 = \frac{9}{2} \div 6 = \frac{9}{2} \div \frac{6}{1} = \frac{\cancel{9}^{3}}{2} \times \frac{1}{\cancel{6}^{2}} = \frac{3}{4}$$

• Find (i) $\frac{7}{9} \div 4$ (ii) $\frac{3}{4} \div 9$ (iii) $4\frac{1}{2} \div 6$ (iv) $2\frac{1}{5} \div 3$

Division of a fraction by another fraction

Now, let us find $\frac{2}{3} \div \frac{1}{6}$ by an activity

1. Draw a rectangle and divide it into three equal parts.

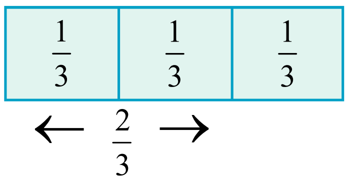


fig. 1

2. Again divide each smaller rectangle into two equal parts and get 6 smaller equal parts.

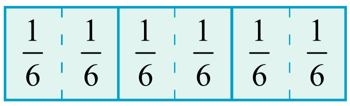


fig. 2

Each part of the rectangle in Figure-1 represents the fraction = $\frac{1}{3}$

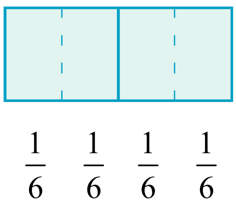


fig. 3

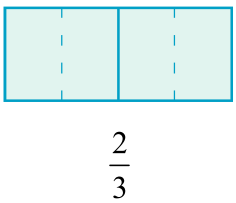


fig. 4

So, fraction $\frac{2}{3}$ is represented by two equal parts.

3. Each part in Fig (2) represents the fraction = $\frac{1}{6}$

4. Take two equal parts of Fig (2) we get Fig (3)

$\frac{2}{3} \div \frac{1}{6}$ means, the number of $\frac{1}{6}$ that are contained in $\frac{2}{3}$.

There are four $\frac{1}{6}$'s in $\frac{2}{3}$.

So, $\frac{2}{3} \div \frac{1}{6} = 4$

that is $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{2 \times \cancel{6}^2}{\cancel{3} \times 1} = \frac{4}{1} = 4$

Similarly, i) $\frac{3}{5} \div \frac{4}{9} = \frac{3}{5} \times$ reciprocal of $\frac{4}{9} = \frac{3}{5} \times \frac{9}{4} = \frac{27}{20}$

(ii) $2\frac{1}{2} \div \frac{3}{5} = \frac{5}{2} \div \frac{3}{5} = \frac{5}{2} \times$ reciprocal of $\frac{3}{5} = \frac{5}{2} \times \frac{5}{3} = \frac{25}{6}$

Exercise - 5.3

1. Find the reciprocal of each of the following fractions.

   (i) $\frac{5}{9}$ (ii) $\frac{12}{7}$ (iii) $2\frac{1}{5}$ (iv) $\frac{1}{8}$ (v) $\frac{13}{11}$ (vi) $\frac{8}{3}$

2. Simplify

   (i) $15 \div \frac{3}{4}$ (ii) $6 \div 1\frac{4}{7}$ (iii) $3 \div 2\frac{1}{3}$ (iv) $\frac{4}{9} \div 15$ (v) $4\frac{3}{7} \div 14$

3. Find

   (i) $\frac{4}{9} \div \frac{2}{3}$ (ii) $\frac{4}{11} \div \frac{8}{11}$ (iii) $2\frac{1}{3} \div \frac{3}{5}$ (iv) $5\frac{4}{7} \div 1\frac{3}{10}$

4. The product of two numbers is $25\frac{5}{6}$. If one of the number is $6\frac{2}{3}$, find the other.

5. By what number should $9\frac{3}{4}$ be multiplied to get $5\frac{2}{3}$?

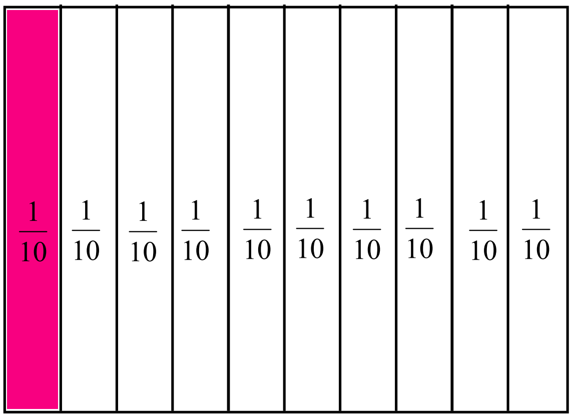
6. A bucket contains $34\frac{1}{2}$ litres of water. How many times do you get $1\frac{1}{2}$ litres of water?

7. The cost of $3\frac{3}{4}$ Kg. of sugar is ₹$121\frac{1}{2}$. Find its cost per 1Kg.

8. The length of a rectangular field is $12\frac{1}{4}$ m. and its area is $65\frac{1}{3}$ m^2.

   Find its breadth.

5.4 Decimal numbers or Decimal Fractions



Decimal:

A decimal is another way of expressing a fraction.

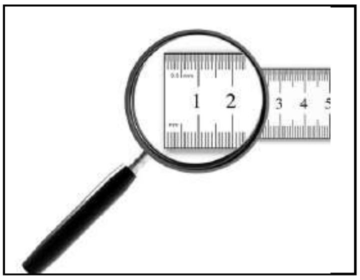
Tenths:

The fraction $\frac{1}{10}$ is written as 0.1 in decimal form. Thus, the shaded part can be read as zero point one: similarly

$\frac{9}{10}$ or 9 tenths	$\frac{6}{10}$ or 6 tenths	$\frac{4}{10}$ or 4 tenths
0.9 or zero point nine	0.6 or zero point six	0.4 or zero point four

The dot or the point between the two digits is called the decimal point.

Whole numbers and the decimal numbers can also be combined as shown in the figure.

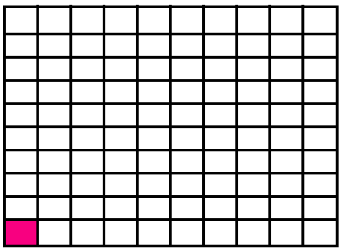


The adjacent picture shows a ruler. A length of 2 cm + 4 parts out of 1 cm. Since each cm is further divided into 10 equal parts (as can be seen), the above given number can be written as 2.4.

$2 \text{ cm} + \frac{4}{10} \text{ cm} = 2\text{cm} + 0.4 \text{ cm} = 2.4 \text{ cm}$

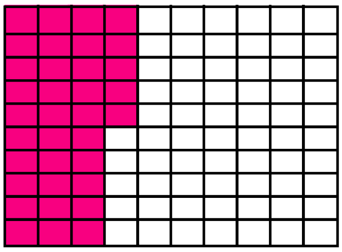
2.4 is also a ***decimal number*** and is ***read as Two Point Four.***

Hundredths: Now, let us divide a whole into 100 equal parts.



Each part out of 100 equal parts is $\frac{1}{100}$ or 0.01.

0.01 is 1 hundredth

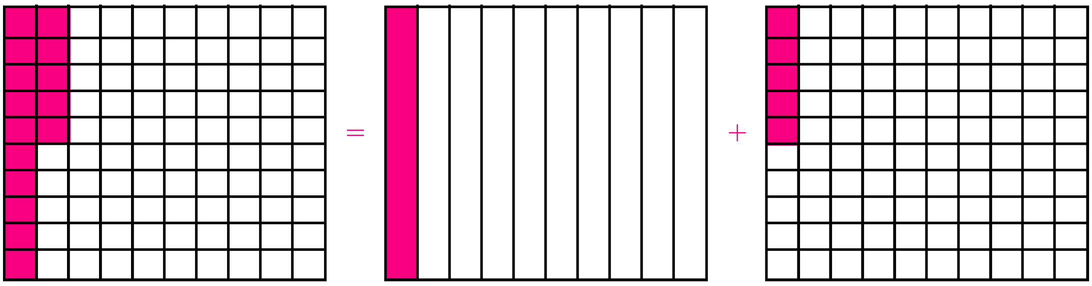


35 parts out of 100 equal parts is $\frac{35}{100}$ or 0.35.

0.35 is 35 hundredths

We read 0.01 as zero point zero one, 0.35 as zero point three five.

0.15 is 15 hundredths or 1 tenth 5 hundredths.



$$0.15 = \frac{15}{100} = \frac{10}{100} + \frac{5}{100} = \frac{1}{10} + \frac{5}{100}$$

Similarly: $5\frac{87}{100} = 5.87 = 5 + \frac{8}{10} + \frac{7}{100}$

$6\frac{42}{100} = 6.42 = 6 + \frac{4}{10} + \frac{2}{100}$

Number of digits after the decimal point is called the number of decimal places.

So, a decimal number has two parts – the whole number part and the decimal part separated by a decimal point.

Whole number part greater than or equal to "0" → **4.73** ← Decimal part is always less than one

↑ Decimal point.

In the above number, there are 2 decimal places after the whole number part. Whenever there is no whole number part, that can be represented with zero.

Ex: $\boxed{.69}$ will be written as $\boxed{0.69}$

Thousandths :

If we divide a whole into thousand equal parts, then each part of the whole represents one thousandth.

Thus, $\frac{1}{1000}$ = 0.001 or one thousandth. *It is read as zero point zero zero one.*

The decimal 0.235 = (235 thousandths) represents 235 parts out of 1000 parts.

$$\text{Also } 0.235 = \frac{2}{10} + \frac{3}{100} + \frac{5}{1000}$$

Similarly, 47.107 is forty seven and one tenths seven thousandths and is read as fourty seven point one zero seven.

$$47.107 = 47 + \frac{1}{10} + \frac{0}{100} + \frac{7}{1000}$$

Notice the similar patterns,

$\frac{4}{1\underline{0}} = 0.\underline{4}$ One digit

$\frac{2}{1\underline{00}} = 0.\underline{02}$ two digits

$\frac{429}{1\underline{000}} = 0.\underline{429}$ three digits

$\frac{25}{1\underline{0000}} = 0.\underline{0025}$ four digits

The number of digits after the decimal point in the decimal numeral is equal to the number of zeros after 1 in the denominator of the corresponding common fraction.

CHECK YOUR PROGRESS

Fill in the blanks:

S.No	Fraction	Decimal Number	Read as
1	$\frac{6}{10}$	0.6	Zero point six
2	$\frac{37}{100}$		
3		0.721	
4			Seventeen point two

5.4.1 Place value of Decimals:

The place value chart shown below can also be used to understand decimals. The place value of each digit in the number 3333.333 is shown below.

Digits	3	3	3	3	3	3	3
Place	1000 Thousands	100 Hundreds	10 Tens	1 Ones	$\frac{1}{10}$ Tenths	$\frac{1}{100}$ Hundredths	$\frac{1}{1000}$ Thousandths
Place Value	3000	300	30	3	$\frac{3}{10}$	$\frac{3}{100}$	$\frac{3}{1000}$

Notice that each place has a value one tenth $\frac{1}{10}$ that of the place to its left. The first place to the right of the decimal point is the tenths place, the second place to the right is the hundredths place and so on. The third place after the decimal point tells us how to name the decimal part. Here, it is 333 thousandths.

Using the place value chart, we can write the expanded form of any decimal number. Take the number 5876. 396.

5 8 7 6 . 3 9 6

5×1000 $\quad 8 \times 100$ $\quad 7 \times 10$ $\quad 6 \times 1$ $\quad 3 \times \frac{1}{10}$ $\quad 9 \times \frac{1}{100}$ $\quad 6 \times \frac{1}{1000}$

5876.396 = 5 thousands + 8 hundreds + 7 tens + 6 ones + 3 tenths + 9 hundredths + 6 thousandths.

$$= 5 \times 1000 + 8 \times 100 + 7 \times 10 + 6 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 6 \times \frac{1}{1000}$$

$$5876\frac{396}{1000} = 5000 + 800 + 70 + 6 + \frac{3}{10} + \frac{9}{100} + \frac{6}{1000} \text{ [fractional form]}$$

$$= 5000 + 800 + 70 + 6 + 0.3 + 0.09 + 0.006 \text{ [Decimal form]}$$

CHECK YOUR PROGRESS

1. Write the place value of the circled digits.

a) 13.26 (6 circled) b) 45.42 (4 circled) c) 15.023 (5 circled)

d) 208. 36 (2 circled) e) 87.243 (4 circled) f) 9.803 (3 circled)

2. Write in decimal form.

a) $700 + 40 + 2 + \frac{1}{10} + \frac{3}{100} + \frac{6}{1000}$

b) $9000 + 800 + 3 + 0.2 + 0.05 + 0.007$

c) $6000 + 400 + 20 + 1 + \frac{2}{10} + \frac{5}{100} + \frac{9}{1000}$

d) $400 + 5 + \frac{1}{10} + \frac{8}{100}$

3. Expand the following into decimal and fractional forms.

a) 164. 238 b) 968.054

5.4.2 Converting fractions into decimals and vice versa:

Fractions whose denominators are 10, 100 or 1000 can be easily converted to decimals by putting the decimal point in the numerator accordingly.

$\frac{6}{10} = 0.6$	$\frac{6}{100} = 0.06$	$\frac{6}{1000} = 0.006$
$\frac{34}{10} = 3.4$	$\frac{34}{100} = 0.34$	$\frac{34}{1000} = 0.034$
$\frac{123}{10} = 12.3$	$\frac{123}{100} = 1.2$	$\frac{143}{1000} = 0.143$
If denominator is10, number of decimal places is 1	*If denominator is100, number of decimal places is 2*	*If denominator is1000, number of decimal places is 3*

For fractions which can be converted into equivalent fractions having denominator 10 or multiples of 10, we apply the same method as above.

5.4.3 Conversion of Simple Fractions into Decimal Fractions

$$\frac{1}{2} = \frac{5}{10} = 0.5 \qquad \frac{1}{4} = \frac{25}{100} = 0.25 \qquad \frac{8}{5} = \frac{16}{10} = 1.6$$

Similarly, $2\frac{4}{5} = 2 + \frac{4}{5} = 2 + \frac{8}{10} = 2 + 0.8 = 2.8$

Decimals can also be converted to fractions as under:

$$0.6 = \frac{6}{10} = \frac{3}{5} \qquad 0.48 = \frac{48}{100} = \frac{12}{25}$$

CHECK YOUR PROGRESS

1.Write fractions as decimals.

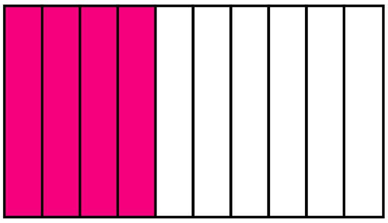
1) $\frac{23}{10} = \boxed{\quad}$ 2) $\frac{6}{100} = \boxed{\quad}$ 3) $\frac{3}{8} = \boxed{\quad}$ 4) $\frac{2}{25} = \boxed{\quad}$

2. Write decimals as fractions in simplest form.

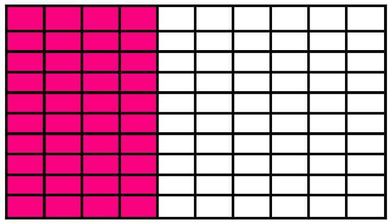
1) $0.2 = \boxed{\quad}$ 2) $0.38 = \boxed{\quad}$ 3) $1.62 = \boxed{\quad}$ 4) $8.1 = \boxed{\quad}$

5.4.4 Equivalent Decimal Fractions

Decimals that have the same part are called equivalent decimals.



4 tenths = 0.4



40 hundredths = 0.40

From the above figures, it is observed that both the shaded parts are equal.

Therefore, $\frac{4}{10} = \frac{40}{100} = \frac{400}{1000}$. Thus, 0.4 = 0.40 = 0.400 = 0.4000 etc.,

This shows that, writing zeros at right-hand end of a decimal number does not change its value.

1.2 = 1.20 = 1.200, 64.27 = 64.270 = 64.2700….

5.4.5 Like and unlike decimal fractions:

- ***Decimals having the same number of decimal places are called like decimals.***

2.7, 1.8, 7.4	1.36, 34.72, 0.07	146.002, 0.729, 1.765
1 decimal place	*2 decimal places*	*3 decimal places*

- ***Decimals having different number of decimal places are called unlike decimals.***

1.7, 4.07, 0.642 are all *unlike decimals.*

Unlike decimals can be converted into like decimals by converting them into their equivalent decimals.

Example-11: Convert 3.6, 235, 0.472 to like decimals.

Solution: The greatest number of decimal places is 3. So, we convert all of them into equivalent decimals with 3 decimal places.

3.6 = 3.600   2.35 = 2.350   0.472

Thus, 3.6, 2.35, 0.472 when converted into like decimals becomes 3.600. 2.350, 0.472

5.4.6 Comparing and ordering decimal fractions.

To compare decimal numbers, proceed as follows.

Step:1 Convert into like decimals.

Step:2 First compare the whole number parts. The number with the greater whole number part is greater.

Step:3 If the whole number parts are the same, compare the tenth's digits. The decimal number having greater tenth's digit is the greater number.

Step:4 If the tenth's digits are also the same, compare the hundredths digits and so on.

Example-12: Compare 5.623 and 5.64

Solution: The numbers are 5. 6 $\underline{2}$ 3

5. 6 $\underline{4}$ 0

Compare digits from left, you see that the hundredths digit differ.

Since 2 hundredth < 4 hundredths. i.e., 5.623 < 5.64

Example-13: Arrange 24.117, 24.118, 29.421 in desending order.

Solution: Comparing whole number parts, you see that 29.421 is greatest.

Compare, 24.117

24. 118

For whole number parts, tenths and hundredths are same. We have to compare thousndths place digits.

7 thousandths < 8 thousandths. So, 24.118 > 24.117

Descending order of given numbers are, 29.421, 24.118, 24.117

Exercise - 5.4

1) Which of the following are unlike decimal fractions ?

(i) 5.03, 6.185 (ii) 42.7, 7.42

(iii) 16.003, 5.301 (iv) 15.81, 1.36

2) Change the following into like decimal fractions.

(i) 0.802, 54.32, 873.274 (ii) 4.78, 9.193, 11.3 (iii) 16.003, 16.9, 16.19

3) Write the following in ascending order.

a) 7.26, 7.62, 7.2 b) 0.464, 0.644, 0.446, 0.664 c) 30.000, 30.060, 30.30

4) Arrange these numbers in descending order.

16.96, 16.42, 16.3, 16.03, 16.1, 16.99, 16.01

5) Fill in the blanks by using appropriate symbols > , < or = .

(i) 0.005___ 0.0005 (ii) 4.353___ 4.2 (iii) 58.3 ___ 58.30

5.5 Addition and subtraction of decimal fractions

The addition and subtraction of decimal fractions can be done in a step wise process.

Step:1 Convert the given decimals into like decimals.

Step:2 Write the decimals in column with the decimal points directly below each other. So, that tenths come under tenths, hundredths come under hundredths and so on.

Step:3 Add or subtract as well as whole numbers.

Step:4 Place the decimal point in the answer directly below.

Example-14: Add 53.08, 5.936. 188.5

Solution: Convert the decimals into like decimals with 3 decimal places and add.

53.08 = 53.080, 5.936, 188.5 = 188.500

$$\begin{array}{r} 53.080 \\ 5.936 \\ (+)\,188.500 \\ \hline \textit{Answer: } \mathbf{247.516} \\ \hline \end{array}$$

Example-15: Sekhar travelled 6 km 40m by bus, 3km 320m by car and the rest 1km 30m on foot. How much distance did he travel in all?

Solution : Distance travelled by bus = 6km 40m = 6.040km

Distance travelled by car = 3km 320m = 3.320km

Distance travelled on foot = 1km 30m = 1.030km

Total distance travelled = 6.040 + 3.320 + 1.030 = 10.390

Therefore, total distance travelled = 10.390km

Example-16: Kavya bought vegetables weighing 10kg Out of this 3kg 500g is onions, 2kg 75 g tomatoes and rest is potatoes. What is the weight of potatoes?

Solution: Weight of onions = 3kg 500g = 3.500 kg

Weight of tomatoes = 2kg 75g = 2.075 kg

Total weight of onion and tomato = 3.500 + 2.075 **= 5.575**

Total weight of vegetables = 10kg

Weight of potatoes = 10.000 kg – 5.575 kg

= 4.425 kg.

5.5.1 Use of decimal fractions – Applications:

Decimal fractions are used in expressing ***Money, Distance, Length, Weight*** *and* ***Capacity***.

Money: 1₹ = 100 Paisa

Therefore, 1Paisa = $\frac{1}{100}$ rupee = 0.01 rupee = ₹ 0.01

We write,

5 Rupees 60 Paisa = ₹ 5 + ₹ 0.60 = ₹ 5.60

We read,

₹ 8.55 as Eight Rupees Fifty Five Paisa

Length: 100 cm = 1m

1cm = $\frac{1}{100}$ m = 0.01 m   25cm = $\frac{25}{100}$m = 0.25m

We write,

5 metres, 60 centimetre as = 5m + 0.60m = 5.60 m

1000 metres = 1 kilometre

1 m = $\frac{1}{1000}$ km = 0.001 km   10m = $\frac{10}{1000}$km = 0.01 km

We write,

8km 460m as 8.460 km; 3km 289m as 3.289 km

Weight: 1000 grams = 1 Kilogram

1 g = $\frac{1}{1000}$ kg = 0.001 kg; 5g = $\frac{5}{1000}$kg = 0.005 kg

We write,

(i) 7 kg 450 g = 7.450 kg (ii) 75 kg 90g = 75.090kg

We read,

(i) 10.004 kg as Ten Kilograms Four Grams

(ii) 7.230 kg as Seven Kilograms Two Hundred Thirty Grams.

Capacity: 1000 millilitres = 1 Litre

1 ml = $\frac{1}{1000}$ = 0.001 *l*   25 ml = $\frac{25}{1000}$ = 0.025 *l*

We write,

(i) 5*l* 420 m*l* = 5.420 *l*   (ii) 15*l* 56 m*l* = 15.056 *l*

We read,

(i) 8.070 Litre as Eight Litres Seventy Millilitres.

(ii) 12.300 Litre as Twelve Litres Three Hundred Millilitres.

Exercise - 5.5

1) Add the following:

(i) 5.702, 5.2, 6.04 and 2.30 (ii) 40.004, 44.444, 40.404 and 4.444

2) Do the following:

(i) 426.326 – 284.482 (ii) 5 – 3.009 (iii) 2.107 – 0.31

3) Akshara bought 3m 40cm cloth for her shirt and 1m 10cm cloth for skirt. Find the total cloth bought by her.

4) Write in decimals using the units written in brackets.

i) 90 rupees 75 paisa (₹) *ii)* 49m 20cm (m) *iii)* 12kg 450g (kg) *iv)* 50*l* 500 ml(l)

5) Convert into decimals and add.

(i) 58kg 100g ; 60kg 350 g (ii) 80m 15 cm ; 72m 30 cm

6) Convert into decimals and subtract.

(i) 14 kg 720g from 16 kg 744g *(ii)* 11 12 ml from 21 20 ml

Unit Exercise

1. The sum of two fractions is $5\frac{3}{9}$. If one fraction is $2\frac{3}{4}$, then find the other fraction.
2. A rectangle sheet of paper is of length $12\frac{1}{2}$ cm and breadth $10\frac{2}{3}$ cm. Find its perimeter.
3. Simplify $\left(3\frac{1}{6}-1\frac{1}{3}\right)+\left(4\frac{1}{6}-2\frac{1}{3}\right)$.
4. By what number should $3\frac{1}{16}$ be multiplied to get $9\frac{3}{16}$?
5. The length of the staircase is $5\frac{1}{2}$ m. If one step is set at $\frac{1}{4}$ m, then how many steps will be there in the staircase?
6. Simplify 23.5 – 27 + 35.4 – 17.
7. Sailaja bought 3.350kg of Potatoes, 2.250kg of Tomatoes and some onions. If the weight of the total items are 10.250kg then find the weight of onions.
8. What should be subtracted from 7.1 to get 0.713?

Points to Remember

1) $Fraction \times Fraction = \dfrac{\text{Product of Numerators}}{\text{Product of Denominators}}$

2) $\dfrac{1}{a}$ of b means $\dfrac{1}{a} \times b$

3) Multiplication of fractions

- It we multiply two proper fractions then their product will be less than the given fractions.
- If we multiply two improper fractions, then their product will be greater than the given fractions.
- The product of proper and an improper fraction is less than the improper fraction and greater than the proper fraction.

4) If we have two non-zero number fractions whose product is one, these fractions are said to be reciprocal to each other.

5) Division of Fractions

i) If we have to divide the whole number with a proper or an improper fraction, then we will multiply that whole number with the reciprocal of the given fraction.

ii) If we have to divide the whole number with the mixed fraction, then we will convert into improper fraction and then multiply its reciprocal with the whole number.

iii) To divide the fraction with a whole number, we have to take the reciprocal of the whole number and then divide it with the whole number as usual.

iii) To divide a fraction with another fraction, we have to multiply the first fraction with the reciprocal of the second fraction.

6) Decimal Numbers:-

Fractions which has denominator 10, 100, 1000 .... etc. are called decimal fractions.

A decimal number is a number with a decimal point.

Addition and subtraction of decimal fractions.

Uses of decimal fractions.

T6Z5Z3

CHAPTER 6

Basic Arithmetic

Learning Outcomes:-

The students are able to

- explain the concept and terms of ratio.
- recall the symbol of ratio and express a ratio in its simplest form.
- explain the concept and terms of proportion.
- read the symbol of proportion.
- use the principle of proportion involving the ratios in solving problems.
- use unitary method in solving various daily life problems.
- explain the concept of percentage.
- convert percentage into other forms.

L3J7C6

Content Items

6.0 Introduction

Generally, a comparison is done in two ways.

(1) By finding the difference in the magnitudes of two quantities. This is known as the ***comparison by difference.***

For example, Ramu and Siva are brothers. Their weights are respectively 56 kg and 48 kgs. Then, we say that Ramu's weight is greater than Siva's weight by 8 kg. We compared the weights of Ramu and Siva by finding difference between them. Such a comparison is known as the comparison by difference.

(2) ***Comparison by division.***

For example, a car costs ₹ 6.6 lakhs and another car costs ₹ 26.4 lakhs. If we calculate the difference between the prices, it is ₹ 19.8 lakhs and if we compare by division, the cost of second car is 4 times the cost of the first one.

Thus, in certain situations, ***"Comparison by division makes better sense than comparison by taking the difference".***

6.1 Ratio

What is a ratio ? It is a way to compare two quantities of same kind. When we compare two quantities of the same kind by division, we say that we form a ratio of those two quantities.

Thus, we say that the ratio of the weight of Ramu to that of Siva is $\frac{56}{48}$.

We use the symbol ":" (isto) to express a ratio.

$\therefore$ The ratio of the weight of Ramu that of Siva is $\frac{56}{48}$. It can also be written as 56 : 48 and read as "56 is to 48" or "56 to 48".

The ratio of two numbers 'a' and 'b' (b $\neq$ 0) is a $\div$ b or $\frac{a}{b}$ and is denoted by a : b (a is to b).

In the ratio a : b, the quantities (numbers) 'a' and 'b' are called terms of the ratio.

'a' is called the "first term" or "antecedent" and 'b' is called the "second term" or "consequent".

We know that a fraction does not change when its numerator and denominator are multiplied or divided by the same non-zero number. So, a ratio does not alter, if its first and second terms are multiplied or divided by the same non-zero number.

$60 : 70 = 60 \times 2 : 70 \times 2 = 120 : 140$ (Multiplying the first and second terms by 2)

$56 : 48 = 56 \div 8 : 48 \div 8 = 7 : 6$ (Dividing the first and second terms by 8)

6.1.1 Ratio in the simplest form

A ratio a : b is said to be in its simplest form if its antecedent 'a' and consequent 'b' have no common factors other than '1'. A ratio in the simplest form is also called the ratio in the lowest terms. Usually, ***a ratio is expressed in its simplest form.***

The ratio 56 : 48 is not in the simplest form, because '8' is a common factor to both the terms. The simplest form of this ratio is 7 : 6 (Dividing the first and second terms by 8).

Note-1 : To find the ratio of two quantities, they must be expressed in the same units.

Note-2 : Ratio has no units or it is independent of the units used in the quantities compared.

Note-3 : The order of the terms in a ratio a : b is very important. The ratio 3 : 4 is different from the ratio 4 : 3.

Consider the ratio 8 : 3.

We know that multiplying "a fraction by a number does not change the value of the fraction".

$$8 : 3 = \frac{8}{3}$$

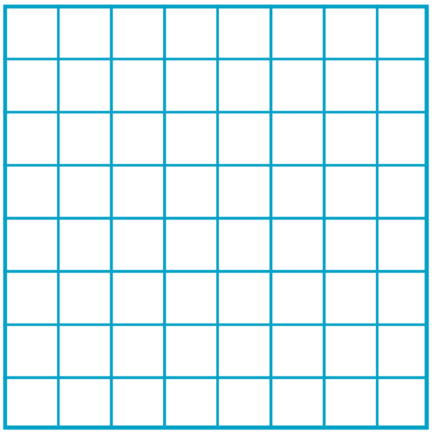
Multiply with 2. We have $\frac{8}{3} = \frac{8 \times 2}{3 \times 2} = \frac{16}{6}$

Multiply with 5. We have $\frac{8}{3} = \frac{8 \times 5}{3 \times 5} = \frac{40}{15}$

Thus, $\frac{16}{6}$, $\frac{40}{15}$, etc. are ratios equivalent to the ratio 8 : 3. What is your observation ?

"A ratio obtained by multiplying or dividing the antecedent and consequent of a given ratio by the same number is called an equivalent ratio".

LET'S DO ACTIVITY



Take a square ruled paper. Throw a dice and note the number on the dice.

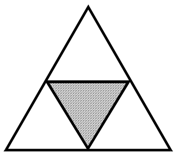
Fill that number of squares with your favourite colour.

Ask your friend to throw the dice and colour as many squares as the number on the dice with some other colour.

1. Find the ratio of those differently coloured squares.
2. Find the ratio of number of squares coloured by you to the total number of squares coloured.
3. Find the ratio of number of squares coloured by your friend to the total number of squares coloured.
4. Can you find any other ratios in this activity ? Think and discuss with your friend.

CHECK YOUR PROGRESS

1. Express the terms 45 and 70 by using ratio symbol.
2. Write antecedent in the ratio 7 : 15.
3. Write the consequent in the ratio 8 : 13
4. Express the ratio 35 : 55 in the simplest form.
5. In the given figure, find the ratio of
   (i) Shaded part to unshaded parts.
   (ii) Shaded part to total parts.
   (iii) Unshaded parts to total parts.



6. Express the following in the form of ratio.
   (a) The length of a rectangle is triple its breadth.
   (b) In a school, the workload of teaching 19 sections has been assigned to 38 teachers.

Example-1 : Express the terms 150 and 400 in simplest form by using ratio symbol.

Solution : Given numbers are 150 and 400.

Ratio of the given numbers = 150 : 400 = 15 : 40 = 3 : 8

$\therefore$ Simplest form of the ratio = 3 : 8

Alternate Method :

Given numbers are 150 and 400.

Comparison by division = $\frac{150}{400} = \frac{15}{40} = \frac{3}{8}$

$\therefore$ Simplest form of the ratio of the given numbers = 3 : 8

Example-2 : Express the following ratios in their simplest form.

(i) 28 : 84 (ii) 250 grams to 5 kgs

(iii) 24 minutes to 3 hours (iv) 200 ml. to 3 litres

Solution : (i) Given 28 : 84

28 $= 1 \times 28$

$= 2 \times 14$

$= 4 \times 7$

Factors of 28 are 1, 2, 4, 7, 14, 28

84 $= 1 \times 84$

$= 2 \times 42$

$= 3 \times 28$

$= 4 \times 21$

$= 6 \times 14$

$= 7 \times 12$

Factors of 84 are 1, 2, 4, 6, 7, 12, 14, 21, 28, 42, 84.

Common factors of 28 and 84 are 1, 2, 4, 7, 14, 28

$\therefore$ H.C.F. of 28 and 84 is 28

So, divide both terms by their HCF (28). Then, we get

$28 \div 28 : 84 \div 28 = 1 : 3$

or

Given $= 28 : 84$

$= 14 : 42$ (dividing both terms by 2)

$= 7 : 21$ (dividing both terms by 2)

$= 1 : 3$ (dividing both terms by 7)

$\therefore$ Required ratio $= 1 : 3$

(ii) Given 250 grams to 5 kgs

1 kg $=$ 1000 grams

250 grams to 5 kgs $= 250 : 5 \times 1000$

$= 250 : 5000$

$= 25 : 500$

$= 5 : 100$

$= 1 : 20$

$\therefore$ Required ratio $= 1 : 20$

(iii) 24 minutes to 3 hours

1 hour $=$ 60 minutes

3 hours $= 3 \times 60 = 180$ min.

24 minutes to 3 hours $= 24 : 180$

$= 12 : 90$

$= 4 : 30$

$= 2 : 15$

(iv) 200 ml. to 3 litres

We know that 1 litre $=$ 1000 ml.

$\therefore$ 3 litres $= 3 \times 1000 = 3000$ ml.

200 ml. to 3 litres $= 200 : 3000$

$= 2 : 30$

$= 1 : 15$

Example-3 : Find the ratio of the price of coffee to that of tea when coffee costs ₹ 36 per 100 gm and tea costs ₹ 240 per half kg.

Solution : In order to compare the price of coffee with that of tea, we must first find the cost of same quantity of each of them. Find the cost of 1 kg of each of the two items.

We have 1 kg $=$ 1000 gr $=$ 10×100 gr

Cost of 100 gr coffee $=$ ₹ 36

$\therefore$ Cost of 1 kg coffee $=$ ₹ $36 \times 10 =$ ₹ 360

It is given that the cost of $\frac{1}{2}$ kg tea $=$ ₹ 240

$\therefore$ Cost of 1 kg tea $= 2\ (240) =$ ₹ 480

Prices of coffee to tea $=$ Cost of 1 kg Coffee : Cost of 1 kg tea

$= 360 : 480$

$= 36 : 48$

$= 9 : 12$

$= 3 : 4$

6.1.2 Comparison of Ratios :

How to compare two or more given ratios ? Let us observe the following example.

Example-4: Compare the two ratios 5 : 8 and 2 : 9.

Solution : Writing the given ratios as fractions, we have $5 : 8 = \frac{5}{8}$ and $2 : 9 = \frac{2}{9}$.

Now, find the L.C.M. of 8 and 9, which is equal to $8 \times 9 = 72$.

Making the denominator of each fraction equal to 72, we have

$$\frac{5}{8} = \frac{5\times 9}{8\times 9} = \frac{45}{72} \text{ and } \frac{2}{9} = \frac{2\times 8}{9\times 8} = \frac{16}{72}$$

Clearly, $45 > 16 \therefore \frac{45}{72} > \frac{16}{72}$

This follows that $\frac{5}{8} > \frac{2}{9}$ or $(5:8) > (2:9)$

Now we can generalize the steps (Algorithm)

1. Write the given ratios.
2. Express each of them in the form of a fraction in the simplest form.
3. Find the L.C.M. of denominators of the two fractions.
4. Make the denominator of each fraction equal to the obtained L.C.M. For this multiply both numerator and denominator with appropriate number.
5. Now compare the numerators of the fractions obtained above. The fraction having larger numerator will be larger than other.

Example-5: Divide ₹ 5,600 in the ratio 3 : 4 between Lalitha and Sekhar.

Solution: Given amount = ₹ 5,600

Given ratio = 3 : 4

Sum of the terms in the ratio = 3 + 4 = 7

Lalitha's share $= \frac{3}{7} \times 5600 = 3 \times 800 =$ ₹ 2400

Sekhar's share $= \frac{4}{7} \times 5600 = 4 \times 800 =$ ₹ 3200

Example-6: Find two equivalent ratios of 6 : 15

Solution : Given ratio $= 6 : 15 = \frac{6}{15}$

Multiplying both terms by 3, we get

$$\frac{6}{15} = \frac{6 \times 3}{15 \times 3} = \frac{18}{45} = 18:45$$

Dividing both terms by 3, we get

$$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5} = 2:5$$

Two equivalent ratios are 18:45 and 2:5.

Example-7: Fill in the boxes with suitable numbers.

(i) $\frac{12}{18} = \frac{\square}{36} = \frac{2}{\square}$ (ii) $\frac{16}{\square} = \frac{\square}{5} = \frac{8}{10}$

Solution : (i) In oder to find the first missing number, we consider the denominators 18 and 36.

We know that $18 \times 2 = 36$

So, we multiply the numerator also with 2, We get $12 \times 2 = 24$.

$\therefore$ First missing number in the box is 24.

For the second missing number, we consider the numerators 12 and 2

$12 \div 2 = 6$

So, we divide the denominator 18 by 6 to get the second missing number.

i.e $\quad 18 \div 6 = 3$

$\therefore$ Second missing number in the box is 3.

(ii) Given $\dfrac{16}{\square} = \dfrac{\square}{5} = \dfrac{8}{10}$

Compare $\dfrac{16}{\square} = \dfrac{8}{10}$

Now consider the numerators 16 and 8

We get 16 by multiplying 8 with 2. $8 \times 2 = 16$

So, $10 \times 2 = 20$ is the first missing number.

Second missing number is 4. How ? Explain.

"In these problems what principle is used in finding the missing number ?"

- Which ratio is larger in the following pairs ?

  (a) 5 : 4 or 9 : 8 (b) 12 : 14 or 16 : 18

  (c) 8 : 20 or 12 : 15 (d) 4 : 7 or 7 : 11

- Find three equivalent ratios of 12 : 16

Exercise - 6.1

1. Express the following in the terms of ratios.

   (i) The length of a rectangle is 5 times to its breadth.

   (ii) For preparing coffee, 2 cups of water require to 1 cup of milk.

2. Express the following in the simplest form

   (i) 24 : 9 (ii) 144 : 12 (iii) 961 : 31 (iv) 1575 : 1190

3. Write the antecedents and consequents of the following ratios

   (i) 36 : 73 (ii) 65 : 84 (iii) 58 : 97 (iv) 69 : 137

4. Find the ratios of the following in their simplest form.

   (i) 25 minutes to 55 minutes (ii) 45 seconds to 30 minutes

   (iii) 4 m. 20 cm to 8 m. 40 cm (iv) 5 litres to 0.75 litre

   (v) 4 weeks to 4 days (vi) 5 dozen to 2 scores (1 score = 20 items)

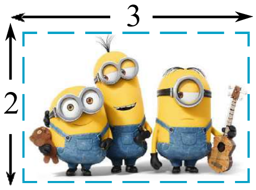
5. Rahim works in a software company and earns ₹.75,000/- per month. He saves ₹.28,000/- per month from his earnings. Find the ratio of

   (i) His savings to his income (ii) His income to his expenditure

   (iii) His savings to his expenditure

6.2 Proportion

Observe the figures. What difference do you find in the figures?



Original Figure

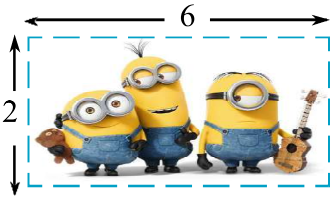


Fig:I

Fig:II

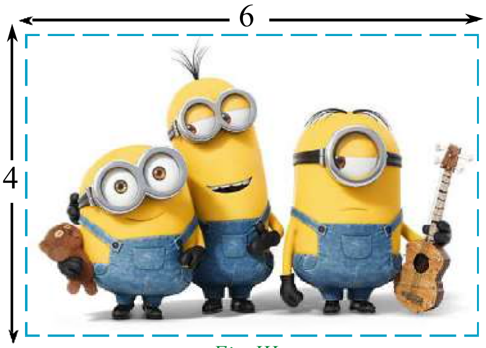


Fig:III

The Fig (i) and (ii) look different and their shapes have changed. Fig (iii) is enlarged but it looks alike. This is because there is a change in the size, but not in the shape.

Fill the table by observing the above pictures.

Picture	Length	Breadth	Ratio of length to breadth	Simplest form
Original	3	2	3 : 2	3 : 2
Fig (i)	6	2	6 : 2	3 : 1
Fig (ii)	3	4	3 : 4	3 : 4
Fig (iii)	6	4	6 : 4	3 : 2

Which figure is good to see ? Why ?

By observing the above table what can you say ? Which fig. ratio is equal to the original fig. ratio ? Ratio of the simplest form in fig. (iii) is equal to the ratio of the original figure.

We can say that fig. (iii) is proportionate to the original fig. and that's why their ratios are same. ***The equality of ratios is called proportion.***

In general, ***if the ratio of 'a' and 'b' is equal to the ratio of 'c' and 'd', we say that they are in proportion. This is represented as a : b :: c : d (read as 'a' is to 'b' is as 'c' is to 'd' or a to b as c to d) or a : b = c : d.***

HISTORICAL NOTE

The symbol :: (is as to) is introduced by "Oughtrad" in 1628.

Observe the following example.

Stalin sells 2 kg of oranges for ₹ 160 and Krishna sells 5 kg of oranges for ₹ 400. Whose oranges are more expensive ?

Let us find their ratios

	No. of Kgs sold	Price in ₹	Weights ratio	Price ratio
Stalin	2	160	2 : 5	160 : 400 = 16 : 40 = 2 : 5
Krishna	5	400		

From the above table what do you observe ?

Ratio of the weight of oranges = ratio of their cost.

$\therefore$ We can say that 2, 5, 160, 400 are in proportion.

It is written as 2 : 5 :: 160 : 400.

In this proportion, 2 and 400 are called extremes, where as 5 and 160 are called means.

Let us examine.

What is the product of extremes ? $2 \times 400 = 800$

What is the product of means ? $5 \times 160 = 800$

What is your observation?

Product of extremes = Product of means

a, b, c, d are in proportion. i.e. a : b :: c : d.

'a' and 'd' are extremes and 'b' and 'c' are means.

$$\therefore \boxed{a \times d = b \times c}$$

Product of extremes = Product of means

Also we say that ***if the product of extremes is equal to the product of means, then these four numbers are in proportion.***

• Check whether the following terms are in proportion ?

1) 5, 6, 7, 8 2) 3, 5, 6, 10 3) 4, 8, 7, 14 4) 2, 12, 3, 18

Example-8: Find the missing number in the following proportions.

(a) $15 : 19 = \boxed{} : 57$ (b) $13 : \boxed{} = 26 : 36$

Solution: Since given numbers are in proportion, product of extremes = product of means

(a) Let the missing number $= x$, then

$$15 : 19 = x : 57$$

$$15 \times 57 = 19 \times x \text{ or } 19x = 15 \times 57$$

$$\therefore x = \frac{15 \times 57}{19} = 15 \times 3 = 45$$

$\therefore$ Missing number in the proportion = 45

(b) Let the missing number = y, then

$$13 : y = 26 : 36$$

$$13 \times 36 = y \times 26$$

$$\therefore\ y = \frac{13 \times 36}{26} = \frac{36}{2} = 18$$

$\therefore$ Missing number in the proportion = 18

Example-9: Venkat sells 25 kgs of rice for ₹ 1200. Rahim sells 75 kgs of rice for ₹ 3,600. Are the ratios in proportion ?

Solution :

Method-1 Ratio of weights of rice = 25 : 75 = 1 : 3

Ratio of cost of rice = 1200 : 3600 = 12 : 36 = 1 : 3

Both ratios are equal, so given ratios are in proportion.

Method-2 Here two ratios are 25 : 75 and 1200 : 3600

Product of extremes = 25×3600 = 90,000

Product of means = 75×1200 = 90,000

Since product of extremes = product of means,

25, 75, 1200, 3600 are in proportion.

Exercise - 6.2

1. Check whether the following are in proportion ? or not ?

(a) 10, 12, 15, 18 (b) 11, 16, 16, 21

(c) 8, 13, 17, 19 (d) 30, 24, 20, 16

2. Write true or false for each of the following.

(a) 4 : 2 :: 14 : 7 (b) 21 : 7 :: 15 : 5

(c) 13 : 12 :: 12 : 13 (d) 5 : 6 :: 7 : 8

3. Check whether the following form a proportion ? Write middle terms and extremes where the ratios form a proportion.

(a) 15cm, 1m and ₹ 45, ₹ 300 (b) 20ml, 2ℓ. and ₹ 100, ₹ 10,000

4. Find the missing numbers in the following proportions.

(a) 8 : 12 :: $\square$: 48 (b) 15 : $\square$ :: 105 : 98 (c) 34 : 102 :: 27 : $\square$

6.3 Unitary Method

Consider the following situations.

1. The cost of 5 pens is ₹ 300. What is the cost of 3 pens ?
2. A motor bike requires 3 litres of petrol to cover 135 km. How many litres of petrol is required to cover 90 km ?

In our daily life we face this kind of situations. How would you solve these ?

In situation (1), the cost of 5 pens (₹ 300) is given. We have to find out the cost of 3 pens.

First we find the cost of 1 unit (pen) by dividing the cost by the number of pens bought i.e., $\frac{300}{5}$ = 60. Cost of one pen is ₹ 60.

We multiply the cost of one unit by required number of units (3 pens) i.e., ₹ 60x3=₹ 180

In the above example, what we have done ? First, we find the cost of one unit and then find the required number of units. This method is known as unitary method.

"The method in which first we find the value of one unit and then the value of required number of units is known as unitary method".

LET'S EXPLORE

Read the table and fill in the boxes

Weight	Cost of Tomato	Cost of Potato
5 kg	₹ 75	₹ 60
1 kg	₹ 15	$\boxed{}$
3 kg	$\boxed{}$	$\boxed{}$

Prepare two similar problems and ask your friend to solve them.

Example-10: If the cost of 1dozen soaps is ₹ 306, what will be the cost of 15 such soaps ?

Solution : We know that 1 dozen = 12 items

$\therefore$ Cost of 12 soaps = ₹ 306

Cost of 1 soap = $\frac{306}{12}$ = ₹ 25.50

Cost of 15 soaps = 15 × 25.50

= ₹ 382.50

Example-11: If the cost of 24 pencils is ₹ 72, then find the cost of 15 pencils ?

Solution : Cost of 24 pencils $=$ ₹ 72

Cost of 1 pencil $= \frac{72}{24} =$ ₹ 3

Cost of 15 pencils $= 15 \times 3 =$ ₹ 45/-

Example-12: A car travels 175 km in $3\frac{1}{2}$ hours

(a) How much time is required to cover 75 km with the same speed ?

(b) Find the distance covered in 2 hours with the same speed.

Solution : Given that a car travels 175 km in $3\frac{1}{2}$ hours. i.e., $3\frac{1}{2}$ hours $= \frac{7}{2}$ hours

(a) 175 km of distance covered in $\frac{7}{2}$ hours

1 km of distance is covered in $\frac{1 \times \frac{7}{2}}{175}$ hrs $= \frac{7}{2 \times 175}$ hrs.

75 km of distance is covered in $75 \times \frac{7}{2 \times 175}$ hrs.

$= 3 \times \frac{7}{2 \times 7} = 3 \times \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$ hrs.

$\therefore$ 75 km of distance is covered in $1\frac{1}{2}$ hrs.

(b) In $3\frac{1}{2} = \frac{7}{2}$ hrs. distance covered $=$ 175 km

distance covered in 2 hrs $= \frac{2 \times 175}{\frac{7}{2}}$

$= \frac{2 \times 175 \times 2}{7}$

$= 2 \times 25 \times 2$

$=$ 100 km

distance covered in 2 hrs $=$ 100 km

Exercise - 6.3

1. If the cost of 3 apples is ₹ 60/-, then find the cost of 7 apples.
2. Uma bought 8 books for a total of ₹ 120. How much would she pay for just 5 books ?
3. The cost of 5 fans is ₹ 11,000/-. Find the number of fans that can be purchased for ₹ 4,400/-.
4. A car is moving at a constant speed covers a distance of 180 km in 3 hours. Find the time taken by the car to cover a distance of 420 km at the same speed.
5. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km ?

6.4 Percentage

Generally we observe the following in our daily life.

- A shop keeper offers as summer sale 25% flat discount on cotton clothes.
- This year our school SSC pass percentage is 92.
- Interest rate on home loans is increased to 9.75%.

The word ***'percent'*** means ***'out of hundred'***. You can therefore consider each 'whole' as broken up into 100 equal parts, each one of which is a single percent.

The symbol ***"%"*** is used to ***represent percent.***

1% (read as one percent) means 1 out of 100 and 29% (29 percent) means 29 out of 100.

We can represent percent as a fraction or a decimal or a ratio.

For example 1% $= \frac{1}{100}$ or 0.01 or 1 : 100 29% $= \frac{29}{100}$ or 0.29 or 29 : 100

• Represent the following in other forms.

Number	Percent	Fraction	Decimal	Ratio
67	67%			
17		$\frac{17}{100}$		
29			0.29	

HISTORICAL NOTE

Percentage is a term from Latin, meaning 'out of one hundred'.

How to convert a percent into a fraction ?

1. Obtain the given percent. Let it be *58%*.
2. Remove the % sign and divide the number by 100.

For example, $58\% = \frac{58}{100} = \frac{29}{50}$.

How to convert a fraction into a percent ?

1. Obtain the fraction. Let it be $\frac{8}{25}$.
2. Multiply the fraction by 100 and place the '%' symbol after it.

For example, $\frac{8}{25} = \frac{8}{25} \times 100 = 8 \times 4 = 32\%$

Example-13: Convert the following into other forms (i.e.) percentage form, fraction form, ratio form and decimal form.

(a) 55% (b) $\frac{4}{25}$ (c) 0.125 (d) $3\frac{3}{4}$ (e) 3 : 16

Solution : (a) $55\% = \frac{55}{100} = \frac{11}{20} \longrightarrow$ fraction form

$\frac{55}{100} = \frac{11}{20} = 11 : 20 \longrightarrow$ ratio form

$\frac{55}{100} = 0.55 \longrightarrow$ decimal form

$\therefore 55\% = \frac{11}{20} = 11 : 20 = 0.55$

(b) Given number $\frac{4}{25}$ is in fraction form

$\frac{4}{25} = \frac{4}{25} \times 100 = 4 \times 4 = 16\% \longrightarrow$ percentage form

$\frac{4}{25} = 4 : 25 \longrightarrow$ ratio form

$\frac{4}{25} = \frac{4 \times 4}{25 \times 4} = \frac{16}{100} = 0.16 \longrightarrow$ decimal form

$\therefore \frac{4}{25} = 16\% = 4 : 25 = 0.16$

(c) Given number 0.125 is in decimal form

$$0.125 = \frac{0.125 \times 1000}{1000} = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8} \longrightarrow \text{fraction form}$$

$$0.125 = \frac{1}{8} = \frac{1}{8} \times 100 = \frac{25}{2} = 12\frac{1}{2}\% \longrightarrow \text{percentage form}$$

$$0.125 = \frac{1}{8} = 1{:}8 \longrightarrow \text{ratio form}$$

$$\therefore 0.125 = \frac{1}{8} = 12\frac{1}{2}\% = 1:8$$

(d) Given number $3\frac{3}{4}$ is in fraction from

$$3\frac{3}{4} = \frac{15}{4} = 15:4 \longrightarrow \text{ratio form}$$

$$3\frac{3}{4} = \frac{15}{4} \times 100 = 15 \times 25 = 375\% \longrightarrow \text{percentage form}$$

$$3\frac{3}{4} = \frac{15}{4} = 3.75 \longrightarrow \text{decimal form}$$

$$\therefore 3\frac{3}{4} = \frac{15}{4} = 15:4 = 375\% = 3.75$$

(e) Given number3:16 is in ratio form

$$3:16 = \frac{3}{16} \longrightarrow \text{fraction form}$$

$$3:16 = \frac{3}{16} \times 100 = \frac{3}{4} \times 25 = \frac{75}{4} = 18.75\% \longrightarrow \text{percentage form}$$

$$3:16 = \frac{3}{16} = 0.1875 \longrightarrow \text{decimal form}$$

$$\therefore 3:16 = \frac{3}{16} = 18.75\% = 0.1875$$

Example-14: Find

(a) 24% of 25 kg (b) $5\frac{1}{2}$ of ₹ 2400

Solution:

(a) 24% of 25 kg $= \frac{24}{100} \times 25 = \frac{24}{4} = 6k.g$

(b) $5\frac{1}{2}$ of ₹ 2400 $= \frac{11}{2} \times \frac{1}{100} \times 2400 = 11 \times 12 =$ ₹132

Example-15: Express 12 hours as a percent of 4 days

Solution: We have, one day = 24 hours

4 days = 4×24 = 96 hours

Let $x\%$ of 4 days = 12 hours

$\Rightarrow$ $x\%$ of 96 hours = 12 hours

$$\Rightarrow \frac{x}{100} \times 96 = 12$$

$$\Rightarrow x = \frac{12 \times 100}{96} = \frac{100}{8} = \frac{25}{2} = 12\frac{1}{2}$$

Hence $12\frac{1}{2}\%$ of 4 days is equal to 12 hours.

Example-16: 60% of the population in Vemavaram village are females. If the total population in the village is 2,400, find the population of males in Vemavaram village.

Solution: Population in Vemavaram village = 2,400

60% of the population $= \frac{60}{100} \times 2{,}400 = 1{,}440$

$\therefore$ Number of females = 1,440.

Number of males = Total population – Number of females

= 2,400 – 1,440 = 960

Alternate method:

The population of Vemavaram = 2,400

The population of females in Vemavaram = 60%

i.e. the rest are males

i.e. 100% – 60% = 40% are males

The population of males = 40% of 2,400 = $\frac{40}{100} \times 2{,}400$ = $40 \times 24 = 960$

∴ The population of males in Vemavaram = 960

Exercise - 6.4

1. Express each of the following percents as fractions in the simplest form.

   (a) 15% (b) 35% (c) 50% (d) 75%

2. Express each of the following fractions as percents.

   (a) $\frac{15}{2}$ (b) $8\frac{1}{4}$ (c) $5\frac{3}{4}$ (d) $3\frac{1}{3}$

3. Express each of the following ratios as percents.

   (a) 3:5 (b) 5:8 (c) 2.5:55 (d) 4:36

4. Express each of the following percents as ratios in the simplest form.

   (a) 12% (b) 25% (c) 45% (d) 84%

5. Express each of the following percents as decimals.

   (a) 1% (b) 6% (c) 19% (d) 67%

6. Express each of the following decimals as percent.

   (a) 0.04 (b) 0.52 (c) 0.125 (d) 0.0006

7. Find the number, which is $12\frac{1}{2}$ % of 75.

8. Pavani secured 85% marks in mathematics paper. If maximum marks in the paper is 80, find the marks secured by her in that paper.

9. Siva spends 78% of his monthly income. If he saves ₹ 7,700/- per month, what is his monthly income?

Unit Exercise

1. Draw a four sided closed figure and divide it into some number of equal parts. Shade the figure with any colour so that the ratio of shaded parts to unshaded parts is 1:3.
2. Ramu spent $\frac{2}{5}$th of his money on a story book. Find the ratio of money spent to the money with him at present.
3. Divide ₹ 72,000 between Kesav and David in the ratio of 5:4.
4. The income of Kumar for 3 months is ₹ 15,000. If he earns the same amount for a month,
   (a) How much will he earn in 5 months?
   (b) In how many months, will he earn ₹ 95,000?
5. The cost of 16 chairs is ₹ 4,800. Find the number of chairs that can be purchased for ₹ 6,600.
7. What percentage of numbers from 1 to 30 has 1 or 9 in the unit's digit?
8. In a college, 63% of students are less than 20 years of age. The number of students more than 20 years of age is $\frac{2}{3}$ of number of students of 20 years of age which is 42. What is the total number of students in the college?

Points to Remember

R1U4P6

1. Comparison of two quantities of same kind is called ratio.

2. Ratio can be expressed by using the symbol ":" and read it as "is to"

3. First term of a ratio is called antecedent and second term is called consequent.

4. A ratio should be expressed in its simplest form.

5. Equality of two ratios is called proportion.

6. In the proportion a:b :: c:d, 'a' and 'd' are called extremes and 'b', 'c' are called means.

7. If four numbers are in proportion, then "the product of extremes is equal to the product of means".

8. The method in which first finding the value of one unit and then the value of required number of units is known as "Unitary method".

9. Percent means 'out of hundred'.

10. We use % symbol to represent percentage.

CHAPTER 7
Introduction to Algebra

C4S5K1

Learning Outcomes:-

The students are able to
- describe, create numeric and geometric patterns of numbers.
- identify the variables for different unknown quantities.
- understand the use of variables in different situations.
- communicate the real-life situations in the algebraic form and vice- versa.
- solve the equation for finding the solution in trial and error method.

7.0 Introduction

Now we begin the study of a branch of mathematics called Algebra. The main feature of algebra is the use of letters or alphabet to represent numbers. Letter can represent any number, not just a particular number. It may stand for an unknown quantity. By learning the method of determining unknowns we develop powerful tools for solving puzzles and many problems in our daily life.

Content Items

Consider the following

Damini and Koushik are playing a game.

Koushik : Hi, Let us play a game to find your age.

Dhamini : But you know my age...

Koushik : Ok, take the age of your friend, who is unknown to me. Do not reveal it.

Dhamini : Alright, I am ready.

Koushik : First, double the age.

Dhamini : Done.

Koushik : Add 5 to the result and tell me the result.

Dhamini : Ok, it is '27'.

Koushik : Good! Your friend's age is 11 years.

Dhamini was surprised. She thought for a while and said 'I know how you found the age'.

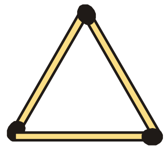
Do you know how it was done? You too can try!!!

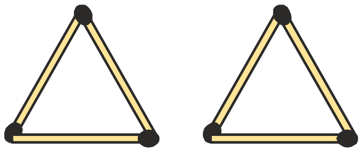
7.1 Patterns - Making Rules

Pattern-1

To make a triangle, 3 match sticks are needed.

To make two triangles, we need 6 match sticks.





Geetha prepared a table that gives the number of match sticks required and the number of triangles formed as shown below. Observe the pattern

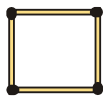
Number of triangles to be formed	1	2	3	4	5	6	...
Number of match sticks required	3	6	9	12	15	18	...
Observation (Pattern)	3×1	3×2	3×3	3×4	3×5	3×6	...

What is the relation between the number of triangles formed and the match sticks needed?

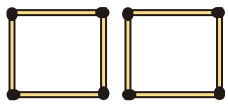
Number of match sticks required = 3 × the number of triangles to be formed.

Pattern-2

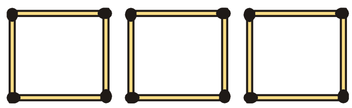
To make a square, 4 match sticks are needed.



We need 8 match sticks to make two squares



We need 12 match sticks to make three squares



Continue and arrange the information in the following table

Number of Squares to be formed	1	2	3	4	....
Number of match sticks required	4	8	12	16	....
Observation (Pattern)	4×1	4×2	4×3	4×4	....

i.e., number of match sticks required = 4 × number of squares to be formed.

7.2 Variable

Let us consider the table of pattern-1

Number of triangles to be formed	1	2	3	4	5	6	...
Number of match sticks required	3	6	9	12	15	18	...
Observation (Pattern)	3×1	3×2	3×3	3×4	3×5	3×6	...

We observe that,

Number of matchsticks required = 3 × No.of triangles to be formed.

From this, we can say that for 7 triangles we require 3 × 7 = 21 sticks.

• ***Can you continue in the similar way for any number of triangles?* Yes.**

If 'n' number.of triangles are to be formed, we need 3 × n matchsticks.

What is ***n*** ?

It may be any number taking values 1, 2, 3, 4, ...

Now the **rule for this pattern** can be taken as

Number of matchsticks needed = 3 × n, where n is the number of triangles to be formed.

Here, the value of **3** remains the same and **n** may be any number like 1, 2, 3, 4,.... In this rule **3** is **Constant.** As the value of **n** changes we call it as a **variable.**

Observe the ***pattern-2*** of squares. If x is the number of squares to be formed, than the number of matchsticks needed are **4** × x.

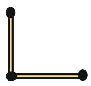
Can we say the rule for this pattern is **Number of matchsticks needed = 4 × x, where x is the number of triangles to be formed ?** **Yes, you are Right !**

Here **4** is **constant** and x is **variable.**

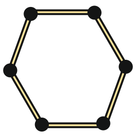
We use letters (small case of English alphabet) m, n, x, y, p, q, l etc., to denote **variables.**

A variable is a letter used to stand for a number. Observe that we have used letters x,n in the examples above to represent numbers.

LET'S EXPLORE

• Arrange 2 matchsticks to form the shape 

Continue the same shape for 2 times, 3 times and 4 times. Frame the rule for repeating the pattern.

• Rita took matchsticks to form the shape 

She repeated the pattern and gave a rule

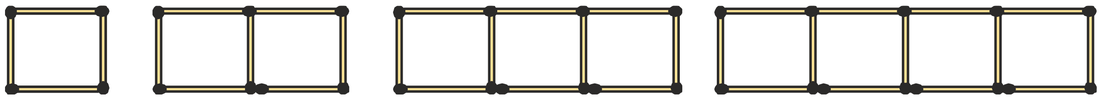
Number of matchsticks needed = 6.y, where y is the number of shapes to be formed.

Is it correct? Explain.

What is the number of sticks needed to form 5 such shapes?

More Patterns

Consider the match stick pattern.



Shape-1 S_1 Shape-2 S_2 Shape-3 S_3 Shape-4 S_4

Shape	S_1	S_2	S_3	S_4
No. of matchsticks	4	7	10	13

Observe the following

$S_1 = 4$

$S_2 = 7 \quad = 4 + 3 \quad$ or $\; S_1 + 3$

$S_3 = 10 \quad = 7 + 3 \quad$ or $\; S_2 + 3$

$S_4 = 13 \quad = 10 + 3 \quad$ or $\; S_3 + 3$

Can we say $S_5 = S_4 + 3$? Yes, $S_5 = 13+3 = 16$

What is S_{10} ? It is $S_9 + 3$. But how to get S_9 ?

Let us change the pattern

$S_1 = 4 \quad = 3+1 \quad = (1\times3)+1$

$S_2 = 7 \quad = 6+1 \quad = (2\times3)+1$

$S_3 = 10 \;= 9+1 \quad = (3\times3)+1$

$S_4 = 13 \;= 12+1 = (4\times3)+1$

Now $S_5 = (5\times3)+1 = 15+1 = 16$

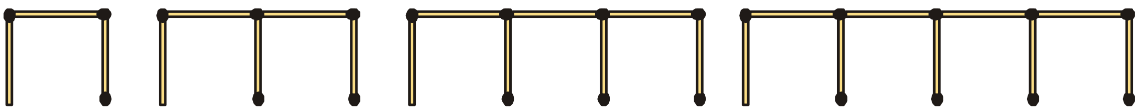
Now the ***rule for this pattern is, Number of matchsicks used in shape-n is*** $S_n = (n\times3)+1=3n+1$

Number of sticks used in shape-10 is $\quad S_n = S_{10}$

$3n+1=(3\times10)+1 = 30+1 = 31$. Got it!

LET'S EXPLORE

A line of shapes is constructed using matchsticks.



Shape-1   Shape-2   Shape-3   Shape-4

(i) Find the rule that shows how many sticks are needed to make a line of such shapes?

(ii) How many match sticks are needed to form shape-12 ?

Exercise - 7.1

1. Find the rule which gives the number of match sticks required to make the following matchstick patterns.
   (i) A pattern of letter 'T'   (ii) A pattern of letter 'E'   (iii) A pattern of letter 'Z'
2. Make a rule between the number of blades required and the number of fans (say n) in a hall?
3. The cost of one pen is ₹ 7 then what is the rule for the cost of 'n' pens.
4. The rule for purchase of books is that the cost of q books is ₹ 25q, then find the price of one book.
5. Harshini says that she has 5 biscuits more than Padma has. How can you express the relationship using the variable 'y'?

7. 3 Expressions with variables

In arithmetic we come across statements like 5 + 4, 11 - 9 etc. These are ***expressions using numbers.***

Observe the following

Ram says that he has scored five marks more than Tony. Can you find the marks scored by Ram ? Here we do not know the marks of Tony. We proceed by supposing Tony's marks. Suppose Tony scored 45 marks. Then marks scored by Ram would be 45+5= 50.
If Tony scored 56 marks. Then marks scored by Ram would be 56+5= 61.
Now let us suppose Tony scored 'x' marks. Then the marks scored by Ram would be x+5.
x+5 is known as an expression in variable 'x'.

In fact, we have seen expressions like 2m, 3y, 4z, 2s+1, 3s+1, 8p, n+3, p-3 in the earlier discussion. Those expressions are obtained by using operations of addition, subtraction, multiplication and division of variables. For example, the expression 'p-3' is formed by subtracting 3 from the variable 'p' and the expression '8 p' is formed by multiplying the variable 'p' by '8'.

We know that variables can take different values; they have no fixed value, but they are numbers. That is why operations of addition, subtraction, multiplication and division can be done on them.

Example-1: Number of pencils with Rama is 3 more than Rahim. Find the number of pencils Rama has in terms of what Rahim has?

Solution: If Rahim has 2 pencils then Rama has 2 + 3 = 5 pencils.

If Rahim has 5 pencils then Rama has 5 + 3 = 8 pencils.

We do not know how many pencils Rahim has.

But we know that Rama's pencils = Rahim's pencils + 3

If we denote the number of pencils Rahim has as n, then the number of pencils of Rama are n+3
Here n = 1, 2, 3 ....therefore 'n' is a variable.

Example-2: Hema and Madhavi are sisters. Madhavi is 3 years younger to Hema. Write Madhavi's age in terms of Hema's age?

Solution:

Given that Madhavi is younger to Hema by 3 years, if Hema is 10 years old then Madhavi is 10-3 = 7 years old.

If Hema is 16 years old, Madhavi is 16-3= 13 years old.

Here we don't know the exact age of Hema. It may take any value. So let the age of Hema be 'p' years, then Madhavi's age is "p - 3" years.

Here 'p' is also an example of a variable. It takes different values like 1,2,3………

As you would expect when 'p' is 10, 'p-3' is 7 and when 'p' is 16, p-3 is 13.

Example-3: Write statement for the following expressions: (i) $2p$ (ii) $7 + x$

Solution: (i) Raju has twice the money than seema. (ii) I have 7 marbles more than Dilip.

Example-4: Madhu plants 5 more groundnut seeds than bean seeds. How many groundnut seeds does he plant (take number of bean seeds as 'm')?

Solutioin: Let the number of bean seeds = m, Therefore number of groundnut seeds = 'm+ 5'

CHECK YOUR PROGRESS

Fill the following tables as instructed. One is shown for you.

S. No.	Expression	Verbal Form
1	y + 3	Three more than y
2	2x - 1	
3	5Z	
4	$\frac{m}{2}$	

Exercise - 7.2

1. Write the expressions for the following statements.
   (i) 5 is added to three times z
   (ii) 9 times 'n' is added to '10'
   (iii) 16 is subtracted from two times 'y'
   (iv) 'y' is multiplied by 10 and then x is added to the product
2. Peter has 'p' number of balls. Number of balls with David is 3 times the balls with Peter. Write this as an expression.
3. Sita has 3 more note books than Geetha. Find the number of books that Sita has. Use any letter for the number of books that Geetha has.
4. Cadets are marching in a parade. There are 5 cadets in each row. What is the rule for the number of cadets, for a given number of rows? Use '*n*' for the number of rows.

7.4 Rules from Geometry / Mensuration

Perimeter of a square

We know that perimeter of a polygon is the sum of the lengths of all its sides. A square has 4 sides and they are equal in length.

Therefore the perimeter of a square is

Sum of the length of the sides of the square = 4 × length of the sides = 4 × s = 4s

Thus we get the rule for the perimeter of the square. The length of the square can have any value, its value is not fixed. It is a variable.

The use of the variable allows us to write the general rule in a way that is concise and easy to remember. What would be the rule for perimeter of an equilateral triangle?

LET'S EXPLORE

1. Find the general rule for the perimeter of a rectangle. Use variables '*l*' and 'b' for length and breadth of the rectangle respectively.
2. Find the general rule for the area of a square by using the variable 's' for the side of a square.

Rule from Arithmetic

Observe the following number pattern 2, 4 , 6 , 8 , 10 , ...

To find the nth term in the given pattern, we put the sequence in a table.

Even Number	2	4	6	8	10	12	14	16	18	20
Pattern	2×1	2×2	2×3	2×4	2×5	.....	2×7	.....	2×9	.....

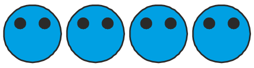
From the table it is clear that the first even number is 2×1, the second even number is 2×2 and so on. Using the above logic, we can fill up the blanks in the table and find the pattern for 'nth' even number. It is $2 \times n$ i.e., '$2n$'.

So the nth term of the pattern 2,4,6,8,10,…… is $2n$.

- Find the nth term in the following sequences

  (i) 3 , 6 , 9 , 12 , … (ii) 2 , 5 , 8 , 11 , … (iii) 1, 4, 9, 16, …

7.5 Simple Equations

Observe the face pattern. 

Now the number of black stickers required is given by the rule $2m$, if m is taken to be the number of faces to be formed.

We can find the number of stickers required for a given number of faces. What about the other way? How to find the number of faces formed when the number of stickers are given?

For example, to find the number of faces (i.e., m) for the given number of stickers 12. we know $2m = 12$. The condition to be satisifed that 2 times m must be 12 is an example of an equation. Our question can be answered by observing the table.

m	$2m$	Condition satisfied? Yes/No.
2	4	No
3	6	No
4	8	No
5	10	No
6	12	Yes
7	14	No

The equation $2m = 12$ is satisfied only when $m = 6$.

CHECK YOUR PROGRESS

Complete the table and find the value of 'p' for the equation $\frac{p}{3} = 4$

'p'	$\frac{p}{3} = 4$	Condition satisfied? Yes/No.
3		
6		
9		
12		

7.5.1 L.H.S and R.H.S of an Equation

If we observe $2m = 12$, we can find that equation has sign of equality between its two sides. The value of expression to the left of the equality sign in an equation is called ***Left Hand Side (LHS)*** and the value of which is on the right side of the equality sign is called ***Right Hand Side (RHS).***

An equation says that the value of the LHS is equal to the value of RHS. This condition of an equation is often compared with a simple balance with equal weights on both pans.

If LHS is not equal to RHS, we do not get an equation. For example 4 + 5 on one side and 7 on the other side is not an equation. We would write $4 + 5 \neq 7$ or $4 + 5 > 7$. Similarly $x + 5 > 6$, $y - 1 < 10$ do not represent equations.

CHECK YOUR PROGRESS

1. Write LHS and RHS of following simple equations.

   (i) $2x + 1 = 10$ (ii) $9 = y - 2$ (iii) $3p + 5 = 2p + 10$

2. Write any two simple equations and write their LHS and RHS.

7.5.2 Solution of an equation (Root of the equation)- Trial & Error Method

At the beginning of the chapter. we observed a conversation between Damini and Kowshik. In that conversation Damini said that the result was 27 and Kowshik told her friend's age as 11 years.

Let us see how he got the answer.

Let Damini's friend's age be 'x' years. By doubling it, we get '$2x$'. After adding 5 to it, it becomes '$2x + 5$'.

Therefore, the result is '$2x + 5$'. Damini said that result was 27.

This tells us $\mathbf{2x + 5 = 27}$

Let us take the above equation $2x + 5 = 27$ is the condition to be satisfied by 'x'.

Here, 'x' is a variable and can take any value like 1, 2, 3, ….

If $x = 1$, then the value of $2x + 5 = 2 \times 1 + 5 = 7 \neq 27$

If $x = 2$, then the value of $2x + 5 = 2 \times 2 + 5 = 9 \neq 27$

If $x = 3$, then the value of $2x + 5 = 2 \times 3 + 5 = 11 \neq 27$, and so on

Writing 1,2,3 ..... in the place of 'x' is called "**Substitution**".

Let us continue giving values 4, 5, 6...... to x in $2x+5$

If $x = 9$, then the value of $2x + 5 = 2\times9+5 = 23 \neq 27$

If $x = 10$, then the value of $2x + 5 = 2\times10+5 = 25 \neq 27$

If $x = 11$, then the value of $2x + 5 = 2\times11+5 = 27 = 27$

If $x = 12$, then the value of $2x + 5 = 2\times12+5 = 29 \neq 27$

From the above it is obvious that when $x = 11$, the both LHS and RHS are equal.

Therefore $x = 11$ is called the ***solution*** of equation $2x + 5 = 27$.

The method we followed in the above is called ***Trial and Error Method.***

Solution of an equation is the value of the variable for which LHS and RHS are equal. The solution is also called as root of the equation.

Algebra is a powerful tool for solving puzzles, riddles and problems in our daily life.

LET'S EXPLORE

Observe for what value of m, the equation $3m = 15$ has both LHS and RHS become equal.

Exercise - 7.3

1. Identify which of the following are equations.

(i)	$x - 3 = 7$	(ii)	$l + 5 > 9$	(iii)	$p-4 < 10$
(iv)	$5 + m = -6$	(v)	$2s - 2 = 12$	(vi)	$3x +5$

2. Write LHS and RHS of the following equations.

(i)	$x-5 =6$	(ii)	$4y = 12$	(iii)	$2z + 3 = 7$

3. Solve the following equation by Trial & Error Method.

(i)	$x + 3 = 5$	(ii)	$y - 2 = 7$	(iii)	$a+ 4 = 9$

Unit Exercise

1. The cost of one fan is ₹ 1500. Then what is the cost of 'n' fans ?
2. Srinu has number of pencils. Raheem has 4 times the pencils as of Srinu. How many pencils does Rahim has? Write an expression.

3. Parvathi has 5 more books than Sofia. How many books are with Parvathi? Write an expression choosing any variable for number of books.

4. Which of the following are equations ?

i) $10 - 4P = 2$ *ii)* $10 + 8 <= -22$ *iii)* $x + 5 = 8$ *iv)* $m + 6 = 2$

v) $22x - 5 = 8$ *vi)* $4k + 5 > -100$ *vii)* $4p + 7 = 23$ *viii)* $y < -4$

5. Write L.H.S and R.H.S of the following equations:

i) $7x + 8 = 22$ *ii)* $9y - 3 = 6$ *iii)* $3k - 10 = 2$ *iv)* $3p - 4q = -19$

6. Solve the following equations by trail and error method.

i) $x - 3 = 5$ *ii)* $y + 6 = 15$ *iii)* $\frac{m}{2} = -1$ *iv)* $2k - 1 = 3$

Points to Remember

1. We can make different patterns and shapes using matchsticks. The relation between the patterns and number of sticks used is observed to frame rules.

2. A variable takes different values. Its value is not fixed.

3. We may use any letter (small case alphabets) a, b, m, n, p, q, x, y, z etc., to represent a variable.

4. A variable allows us to express relations in any practical situation.

5. Variables are numbers, although their values are not fixed. We can do operations on them just as in the case of fixed numbers.

6. We can form expressions with variables using different operations. Some examples are $2m$, $3s+1$, $8p$, $x/3$ etc.

7. Variables allow us to express many common rules of geometry and arithmetic in a more general way.

8. An equation is a condition on a variable. Such a condition limits the values the variable can have.

9. An equation has two sides, L.H.S. and R.H.S., on both sides of equality sign.

10. The LHS of an equation is equal to its RHS only for definite values of the variable in the equation

11. Trial and Error method is useful to solve equations.

C6V7Y7

CHAPTER 8

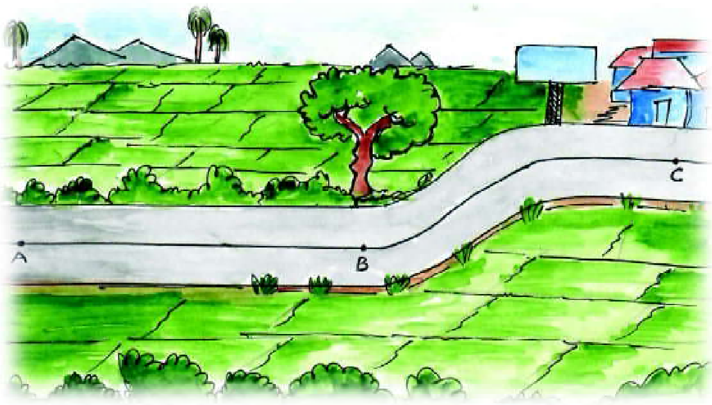
Basic Geometric Concepts

Learning Outcomes:-

The students are able to

- identify and classify point, ray, line and line segment.
- solve the problems on geometric concepts in daily life.
- read and write geometric symbols and concepts.
- draw parallel lines, perpendicular lines and angles.
- measure the lengths of line segments and angles.

P3H2N3



Content Items

8.0 Introduction

Observe the aerial view of the path leading to village from A to B and from B to C. From A to B it is straight where as from B to C it is not straight but curved. The line with end point A and B is called a ***line segment*** and from B to C it is ***curved line.***

• How many end points are there in the line segment AB?

Do you know how to play Carrom Board ?

Two players are playing it. There are 5 coins left on the board.

Coins are as shown in the figure.

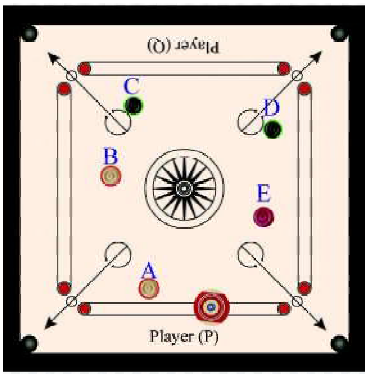
Players P and Q are playing.

Now it is the turn of player P.

i) How many rays are there?

ii) How many coins are close to player Q?

iii) While striking with striker there is a possibility of a coin



touches with any other. Draw all such possibilities in the given picture by means of the line segments.

iv) How many such line segments can be drawn in the picture?

LET'S DO ACTIVITY

Observe the table and their notations and fill the gaps

Figure	Notation	Read as	Notation	Read as
• A	capital letters as A, B, C...	point A	• B	....
A B (ray)	$\overrightarrow{AB}$	ray AB	....	ray XY
A B (segment)	$\overline{AB}$	segment AB	....	segment OP
A B (line)	$\overleftrightarrow{AB}$	line AB	$\leftrightarrow$	....

Note : We represents the lines with small letters like ℓ, m, n etc. We read them as line ℓ, line m, line n etc.

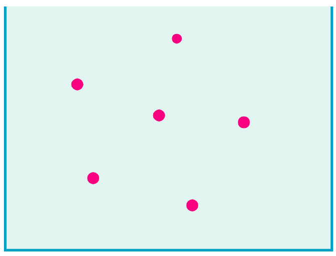
HISTORICAL NOTE

Geometry has a long and rich historical nature. The term 'GEOMETRY' is derived from the greek word 'GEOMETRON'. 'GEO' means earth and 'METRON' means measurement. So, Geometry is the mathematics related to the earth's measurement. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas and volumes which were developed to meet some practical need in surveying, construction, astronomy and various crafts.

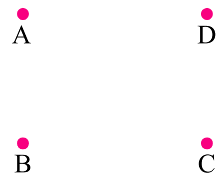
In the ancient India Aryabhata, Brahmagupta were some of the Indian Mathematicians who contributed their works in geometry.

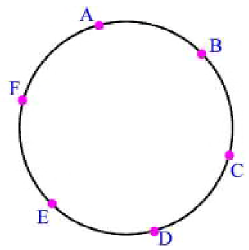
Exercise - 8.1

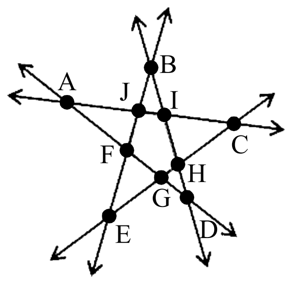
1. In the given figure there are some points marked. Name them.



2. Join the points given below. Name the line segments so formed in the figure.

(i) 

(ii) 



3. Identify the following from the given figure.
   (i) Any six points.
   (ii) Any six line segments. (Starts with G)
   (iii) Any six rays. (With initial point I)
   (iv) Any three lines.

4. Write 'True' or 'False'.
   (i) A line has two end points.
   (ii) Ray is a part of line.
   (iii) A line segment has two end points.
   (iv) We can draw many lines through two points.

5. Draw and Name :
   (i) Line containing point K.
   (ii) Draw a circle and a line such that the line intersects the circle
      a) at no points   b) at one point   c) at two points
   (iii) Can we draw a line and a circle with 3 intersecting points ?

6. List out all capital letters in English alphabet, that you can write with 3 line segments by using all.

8.1 Measuring the length of a given line segment

The length of any line segment can be measured with the help of a scale and divider or compass.

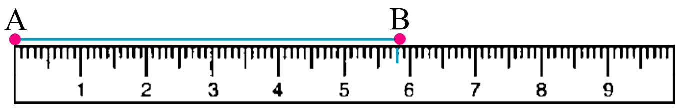
Using Scale:

Step-1: Keep the "0" of the scale to align with one end point A.

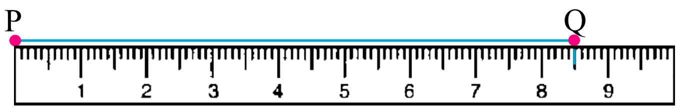
Step-2: Adjust the edge of the scale to align with the entire line segment.

Step-3: Look for the mark on the scale where the other end point B coincides. This reading on the scale gives the length of the line segments.

AB = 5.8 cms



PQ = 8.5 cms



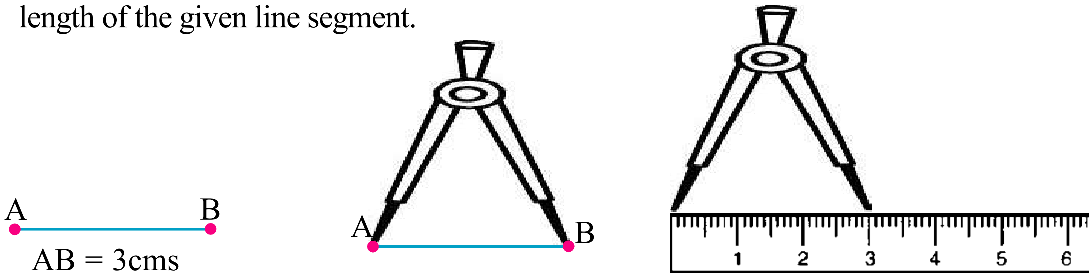
Using Divider or Compass:

Step-1: Open the divider or compass and keep one metallic point on one end of the line segment.

Step-2: Adjust the other metallic point (Pencil point in the place of compass) on the other end of the line segment.

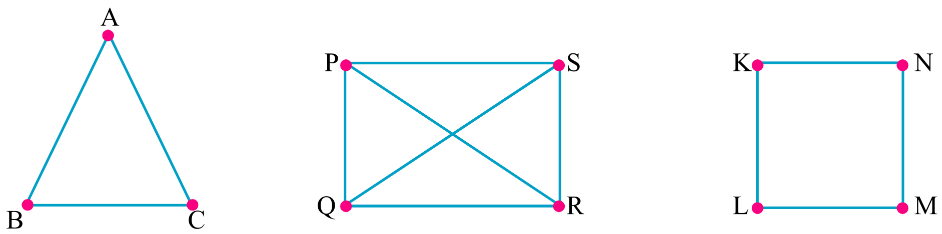
Step-3: Lift the divider or the compass undisturbed and place one metallic end on "0" of the scale and the other end along the marks on the scale.

Step-4: The reading where the other metallic end of the divider (point of the pencil) gives the length of the given line segment.



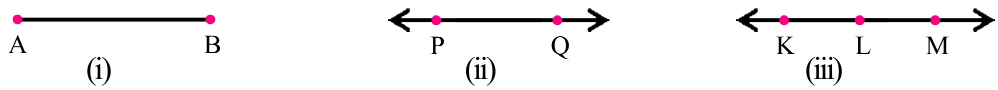
LET'S EXPLORE

Measure the lengths of all line segments in the given figures by using divider and scale. Then compare the sides of the given figures.



Exercise - 8.2

1. Measure the lengths of the given line segments.



2. Draw the following line segments.

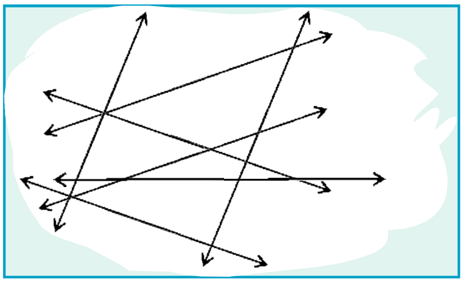
   (i) AB = 6.3 centimeters (ii) MN = 3.6 centimeters

3. Draw PQ = 4.6 cm and extend upto R such that PR = 6 cm.

4. Draw a line segment $\overline{OP}$ with certain length and mark a point Q on it. Check whether $\overline{OP} - \overline{PQ} = \overline{OQ}$.

8.2 Intersecting lines, Parallel lines, Concurrent lines, Perpendicular lines

Usha took a paper and folded it. She unfolded the paper and drew a line through the crease. She repeated the same with different folding, she obtained the lines as given here under.



What do you observe?
Some lines, line segments, points.
Lable all the lines with ℓ, m, n, o, p, q, r. and also the points with A, B, C, etc.
List out the points on the same line.
Identify the points which lie on more than two lines.
What do you observe? Some are crossing one another. But some are not.

Parallel Lines: The lines that never meet even if they are extended any further are called Parallel lines. The lines in your ruled book are parallel lines. If line ℓ is parallel to line *m,* we denote this by writing $\ell \parallel m$. We read this "ℓ is Parallel to *m*".

Intersecting Lines: The lines that cross one another are called intersecting lines and the point at which they intersect is called as ***intersecting point.***

Concurrent Lines: Three or more lines passing through the same point are called concurrent lines. The point is called as ***point of concurrency.***

Perpendicular Lines: If you observe some more intersecting lines in the figure they have different nature. They look like the edges of a paper, black-board, door etc which are adjacent to each other. These such lines are called ***perpendicular lines.*** They are denoted with the symbol '$\perp$' (perpendicular). If line ℓ is perpendicular to line *m* then write this as $\ell \perp$ m and read as *"ℓ is perpendicular to line m".*

- Find the parallel lines in the above figure. Name, write and read them.
- Find the intersecting lines in the above figure. Name, write and read them.
- Find the concurrent lines in the above figure. Name, write and read them.
- Find the perpendicular lines in the above figure. Name, write and read them.

Example-1:

Identify parallel and perpendicular sides in the given figures. Represent by using symbols selecting from "||", "$\perp$"

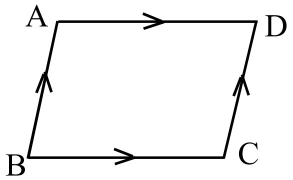


fig. 1

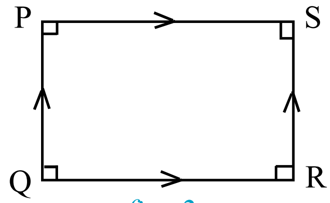


fig. 2

Note: i) Arrow heads in the same direction indicates parallel lines.

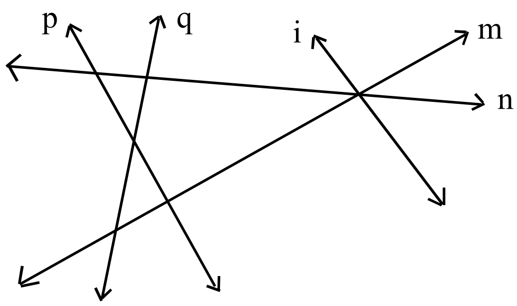
ii) Lines with the symbol "¬" at the intersecting points of the lines represents perpendicularity.

Solution: In fig. 1 $\left.\begin{matrix} BC \parallel AD \\ BA \parallel CD \end{matrix}\right\}$ *Parallel sides* In fig. 2 $\left.\begin{matrix} QR \parallel PS \\ PQ \parallel SR \end{matrix}\right\}$ Parallel sides

In fig. 2 $\left.\begin{matrix} PQ \perp QR \\ QR \perp RS \\ RS \perp SP \\ SP \perp PQ \end{matrix}\right\}$ Perpendicular sides

Example-2 :

Identify intersecting lines and concurrent lines.

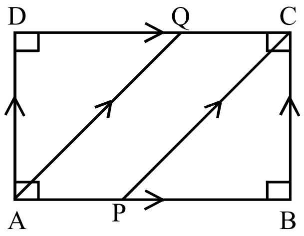


Intersecting lines: p and q; p and m; p and n; q and m; and q and n.

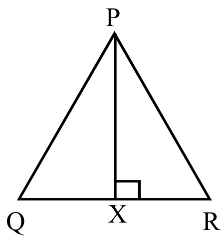
Concurrent lines: i, m and n.

Exercise - 8.3

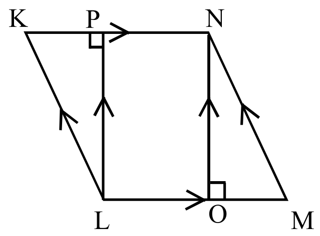
1. Given "$\overline{AB} \parallel \overline{CD}$, $\ell \perp m$". Which are perpendicular? Which are parallel?
2. Write the set of parallels and perpendiculars in the given by using symbols.



(i)

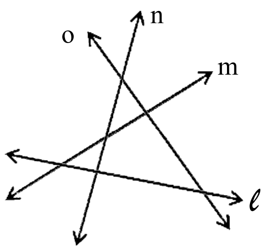


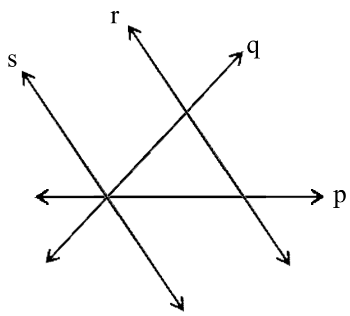
(ii)



(iii)

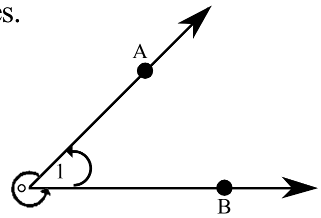
3. From the given figure find out the intersecting lines and concurrent lines.





8.3 Angles, Types, Measuring Angles

In the previous classes you have learnt about angles and types of angles.



Angle

An angle is a figure formed by two rays with a common end point. The ***common end point*** is the angle's ***'vertex'***. The rays are the ***'sides or arms'*** of the angle. The sides of this angle shown here are $\overrightarrow{OA}$ and $\overrightarrow{OB}$. The vertex is O.

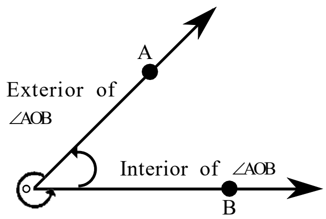
An angle can be named in several different ways

i) by vertex $\angle O$

ii) by a point of each ray and the vertex $\angle AOB$ or $\angle BOA$

iii) by a number $\angle 1$

The ***exterior of an angle*** is the collection of all points outside the angle. And the ***interior of an angle*** is the group of all points between the sides of the angle.

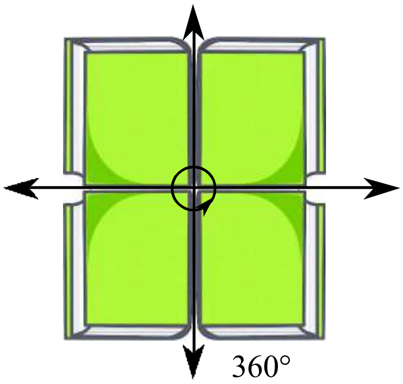


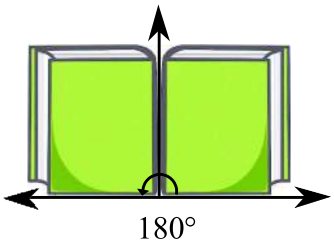
Measurement of an angle is nothing but the magnitude of the rotation of one arm with reference to the other having common vertex.

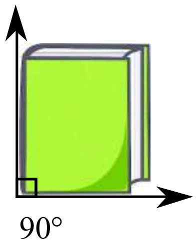
Sexagesimal System: (system related to number sixty)

In the above system the angle made in one complete rotation is considered as 360° (Here "°" is a symbol for degree)

Join four notebooks of the same size as shown in the figure.







Given the angle measurement at the center you know it as 360°. So, one complete rotation = 360°

If you join two books then the arms are straight. The angle at the center is half the complete rotation. It is called straight angle. Straight angle = 180°

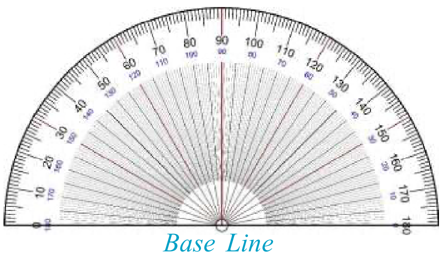
If you consider the edges of one book, its a right angle. Right angle = 90°

Acute angle is the angle less than 90°, Obtuse angle is the angle more than 90° and less than 180°. Reflexive angle is the angle more than 180° and less than 360°.

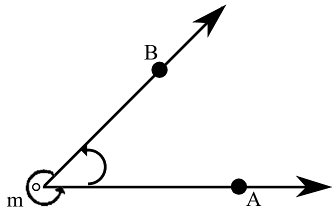
1)	Acute Angle	$< 90°$
2)	Right Angle	$= 90°$
3)	Obtuse Angle	$> 90°$ and $<180°$
4)	Straight Angle	$= 180°$
5)	Reflexive Angle	$> 180°$ and $< 360°$
6)	Complete Angle	$= 360°$

How to measure an angle?

We use a tool called "Protractor" to measure the given angle.



Suppose we want to measure the angle subtended by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ given in the adjacent figure.



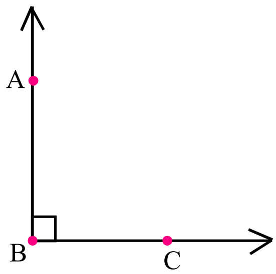
Step-1: Place the protractor such that its base line aligns with OA and mid point of protractor coincides with O.

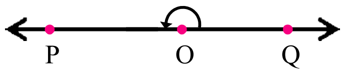
Step-2: Start reading the measurement through the scale printed on the edge of the protractor from the side of the intial ray ($\overrightarrow{OA}$)

Step-3: The reading through which the other ray ($\overrightarrow{OB}$) passes through gives the measurement of the angle $\angle AOB$. Measure of angle $\angle AOB$ is denoted by m$\angle AOB$.

Example-3: Measure the angles given below. Write them using with symbols.

Solution:

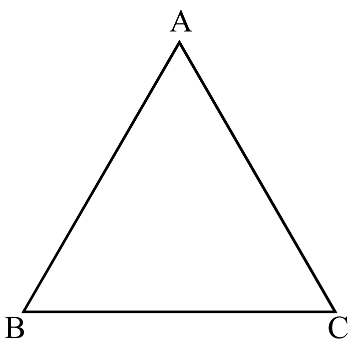


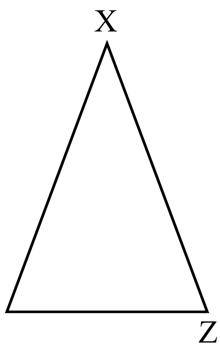


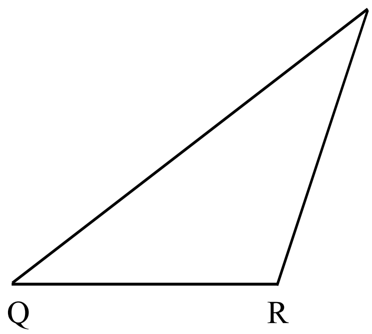
i) m$\angle ABC = 90°$

ii) m$\angle POQ = 180° = 2$ right angles

• Measure the angles at the Vertices.

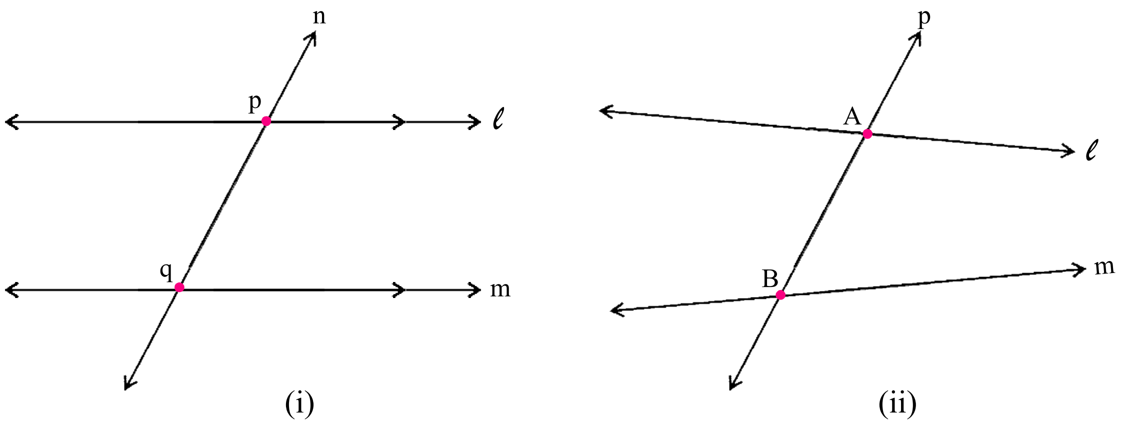






Exercise - 8.4

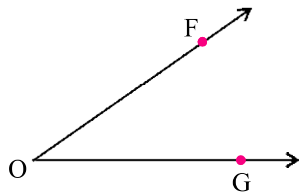
1.



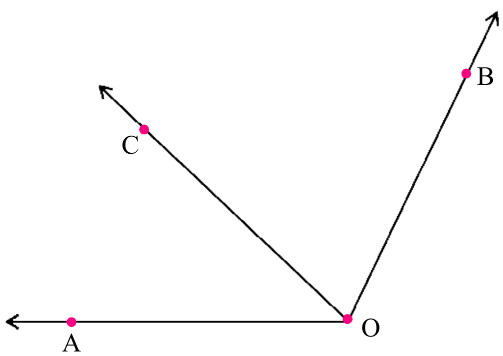
Measure all the angles in the above figures.

2. Sum of which two angles is 180° in each figure?

3. In the given figure measure $\angle FOG$ and draw the same in your note book.



4. In the given figure measure the angles $\angle AOB$, $\angle BOC$.



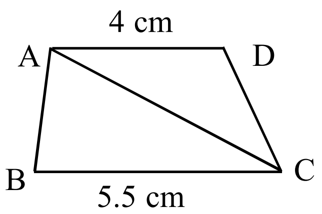
5. Write some acute, obtuse and reflexive angles atleast 2 for each.

Unit Exercise

1. In the given figure, measure the length of AC. Check whether

i) AB+AC > AC

ii) AC >AD -BC



2. Draw a line segment $\overline{AB}$. Put a point C in between $\overline{AB}$. Extend $\overline{CB}$ up to D such that CD > AB. Now check whether AC and BD are equal length.

3. Draw an angle $\angle AOB$ as $m\angle AOB = 40^\circ$ Draw an angle $\angle BOC$ such that $\angle AOC = 90^\circ$ Check whether $m\angle AOB + m\angle BOC = m\angle AOC$.

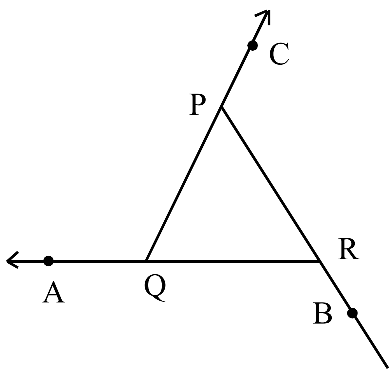
4. Draw an angle $\angle XYZ$ as $m\angle XYZ = 62^\circ$. Measure the exterior angle $\angle XYZ$.

5. Match the following

1. Set square	A) to measure angles
2. Protractor	B) to measure the lengths of line segments
3. Divider	C) to draw parallel lines

6. List out the letters of English alphabet (capital letters) which consist of right angles.

7. Measure the angles $\angle AQP$, $\angle CPR$, $\angle BRQ$



Find $m\angle AQP$, $m\angle CPR$, $m\angle BRQ$

Points to Remember

S2M9U2

1) Points should be denoted with capital letters in English.
2) By using divider and scale, we can measure length of line segment.
3) Parallel lines are the lines which never meet.
4) Intersecting lines are the lines which intersect at one point, the point where they intersect is called intersecting point.
5) If more than three lines pass through one point they are called concurrent lines. The point through which they passes is called point of concurrency.
7) Two rays with a common initial point make two regions. The interior region along with rays is called interior angle.
8) The exterior region along with the rays is called exterior angle.
9) In sexagesimal system angle is measured in degrees.

CHAPTER 9

2D - 3D Shapes

Learning Outcomes:-

The students are able to

- describe geometrical ideas like triangle, quadrilateral and circle with the help of examples in surroundings.
- identify parts of a triangle.
- identify vertices and sides of polygons.
- identify and compare various 3D objects like sphere, cube, cuboid, prism, pyramid, cone and cylinder.
- describe and provide examples of edges, vertices and faces of 3D objects.
- verify Euler's formula: F+V=E+2.

J5V5S4

Content Items

9.0 Introduction:

Observe the world around us. It is full of shapes. Shapes like triangle, circle, rectangle exist only on flat surfaces like paper, board etc., Such shapes are called 2D shapes.

Other shapes like a house, ball, pole exist everywhere else. These cannot be drawn as they are on a flat surface. Such shapes are called 3D shapes.

A 2D or 2-Dimensional shape has two dimensions namely length and the breadth. A 3D, or 3-Dimensional shape has three Dimensions namely length, height and width (depth).

In this chapter, we learn about such geometric shapes.

Plane

Where do you find stars and satellites?

In the space. Astronauts travel in space by rockets which travel in space. A space extends infinitely in all directions and is a set of all points in three dimensions. Tell some smooth surfaces you see in your daily life. Wall, blackboard, paper are some examples. In mathematics, a plane is a flat surface that extends infinitely in all directions.

Plane deals with objects that are flat, such as triangles and lines that can be drawn on a flat piece of paper.

9.1 Polygon – Different Types

You have learnt lines, rays, line segments, angles, open and closed figures in the previous chapter.

Observe the following figures formed by match sticks.

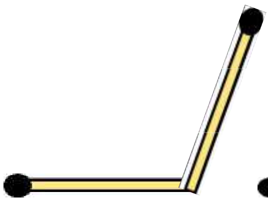


Fig-i

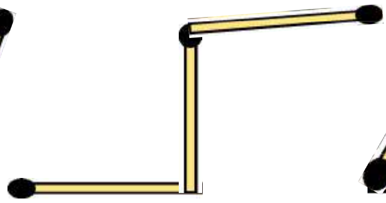


Fig-ii

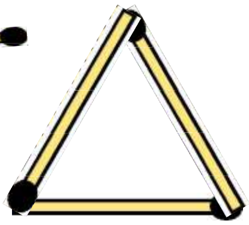


Fig-iii

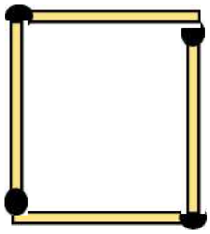


Fig-iv

Fig (i) and (ii) are not closed. But (iii) and (iv) are closed figures.

Can you explain why with two match sticks we cannot make a closed figure?

Observe the following.

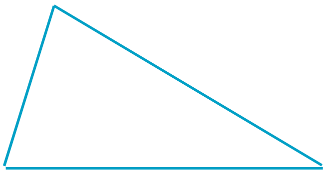


Fig-i

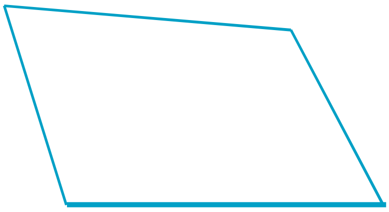


Fig-ii

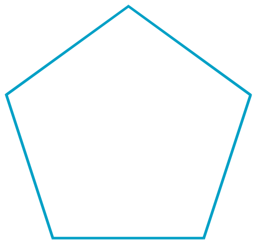


Fig-iii

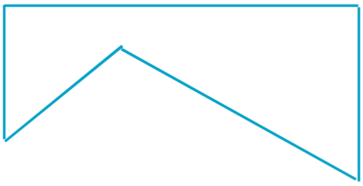


Fig-iv

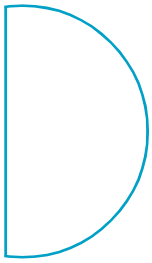


Fig-v

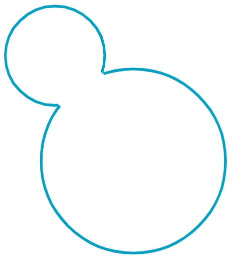


Fig-vi

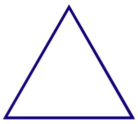
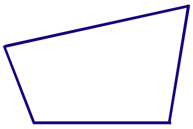
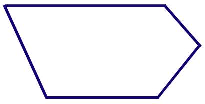
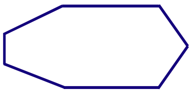
All the above figures are closed, but some figures are closed with line segments on all sides. And some are closed with curved sides. Figures i, ii, iii & iv are called ***polygons*** (***Poly*** = many, ***gons*** = sides).

A figure is a polygon, if it is a closed figure, formed with a definite number of straight line segments.

A polygon is only 2- dimensional closed shape formed with straight lines. A polygon is a closed figure in the plane with at least three line segments and typically four or more.

Depending on the number of line segments involved in the formation of the polygon, they are named as triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, etc.

Study the table given below and observe the names of various types of polygons.

Figure	Number of sides	Name of polygon	
	3	Triangle	(Tri = 3)
	4	Quadrilateral	(Quadri = 4)
	5	Pentagon	(Penta = 5)
	6	Hexagon	(Hexa = 6)
	7	Heptagon / Septagon	(Septa = 7)

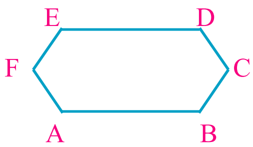
• *Draw six different types of rough sketches of polygons in your notebook.*

In which case, it is not possible to form a polygon?

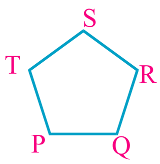
Hence, what is the least number of sides needed to form a polygon?

Obviously three.

Exercise - 9.1

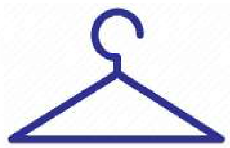


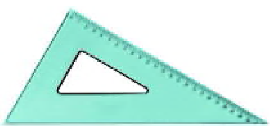
1. What is the name of four sided polygon? Draw the rough sketch.
2. Draw a rough sketch of pentagon.
3. Write all the sides of the given polygon ABCDEF?
4. Write the interior angles of the polygon PQRST.
5. Measure the length of the sides of the polygon PQRST.

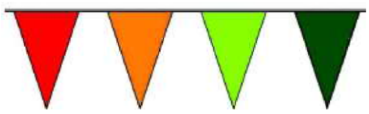


9.2 Triangles

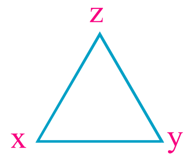
Observe these figures.







The simple closed figure formed by three line segments is called a triangle. The line segments are called sides. A triangle contains ***three sides, three angles and three vertices.*** A triangle is denoted by the symbol "Δ".



Parts of the triangle:

$\overline{XY}$, $\overline{YZ}$, $\overline{ZA}$ are three sides of ΔXYZ.

$\angle XYZ$ or $\angle Y$; $\angle YZX$ or $\angle Z$; $\angle ZXY$ or $\angle X$ are three angles of the given triangle.

These three sides and three angles are called parts of ΔXYZ. Points X, Y and Z are called vertices of the ΔXYZ.

• **Take a card board or a chart and make different sizes of triangles by cutting it.**

Example-1 :

Look at the triangle given below and write its vertices, sides and angles.

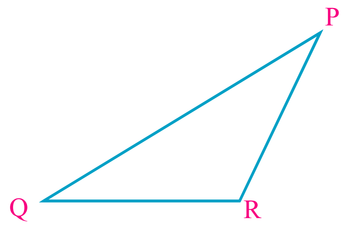
Solution :

ΔPQR is given triangle.

Vertices: P, Q, R

Sides: $\overline{PQ}$, $\overline{QR}$, $\overline{RP}$

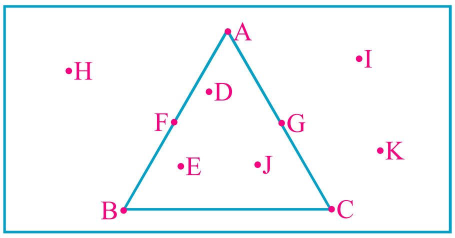
Angles: $\angle P$, $\angle Q$, $\angle R$.



9.2.1 Triangular Region

Observe the triangle and the points marked in the figure.

"Point D lies in side of the triangle" ABC. It is called interior point of the triangle. E and J are other interior points of the triangle.



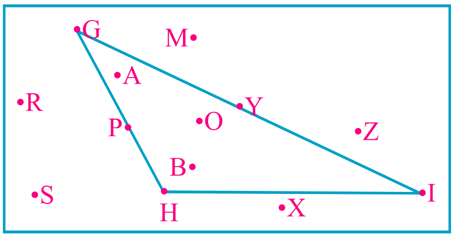
Point A lies on the triangle. B, C, F and G are other points lying on the triangle. I is out side of the triangle. It is called an exterior point of the triangle. H and K are other exterior points of the triangle.

Therefore, a triangle divides the points on the plane into three parts.

1. Interior points of the triangle.
2. Points on the triangles.
3. Exterior points of the triangle.

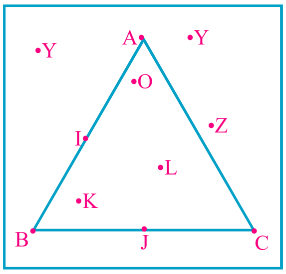
CHECK YOUR PROGRESS

Look at the adjacent figure.

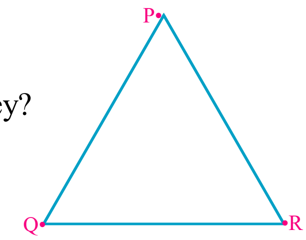


1. Which points are marked in the interior of ΔGHI?
2. Which points are marked on the triangle?
3. Which points are marked in the exterior of ΔGHI?

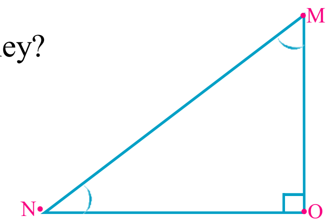
Exercise - 9.2



1. Look at the given triangle and answer the following questions.
   i) Which points are marked in the exterior of the triangle?
   ii) Which points are marked on the triangle?
   iii) Which points are marked in the interior of the triangle?



2. Look at the adjacent figure. Answer the following questions.
   i) How many sides are there in the triangle? What are they?
   ii) How many vertices lie there on the triangle? What are they?
   iii) What is the side opposite to the vertex P?
   iv) What is the vertex opposite to $\overline{PR}$?

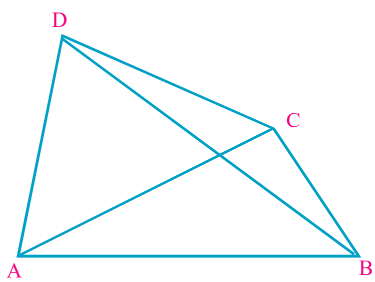


3. Look at the given triangle and answer the following questions.
   i) How many angles are there in the triangle? What are they?
   ii) What is the angle opposite to $\overline{MN}$?
   iii) Where is the right angle in the given triangle ?

9.3 Quadrilateral

Do you know the name of a polygon with four sides?

A polygon with four sides is called a quadrilateral. Quadra means ***four***, ***lateral*** means ***side.***

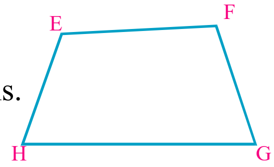


Look at the figure. All the four sides of a quadrilateral may be equal or not. Here ABCD is a quadrilateral. Four line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are called its four sides and $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are its four angles. Points A,B,C and D are its vertices.

The line segments AC and BD are called its ***diagonals*** and represented as $\overline{AC}$ and $\overline{BD}$.

Quadrilaterals are used in graphic art, sculpture, logos, packing, computer programming and web design.

Example-4 :



Look at the given quadrilateral and answer the following questions.

1) What are the adjacent angles of $\angle E$?

2) What is the opposite angle to $\angle G$ in the given quadrilateral?

Solution - 1 : EFGH is given quadrilateral

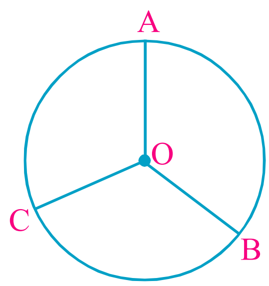
$\angle H$ and $\angle F$ are adjacent angles of $\angle E$.

Solution - 2 : $\angle E$ is the opposite angle to $\angle G$.

9.4 Circle

Look at the following figures.

Keep a round shaped object on a paper and draw along its boundary with pencil. You get a round shape. This will give you an idea of a circle. Such a round shaped figure is a circle. Can you think of some more examples from real life?



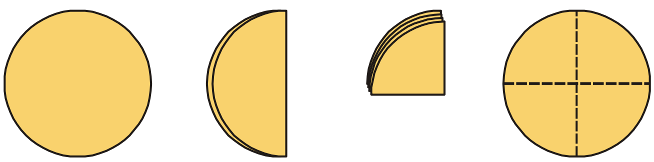
Observe a cycle wheel and measure the length of each spoke. You might conclude that the length of each spoke is same. The point in the middle is the **centre** and the length of curve edge is called **circumference** and the distance from the centre to any point on the circle is the **radius**.

"O" is the centre. $\overline{OA}$, $\overline{OB}$ and $\overline{OC}$ are radii. Measure the radii.

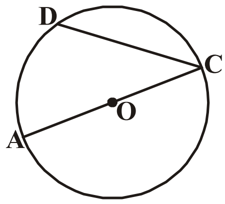
Are all the radii equal?

LET'S DO ACTIVITY

Draw a circle on a paper and cut it along its edge. Fold it into half and again fold it to one fourth to make folding marks as shown.

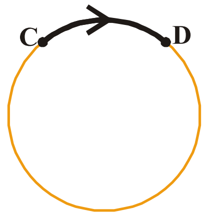


You will observe a point in the middle. Mark this O. This is the centre of the circle. You can also indicate its radius.How many radii can you draw in a circle?

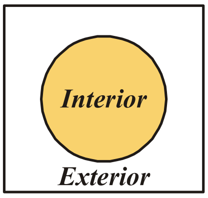


$\overline{AC}$ is a line segment joining two points on the circle.

Is there any other such line segment which joins two points on the circumference? CD is one such line segment. ***A line segment joining two points on the circumference of the circle is called a chord.*** Thus both $\overline{CD}$ and $\overline{AC}$ are chords of the circle. The chord $\overline{AC}$ is a special chord as it passes through the centre 'O'. ***A chord which passes through the centre of a circle is called diameter.*** It is the largest chord.



Look at the figure. The part of the circle between the points C and D is called an **arc** and denoted by $\overset{\frown}{CD}$.



Name the other arc in the figure.

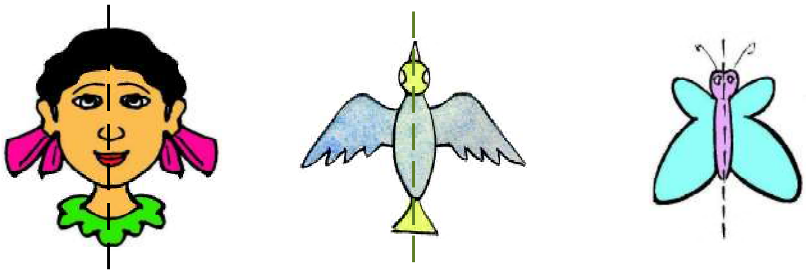
A circle is a simple closed figure. It divides the plane with its boundary as ***interior*** and ***exterior.***

The region in the interior of a circle enclosed by the boundary is called **circular region**.

9.5 Symmetry

Line Symmetry

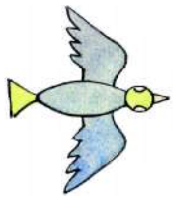
Observe the following figures. What do you notice?



The above figures are beautiful because they are symmetric i.e. if the figure is folded along the given dotted line each part coincides with the other part exactly. It is called ***line symmetry*** and the line along which the paper is folded is called ***line of symmetry*** or ***axis of symmetry***.

Observe the following figures

(i) M (ii) G (iii)



We see that the first and the third figures are symmetric. First figure M has a line of symmetry vertically at its middle and third figure bird has a line of symmetry, horizontally.

Any line along which we can fold a figure so that the two parts of it coincide exactly is called a line of symmetry. It can be horizontal, vertical or diagonal.

• Take a rectangular sheet (like a post-card). Fold it along its length so that one half fits exactly over the other half. Is this fold a line of symmetry? Why?

Open it up now and again fold along its width in the same way. Is this second fold also a line of symmetry? Why?

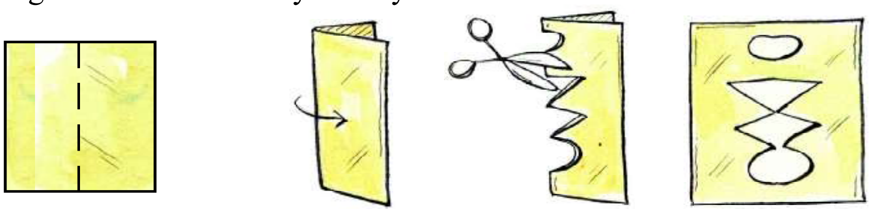
Do you find that these two lines are the lines of symmetry?

Take a square piece of paper. Fold it into half vertically so that the edges coincide. Open the fold and you will find that the two halves made by the fold are congruent. The fold at the centre becomes a line symmetry for the paper. Try to fold the paper at different angles so that it becomes a line of symmetry.

Paper cutting using symmetry

Remember how you decorate your classroom on Independence day or on Republic day, with colour papers cut in various designs. Do you know how to cut these designs?

Take a square paper and fold at the middle vertically. Draw a design on the fold as shown in the figure and cut the paper along the folded edge. When you open you see a symmetric design with one line of symmetry.



Take a square paper and fold at the middle vertically and horizontally. Draw a design on the fold as shown in the figure and cut off the paper on edges. Then open to see a symmetric design with two lines of symmetry.

PROJECT

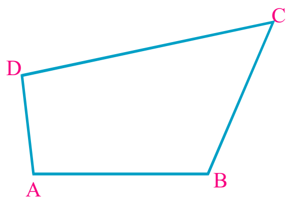
Collect symmetrical figures from your surroundings and prepare a scrap book.

Exercise - 9.3

1. Look at the given quadrilateral and answer the following questions.

i) What are the sides of the given quadrilateral?

ii) What is the opposite side of $\overline{AB}$?



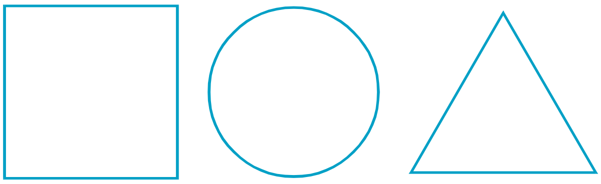
iii) What is the opposite vertex of B?

iv) What is the opposite angle of $\angle C$?

v) How many pairs of adjacent angles are there? What are they?

vi) How many pairs of opposite angles are there? What are they?

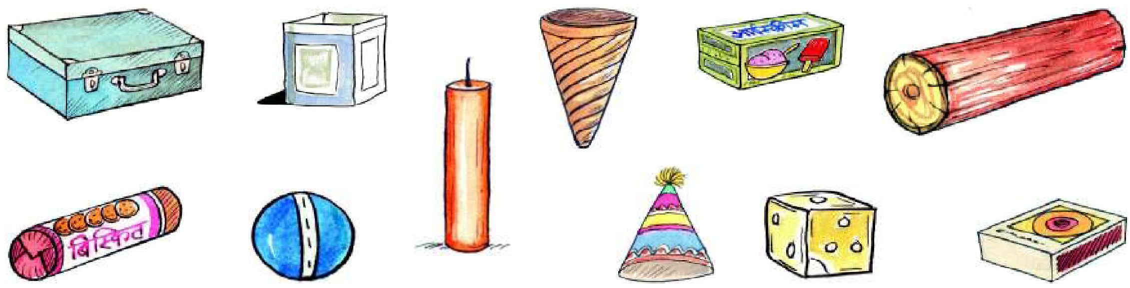
4. Find the number of lines of symmetry in the following.



9.6 Understanding 3D Shapes

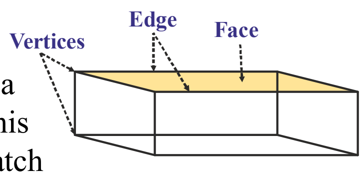
We have learnt about triangles, squares, rectangles etc. in the previous classes. All these shapes are spread in two directions only and thus called ***two-dimensional*** or ***2D shapes.***

Pictures of some objects are given below.



All solid objects like the above, have length, breadth and height or depth. They are thus called ***three dimensional*** or ***3D-shapes.*** Now, we will learn about various 3 dimensional or 3D shapes.

9.6.1 Cuboid

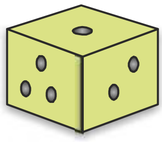


The shapes like a closed match box are examples of a cuboid. Touch with your hand on the top of the match box. This plane surface is the face of match box. How many faces does a match box have?

The sides of the faces are the edges. How many edges are there for a match box ?

The corners of the edges are the vertices of the match box. How many vertices does a match box have? Now take an eraser. Its shape is similar to that of a match box. Touch with your hand along its faces, edges and vertices. Does the eraser have the same number of faces, edges and vertices as that of match box? You will find this to be true. Objects like match boxes, erasers etc. are in the shape of a cuboid and have 6 faces, 12 edges and 8 vertices.

9.6.2 Cube



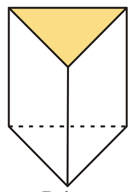
A dice is an example of a cube. Take a dice. Locate its faces, edges and vertices. Count them. How many faces, edges and vertices does a dice have?

You will find that a dice has 6 faces, 12 edges and 8 vertices, same as that of a cuboid. Then what is the difference between a cube and a cuboid? You will find that the length, breadth and height of a cube are all equal, but in a cuboid they are different. Verify this by measuring the length, breadth and height of an eraser and a dice.

9.6.3 Prism

Here is a diagram of a **prism.**

Have you seen it in the laboratory? Two of its faces are in the shape of a triangle. Other faces are either in the shape of rectangle or parallelogram. It is a triangular prism. If the prism has a rectangular base, it is a rectangular prism. Can you recall another name for a rectangular prism?

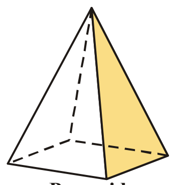


Prism

9.6.4 Pyramid

A **pyramid** is a solid shape with a base and a point vertex, the other faces are triangles. All the triangular faces meet at vertex of the prism.

Here is a ***square pyramid.*** Its base is a square. Can you imagine a triangular pyramid? Draw a rough sketch of it.



Pyramid

9.6.5 Cylinder

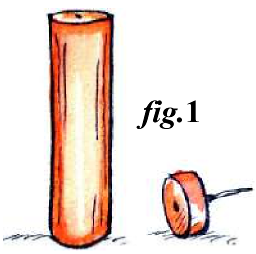
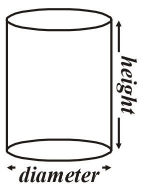


fig.1

Objects like a wooden log, a piece of pipe, a candle, tube light are in the shape of a cylinder. Take a candle. Slice it on the top as shown in the *fig*.1. Lay it down horizontally (*fig*.2). Can you roll it?

Now erect candle up vertically (*fig*.3). Does it roll?

The surface on which the candle rolls is called its ***curved surface.*** The surface on which the candle does not roll, but stands on vertically is the base, which is circular in shape.



Now what are the height and width of the candle? Look at the height and width of the cylinder shown in the figure.

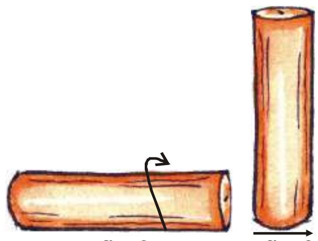
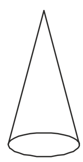


fig.2 *fig*.3

9.6.6 Cone

John wants to buy a special cap for his birthday. He asked Harshini to come along with him. Harshini said that there is no need to go to the market as they can make the cap as shown in the figure. This shape is called cone.

Cone is a three dimensional shape that tappers smoothly from a flat base (generally circle) to a point called vertex.

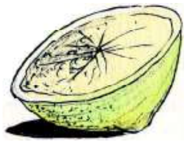


9.6.7 Sphere

Balls, laddoos, marbles etc. are all in the shape of a ***sphere***. They roll freely on all sides.

Can you call a coin a sphere? Does it roll on all its sides? Is the case same with a bangle?

You may have seen lemon in your daily life.



When we cut it horizontally it looks like the shape shown in the figure. The shape of such an object is called ***semisphere / hemisphere.***

Cylinder, cone and sphere have no straight edges.

Identify the shape of these objects and write in the table.

Object	Shape
Match box	
Ball	
Wooden log	
Dice	
Birthday Cap	

9.6.8 Faces, Edges, Vertices - Euler's Formula

We have seen various solids namely cuboid, cube, prism, pyramid, cone, cylinder and sphere. Now let us recall their faces, edges and vertices to strengthen our knowledge about them. Observe the table.

Shapes	Faces (F)	Edges(E)	Vertices (V)	F+V	E+2
Cube	6	12	8	6 + 8 = 14	12+2 = 14
Pyramid	5	8	5	5+5 = 10	8+2 = 10

We understand that $F + V = E + 2$. This relation is called ***Euler's formula.***

Exercise - 9.4

1) Write the shapes of the following.

(i) A brick (ii) A road roller (iii) Foot ball (iv) Joker cap

2) Fill in the blanks.

(i) The shape of heap of grain ___________

(ii) The shape of a dice________________

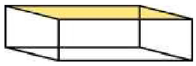
(iii) The shape of a bubble ____________

(iv) The shape of a candle_______________

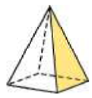
3) Match the following.

a) Pyramid [ ] 1) 

b) Cuboid [ ] 2) 

c) Cylinder [ ] 3) 

d) Cone [ ] 4) 

e) Sphere [ ] 5) 

4) Fill in the table

Shape	No.of faces	No. of vertices	No. of edges
Cube			
Triangular prism			
Square pyramid			
Cuboid			

Verify Euler's Formula for the data in the table.

Unit Exercise

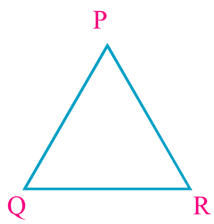
1) Give examples for each shape in the table.

S.No	Sphere	Cylinder	Cube	Cone
1				
2				
3				

2) Look at the adjacent figure and answer the following questions.

(i) What is the name of the triangle?

(ii) Write all sides, angles and vertices of the triangle?

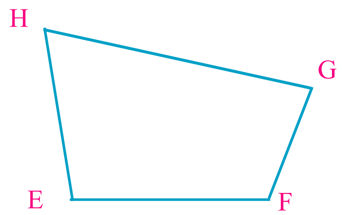


3) Look at the adjacent figure and answer the following questions.

(i) Write the name of this polygon.

(ii) Write the pairs of adjacent sides and adjacent angles.

(iii) Write all vertices, pairs of opposite sides and pairs of opposite angles.



4) Say true or false.

(i) We can locate only one centre in a circle. [ ]

(ii) All chords are called diameters. [ ]

(iii) A square pyramid has squares as its faces. [ ]

Points to Remember

1) A closed figure, formed with a definite number of straight line segments is called a polygon.
2) A triangle is a simple closed figure with three line segments.
3) A triangle has three vertices, three sides and three angles.
4) Interior and exterior of triangle.
5) A simple closed figure with four sides is called a quadrilateral.
6) Line symmetry, multiple line symmetry.
7) 3D shapes (cuboid, cube, prism, pyramid, cone, cylinder, sphere)

N8P8M3

CHAPTER 10

Practical Geometry

Learning Outcomes:-

The students are able to

- estimate the given pair of lines perpendicular or not
- estimate the given pair of lines are angle bisector or not
- explain the constructions of line segment, circle, perpendicular bisector, angle and angle bisector
- draw the line segment, circle, perpendicular bisector, angle and angle bisector

W4K2V6

Content Items

10.0 Introduction

10.1 A line segment

10.2 Construction of a Circle

10.3 Constructing perpendicular bisector of the given line segment

10.4 Construction of angle using protractor

10.5 Constructing a copy of an angle of unknown measure

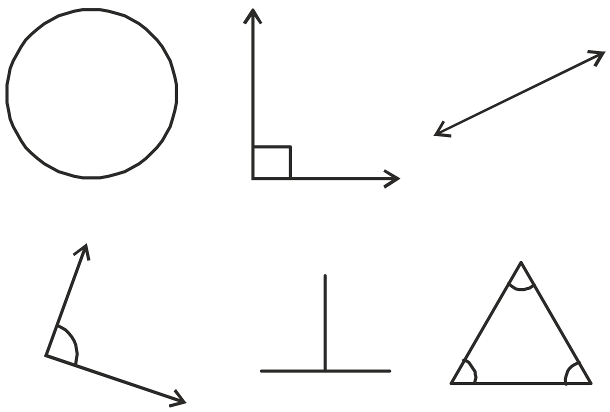
10.6 Drawing parallel lines

10.7 Construction of an angle bisector

10.8 Constructing angles of special measures

10.0 Introduction

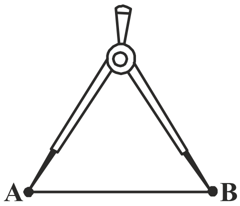
Copy the following shapes in your notebook with a pencil.



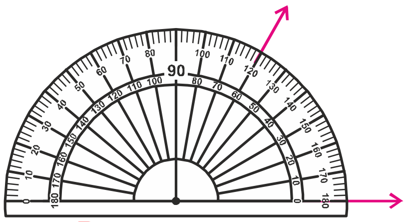
Do they look exactly the same? Measure their sides and angles by ruler and protractor.

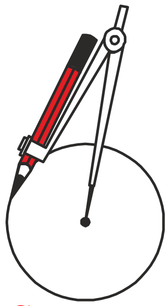
What do you find? You will find their measures are not exactly the same. To make them exactly the same, we need to draw them with accurate sizes. For this, we need to use tools. We will learn to construct such figures, in this chapter by using compasses, ruler and protractor. Ruler, compasses and protractor are our tools. These are all part of our geometry box. Let us observe the geometry box.

What all is there in the geometry box? Besides the ruler, compasses and protractor, we have a divider and set squares. The ruler is used for measuring lines; compasses for drawing arcs, circles; protractor is used to measure angles and the divider to make equal line segments or mark points on a line.



Divider


Protractror


Compass

10.1 A Line Segment

Let A and B be two points on a paper. Then, the straight path from A to B is called a line segment AB, denoted by $\overline{AB}$.

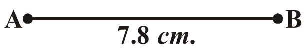
The distance between the points A and B is called the length of AB. Thus, a line segment has a definite length, which can be measured.

10.1.1 Construction of a Line Segment of a given length

We can construct a line segment of given length in two ways.

1. By using ruler: Suppose we want to draw a line segment of length 7.8cm.

We can do it in this way.



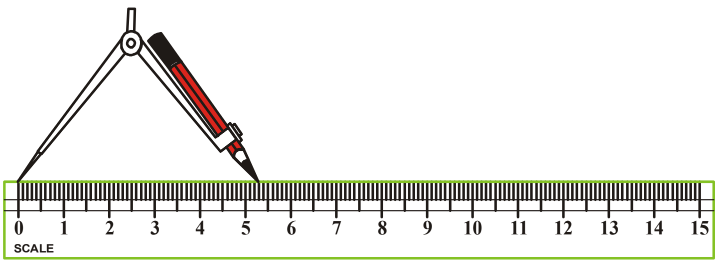
Place the ruler on paper and hold it firmly. Mark a point with a sharp edged pencil against 0 cm mark on the ruler. Name the point as A. Mark another point against 8 small divisions just after the 7 cm mark. Name this point as B. Join points A and B along the edge of the ruler. AB is the required line segment of length 7.8 cm.

2. By using Compasses:

Suppose we want to draw a line segment of length 5.3 cm.

Step-1: Draw a line *l*. Mark a point A on the line *l*.

Step-2: Place the metal pointer of the compasses on the zero mark of the ruler. Open the compasses so that pencil point touches the 5.3 cm mark on the ruler.



***Step-3*:** Place the pointer at A on the line *l* and draw an arc to cut the line. Mark the point where the arc cuts the line as B.

***Step-4*:** On the line *l*, we get the line segment AB of required length.

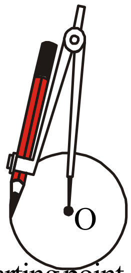
10.2 Construction of a Circle

Look at the wheel shown here. Observe that every point on its boundary is at an equal distance from its centre.

Think of other such objects that are of this shape. Give 5 examples.

How to draw objects and figures having this shape? We can use many things like bangle, bowl top, plate and other things. These are, however, of different sizes. To draw a circle of given radius (the length between the centre and a point on the edge of the circle), we use the compasses.

We use the following steps to construct a circle.



***Step-1*:** Open the compasses for required radius. Let us say for example it is 3.7 *cm*.

***Step-2*:** Mark a point with sharp pencil on the paper. Mark it as O. This is the centre.

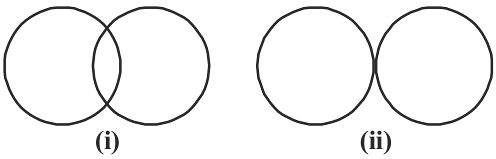
***Step-3*:** Place the pointer of the compasses firmly at O.

***Step-4*:** Without moving its metal point, now slowly rotate the pencil till it comes back to the starting point.

Construct two circles with same radii (plural of radius) in such a way that

(i) the circles intersect at two points.

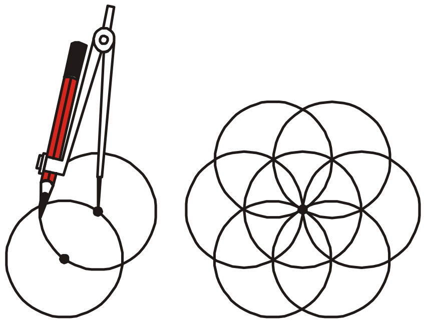
(ii) touch each other at one point only.



Exercise - 10.1

1. Construct a line segment of length 6.9cms using ruler and compass.
2. Construct a line segment of length 4.3cms using ruler.
3. Construct a circle with centre M and radius 4 cm.
4. Draw any circle and mark three points A, Band C such that

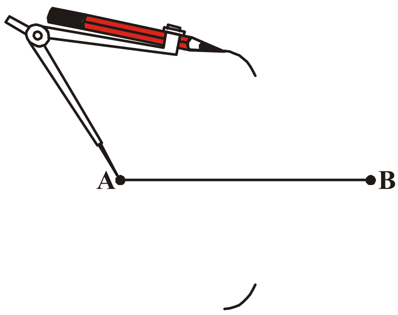
   (i) A is on the circle (ii) B is in the interior of the circle (iii) C is in the exterior of the circle.



Construct a circle of desired radius in your note book. Make a point on it. Put compasses on it and make a circle without changing the radius. It will cut the circumference at two points. On both points repeat the process again, you will get a beautiful picture as shown . Colour it as your wish.

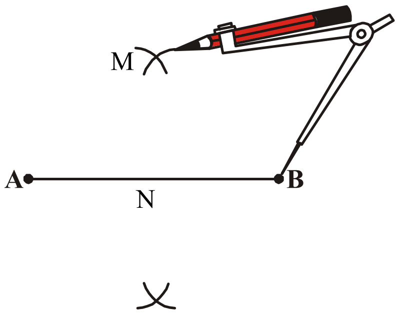
10.3 Constructing Perpendicular Bisector of the given Line Segment

***Step-1*:** Draw a line segment $\overline{AB}$.



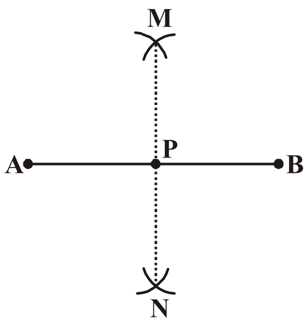
***Step-2*:** Set the compasses as radius with more than half of the length of $\overline{AB}$.

***Step-3*:** With A as centre, draw arcs below and above the line segment.



***Step-4*:** With the same radius and B as centre draw two arcs above and below the line segment to cut the previous arcs. Name the intersecting points of arcs as M and N.

***Step-5*:** Join the points M and N. Then, the line l is the required perpendicular bisector of the line AB. Line l intersects line AB at P.



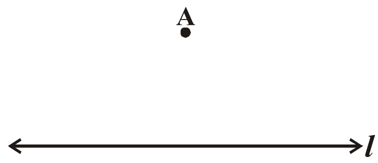
Measure the lengths of $\overline{AP}$ and $\overline{BP}$ in both the constructions. What do you observe ?

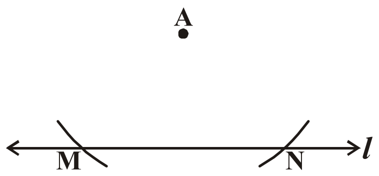
Let's Think

In the construction of perpendicular bisector in step 2, what would happen if we take the length of radius to be smaller than half the length of $\overline{AB}$?

2. Perpendicular to a Line, through a Point which is not on it

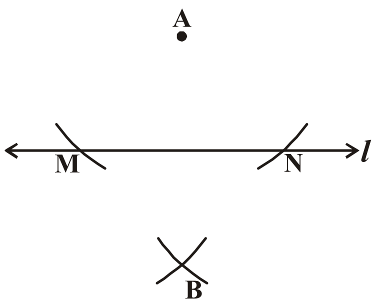
***Step-1*:** Draw a line *l* and a point A not on it.

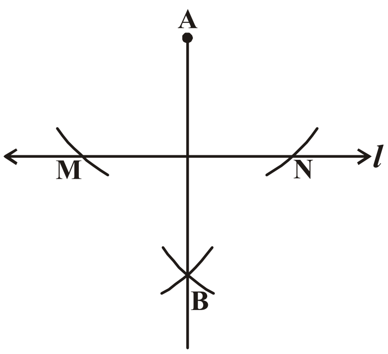




***Step-2*:** With A as centre draw an arc which intersects the given line *l* at two points M and N.

***Step-3*:** Using the same radius or a radius more than half of MN and with M and N as centres construct two arcs that intersect at a point, say B on the other side of the line.

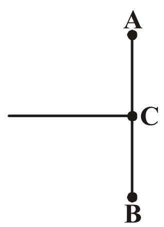




***Step-4*:** Draw a line through A and B. AB is a perpendicular of the given line *l*.

Exercise - 10.2

1. Draw a line segment PQ = 5.8cm and construct its perpendicular bisector using ruler and compass.
2. Ravi made a line segment of length 8.6 cm. He constructed a bisector of AB on C. Find the length of AC and BC.

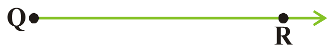


3. Using ruler and compasses, draw AB = 6.4 cm. Locate its mid point by geometric construction.

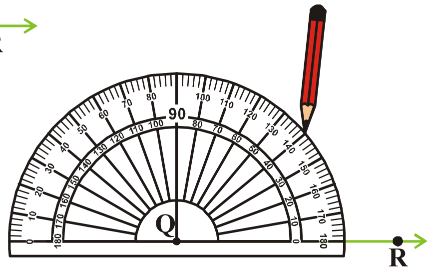
10.4 Construction of Angles using Protractor

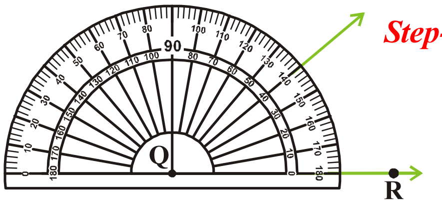
Let us construct $\angle PQR = 40°$.

***Step-1*:** Draw a ray QR of any length.

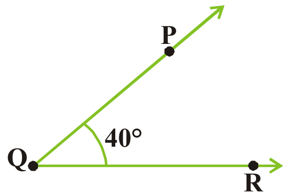


***Step-2*:** Place the centre point of the protractor at Q and the line aligned with $\overrightarrow{QR}$.





***Step-3*:** Mark a point P at 40°.

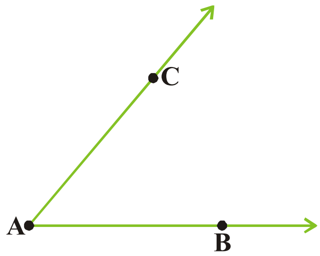


***Step-4*:** Join QP. $\angle PQR$ is the required angle.

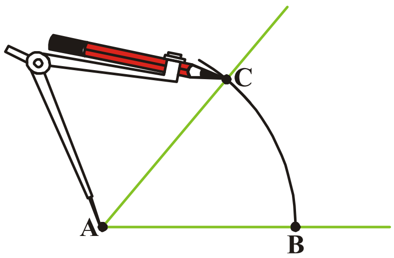
10.5 Constructing a copy of an angle of unknown measure

Suppose an angle (whose measure we do not know) is given and we want to reconstruct this angle.

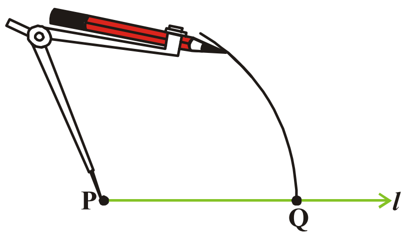
Let $\angle A$ is given, whose measure is not known.



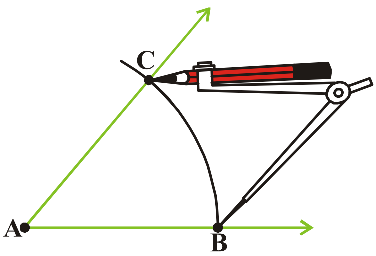
***Step-1*:** Draw a line l and choose a point P on it.



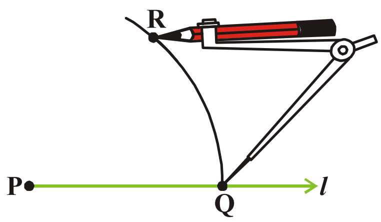
***Step-2*:** Now place the compass at A and draw an arc to cut the rays AC and AB.



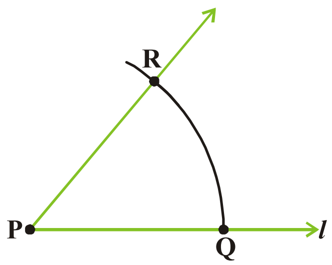
***Step-3*:** Use the same compass setting to draw an arc with P as centre, cutting l at Q.



***Step-4*:** Set your compass with $\overline{BC}$ as the radius.



***Step-5*:** Place the compass pointer at Q and draw an arc to cut the existing arc at R.



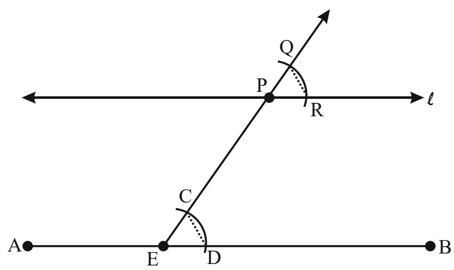
Step-6: Join PR. This gives us ∠RPQ. It has the same measure as ∠CAB.

This means ∠QPR has same measure as ∠BAC.

10.6 Drawing Parallel lines

How to draw a parallel line to the given line, through a given point?

Let $\overline{AB}$ be the line segment and P is the point through which a parallel line in to be drawn to $\overline{AB}$.



Step-1: Draw a line segment $\overline{AB}$.

Step-2: Draw a ray $\overrightarrow{EQ}$ through P (with any convenient angle).

Step-3: Using compass with E as centre draw an arc to cut the ray $\overrightarrow{EP}$ at C and $\overline{AB}$ at D.

Step-4: Without disturbing the compass, draw another arc with P as centre to cut the $\overrightarrow{EP}$ at Q.

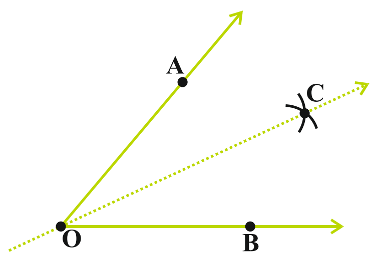
Step-5: With Q as centre, cut the arc drawn through Q at R such that $\overline{CD}$ is equal to $\overline{QR}$.

Step-6: Draw a line ℓ through P and R.

Now the line ℓ will be parallel to $\overline{AB}$.

10.7 Construction to bisect a given angle

By Folding

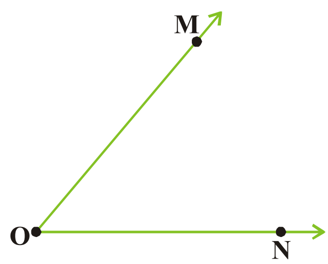


Take a tracing paper. Mark a point O on it. With O as initial point, draw two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. You get ∠AOB. Fold the sheet through O such that the rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ coincide. Let $\overrightarrow{OC}$ be the crease of paper which is obtained after unfolding the paper.

$\overleftrightarrow{OC}$ is clearly a line of symmetry for ∠AOB.

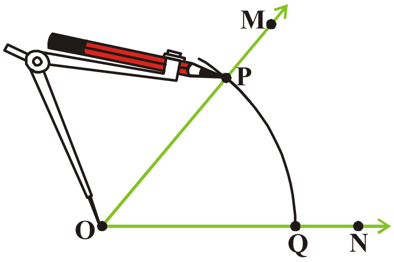
Measure ∠AOC and ∠COB. Are they equal? $\overleftrightarrow{OC}$ the line of symmetry, is therefore known as the angle bisector of ∠AOB.

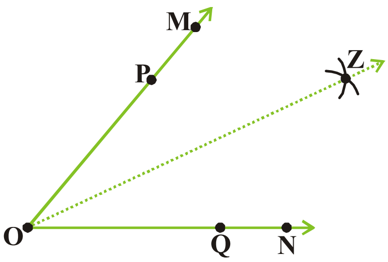
Let an angle say ∠MON be given.



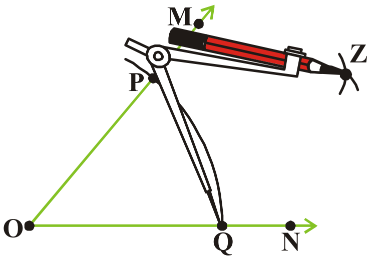
Step-1: With O as centre and any convenient radius, draw an arc $\overset{\frown}{PQ}$ cutting OM and ON at P and Q respectively.

Step-2: With P as centre and any radius slightly more than half of the length of PQ, draw an arc in the interior of the given angle.





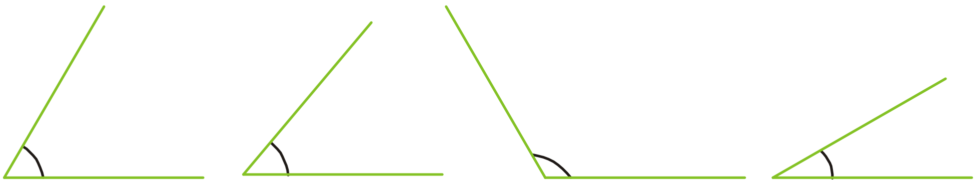
Step-3: With Q as centre and without altering radius (as in step 2) draw another arc in the interior of $\angle MON$. Let the two arcs intersect at Z.



Step-4: Draw ray $\overrightarrow{OZ}$. Then $\overrightarrow{OZ}$ is the desired bisector of $\angle MON$. Observe $\angle MOZ = \angle ZON$.

Exercise - 10.3

1. Construct the following angles with the help of a protractor .
   (i) $\angle ABC = 65°$ (ii) $\angle PQR = 136°$
   (iii) $\angle Y = 45°$ (iv) $\angle O = 172°$
2. Copy the following angles in your note book and find their bisectors:



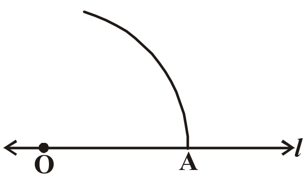
10.8 Constructing angles of special measures

There are some elegant and accurate methods to construct some angles of special sizes which do not require the use of the protractor. A few have been discussed here.

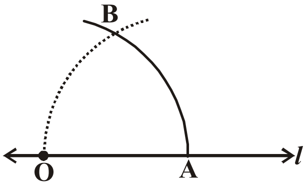
You learnt the construction of any given angle by using a protractor. Now we will learn construction of some angles by using compass only.

10.8.1 Construction of 60° angle

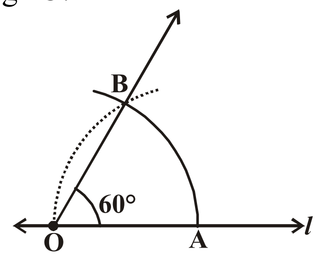
Step-1: Draw a line l and mark a point O on it.



Step-2: Place the pointer of the compass at O and draw an arc of convenient radius which cuts the line l at a point say, A.



***Step-3*:** With the pointer at A (as centre) and the same radius as in the step-2, now draw an arc that passes through O.

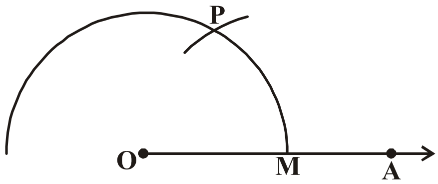


***Step-4*:** Let the two arcs intersect at B. Join OB. We get ∠BOA whose measure is 60°.

10.8.2 Construction of 120° angle

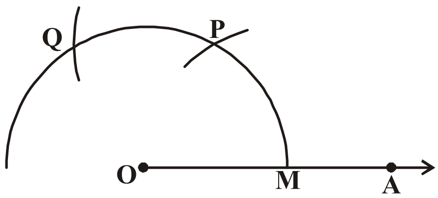
An angle of 120° is nothing but twice of an angle of 60°. Therefore, it can be constructed as follows:

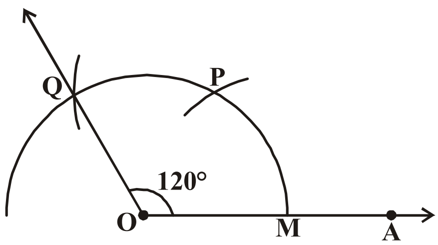
***Step-1*:** Draw any ray OA



***Step-2*:** Place the pointer of the compass at O. With O as centre and any convenient radius draw an arc cutting OA at M.

***Step-3*:** With M as centre and without altering radius (as in step 2), draw an arc which cuts the first arc at P.



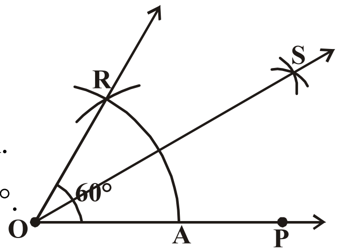


***Step-4*:** With P as centre and without altering the radius (as in step 2) draw an arc which cuts the first arc at Q.

***Step-5*:** Join OQ. Then ∠AOQ is the required angle.

- Construct angles of 180°, 240°, 300°.

10.8.3 Construction of 30°angle



Step-1: Draw an angle of 60° as discussed above. Name it as ∠AOR.

Step-2: Bisect this angle as shown earlier to get two angles each of 30°.

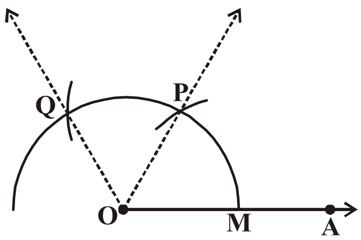
10.8.4 Construction of 90° angle

Look at the given figure

∠AOP = 60°, ∠POQ = 60° and

∠AOQ = 120°

We want to construct an angle of 90°.

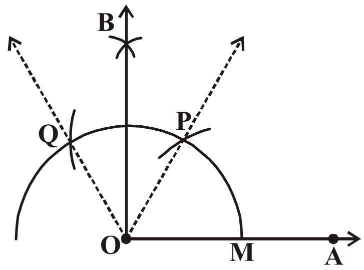


We know that 90° = 60° + 30° and also 90° = 120° - 30°

So, we need to bisect ∠POQ to get an angle of 30°.

∠BOP = 30° and ∠AOB = 90°

Think of one more way to construct a 90° angle.



- Construct an angle of 45° by using compasses.

Exercise - 10.4

1. Construct ∠ABC = 60° without using protractor.
2. Construct an angle of 120° with using protractor and compasses.
3. Construct the following angles using ruler and compasses. Write the steps of construction in each case.
   (i) 75° (ii) 15° (iii.) 105°

Unit Exercise

1. Construct a circle with centre X and diameter 10 cm.
2. Draw four circles of radius 2cm,3cm,4cm and 5cm with the same centre P.
3. Draw the angles given below using a protractor.
   (i) 75° (ii) 15° (iii.) 105°
4. Construct ∠ABC = 50° and then draw another angle ∠XYZ equal to ∠ABC without using a protractor.
5. Construct ∠DEF = 60°. Bisect it, measure each half by using a protractor.

Points to Remember

1. We use the following geometrical instruments to construct shapes:
   (i) A graduated ruler (ii) The compass
   (iii) The divider (iv) Set-squares (v) The protractor
2. Using the ruler and compass, the following constructions can be done.
   (i) A circle, when the length of its radius is known.
   (ii) A line segment, if its length is given.
   (iii) A copy of a line segment.
   (iv) A perpendicular to a line through a point
   (a) on the line (b) not on the line
   (v) The perpendicular bisector of a line segment of given length.
   (vi) An angle of a given measure.
   (vii) A copy of an angle.
   (viii) The bisector of a given angle.
   (ix) Some angles of special measures such as:
   (a) 90° (b) 60° (c) 30° (d) 120°

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CHAPTER 11

Perimeter and Area

Learning Outcomes:-

The students are able to:

- solve the problems involving perimeter and area of rectangle and square from their daily life.
- solve the verbal problems on perimeter and area of a rectangle and square.
- recall the formulae for perimeter and area of rectangle / square and describe the terms.
- establish the relation between the units of perimeter and area.
- solve the problems on perimeter and area involving in various concepts.
- differentiate perimeter and area of a given figure.

H7I3R1

Content Items

11.0 Perimeter

Introduction

Doctor advised Lakshmi to walk two kilometers every day. She wanted to walk in a play – ground nearby. The ground is in the shape of a square of side 100 meters.

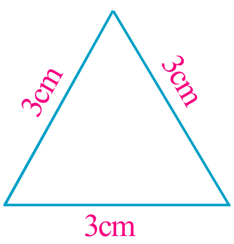
How many times she must walk around the ground to complete two kilometers?

Saleem and Victor bought two rectangular plots. Their plots are 100 m x 80 m and 120m x 60m measurements. They wanted to construct fencing for their plots. If the rate of fencing is fixed per meter who pays more? These types of incidents lead us to find the total length of the edges or sides around the region.

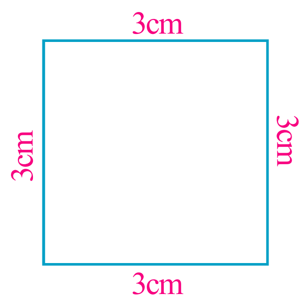
Sum of all the lengths of a polygon is called as its perimeter.

In the previous chapters, we learned about some basic geometric shapes in a plane. These shapes may be with equal sides or may not be with equal sides. For figures with equal sides we can make a formula.

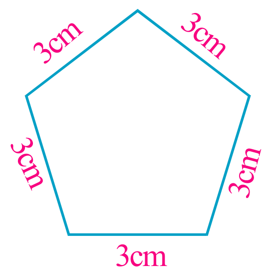
Let us see some examples



3 + 3 + 3
= 3 x 3cm = 9cm



3 + 3 + 3 + 3
= 4 x 3cm = 12cm

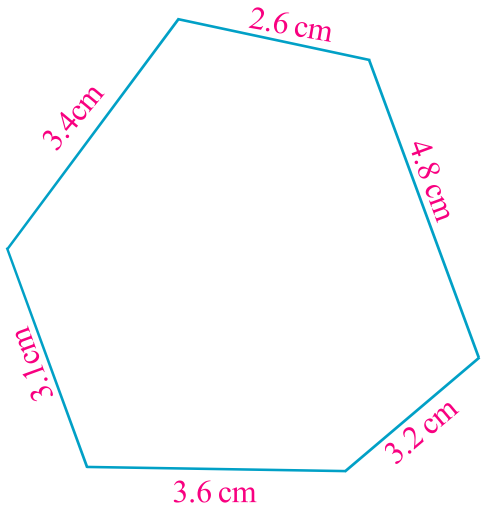


3 + 3 + 3 + 3 + 3
= 5 x 3cm = 15cm

Another example

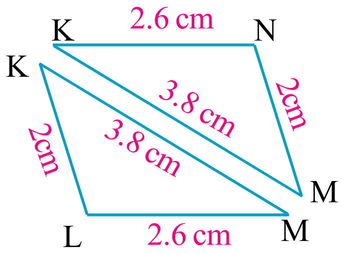
Simhachalam donated his land for a school building. Below is the survey map of his land. Government decided to construct a boundary wall for the school. How much length of the boundary wall is to be constructed?

To find the length, we simply add the lengths of all the sides. The shape of the plot is irregular. Hence we cannot use any specific formula for this.

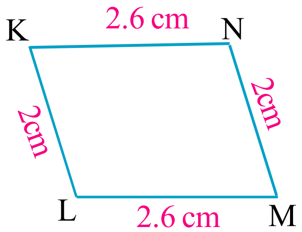


CHECK YOUR PROGRESS

Find the perimeters of the given figures.



(i)



(ii)

In the above figures (i) and (ii), find the perimeter of △KLM,△KMN and □ KLMN.

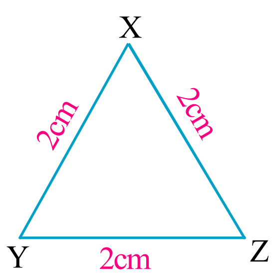
a) Compare the perimeters of △ KLM and □ KLMN.

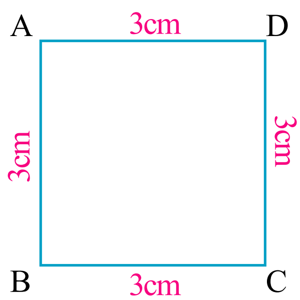
Compare the perimeters of △ KMN and □ KLMN.

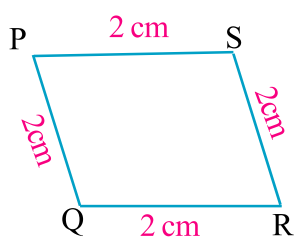
What can you say?

Exercise - 11.1

1) Find the perimeter of the following figures.







i) Check whether the perimeter of $\triangle XYZ$ = 3 x Length of the side?

ii) Check whether the perimeter of $\square ABCD$ = 4 x Length of the side?

iii) Check whether the perimeter of $\square PQRS$ = 4 x Length of the side?

2) Measurements of two rectangular fields are 50m x 30m and 60m x 40m. Find their perimeters. Check whether the perimeters are 2 x length + 2 x breadth.

3) Find the perimeter of

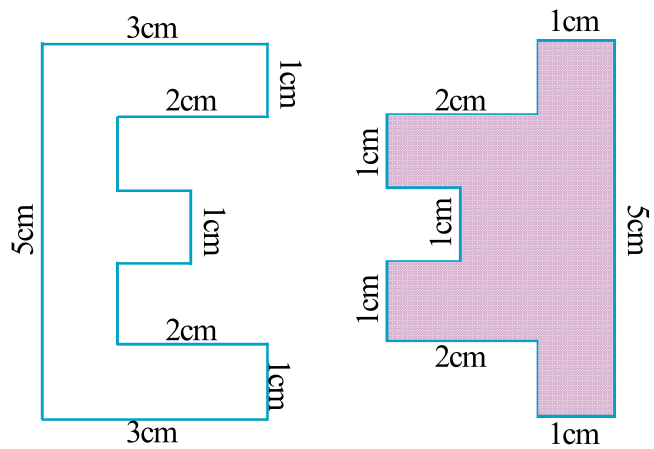
a) An equilateral triangle whose side is 3.5cm.

b) A square whose side is 4.8cm.

4) Length and breadth of top of one table is 160cm and 90cm respectively. Find how much length of beading is required for each table.

5) Manasa has 24cm of metallic wire with her. She wanted to make some polygons with equal sides whose sides are integral without milling into pieces values. Find how many such polygons she can make with the length of 24cm metallic wire?

6) Find the perimeter of the following figures (i) and (ii).



(i) (ii)

7) Statement P : So many rectangles exists with the same perimeter.

Statement Q : So many squares exists with the same perimeter.

Which option is correct?

A) P wrong Q correct

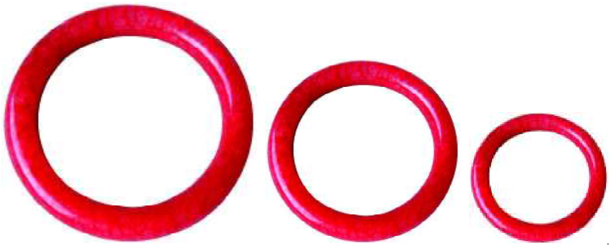
B) P Correct Q wrong

C) P and Q are correct

D) P and Q are wrong.

11.1 Circumference

LET'S DO ACTIVITY



- Take different sizes of bangles or rings.
- Put a bangle or ring in between two straws as shown below.

- Measure the distance ‘d’ between the two straws.
- Mark a point on the bangle and roll.

i) Measure the length that moves in one rotation.

ii) Take 5 bangles of the same type but with ***different measurements of d.***

iii) Repeat the process (i)

iv) Fill the table with observations (measurements)

Bangle Type	Measurement of 'd'	Length of the distance for 1 rotation 'l'	ℓ / d
1)			
2)			
3)			
4)			
5)			

You can observe in the above table that all the values in the column $\frac{l}{d}$ are nearly equal (Constant). That constant is called as **pie (π)** which is nearly equals to $\frac{22}{7}$.

So, $\pi = \frac{l}{d}$ or $l = \pi$ d. Now circumference of the circle is π d

Here d=2r. Hence the circumference of the circle is $2\pi r$

Circumference is represented by the letter C. Therefore C$=2\pi r = \pi$ d

Example-1 : Find the circumference of a circle with radius 7cm. (Take $\pi = \frac{22}{7}$)

Solution : Given radius (r) = 7 cm

Circumference of a circle $= 2\pi$ r (Take $\pi = \frac{22}{7}$)

$$= 2 \text{ x } \frac{22}{\cancel{7}} \text{ x } \cancel{7}^1 = 44 \text{ cm.}$$

Example-2 : If the circumference of the circle is 66cm find its radius.

Solution: Circumference of the circle = 2 π r = 66cm

$$2 \text{ x } \frac{22}{7} \text{ x r = 66 cm dividing both sides with 2 x } \frac{22}{7}$$

$$\frac{2\times\frac{22}{7}\times r}{2\times\frac{22}{7}} = \frac{66}{2\times\frac{22}{7}} \qquad \therefore \text{ radius r} = \frac{\cancel{66}^3\times 7}{2\times\cancel{22}^1} = \frac{21}{2} = 10.5cm$$

LET'S EXPLORE

1) If the radius of a circle is doubled, then, what is the change in its circumference?
2) If the radius is halved, then, what is the change in its circumference?

Exercise - 11.2

1) Find the circumferences of the circles with the radius given below.
   A) 7 cm B) 3.5 cm C) 14 cm
2) Given below are the circumferences of different circles. Find the radius of each circle.
   A) 4.4m B) 176 cm C) 1.54 cm
3) A gold smith has 8.8m of gold wire with him. He has to make gold rings of 2cm radius. How many such rings he can make with it?
4) A wire was bent in the shape of a circle with radius 7cm. If the same wire was again used to make a square, then find its side.

5) In a chemical factory two wheels of different radius were connected with a belt. Radius of the bigger wheel is 21cm and radius of the smaller wheel is 7cm. If the bigger wheel rotates completely 100 times, find out the number of times that the smaller wheel rotates.

6) Mohan is playing with a ring of diameter 14 cm, which is made up of metallic wire. When his brother asked, Mohan stretched the wire and made it as two equal parts. With those parts, he made another two small rings. Find the radius of smaller ring.

7) In designing an iron gril a black smith needed 70 rings with radius of 7cm each. Find how much length of the rod he required, if the wastage is 20 cm.

11.2 Areas of rectangles and squares

Bujji and his father went to buy flooring tiles for their house. They selected a design. The shopkeeper told that a square foot costs ₹125. Bujji's father purchased the tiles. while returning back Bujji asked his father. What is a square foot?

Sometimes length is measured in feet. The place occupied by a square of side 1 foot is called 1 square foot. ***The place occupied by an object is called area of the object. Area is measured in square units.***

Example-3 : Length of a rectangle is 16 cm and breadth is 13 cm. Find the area of the rectangle.

Solution : Given length of the rectangle (l) = 16 cm

Breadth of the rectangle (b) = 12 cm

Area of the rectangle = Length x Breadth = 16x12 = 192 square centimeters.

Example-4:

A rectangular piece with dimensions 12cm x 8cm (Length 12cm, Breadth 8cm) was taken from a square piece of paper whose side is 16 cm. Find the area of the remaining piece.

Solution: Side of the square (s) = 16 cm

Area of the square = s x s = 16 x 16 = 256 square centimeters (sq.cm).

Length and breadth of rectangle (l, b) are = 12cm, 8cm respectively.

Area of the rectangle = l x b = 12 x 8

= 96 sq.cm

Area of the remaining piece = area of a square – area of a rectangle

= 256 – 96 = 160 sq.cm

CHECK YOUR PROGRESS

1) Find the area of a square with side 16 cm.
2) Length and breadth of a rectangle are 16 cm and 12 cm respectively. Find its area.

Let's Think

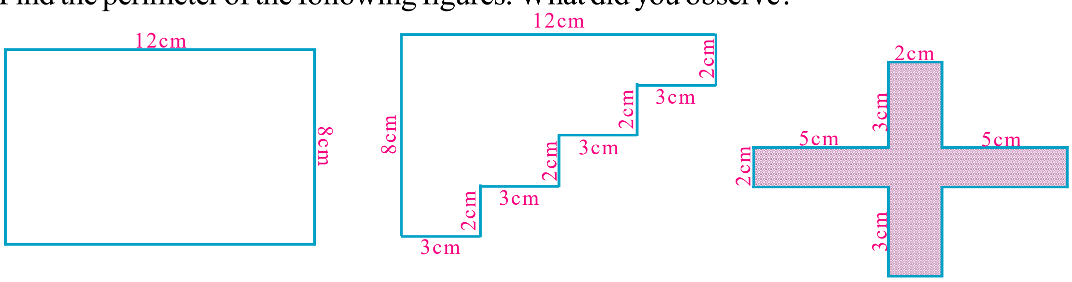
Find the perimeter and area of a square of side 4 cm.
Are these same? Give some examples to support your answer.

Exercise - 11.3

1. Find the area of the rectangle with measurements 15cm and 8cm as length and breadth respectively.
2. Find the area of a square whose perimeter is 64m.
3. Perimeters of a rectangle and square are equal. If the length of the rectangle is 14 cm and the perimeter of the square is 44 cm, find the area of the rectangle.
4. Find the perimeters and areas of the following and answer the questions.
   A) A rectangle with length and breadth as 16 cm and 8cm respectively.
   B) A rectangle with length and breadth as 14 cm and 10cm respectively.
   C) A square with side 12 cm.
      (i) Which of the above perimeters are equal ?
      (ii) Are all these areas equal? If not, which one has the bigger area?

Unit Exercise

1. Find the area of the square whose perimeter is 48cm.
2. If the length of a rectangle is 14cm and its perimeter is 3times of its length. Find its area
3. Find circumference of the circle whose diameter is 14cm.
4. 14cm and 12cm are the length and breadth of a rectangle. If the breadth is increased by 6cm and length is decreased by 6cm, find the difference in areas.
5. Find the perimeter of the following figures. What did you observe?



6. A square sheet of 8cm side was taken and made into 64 equal small squares. Find the perimeter of square sheet and also find the sum of the perimeters of all 64 small squares. What did you observe?

Points to Remember

1. Perimeter of any polygon is sum of the lengths of all sides.
2. Perimeter of an equilateral triangle is $3\times$ length of the side.
3. Perimeter of a rectangle is ($2\times$ length + $2\times$ breadth).
4. Perimeter of a square is $4\times$side.
5. Circumference of a circle is $2\pi r$, where r is the radius.
6. Area of a rectangle is $l\times b$, where l is the length and b is the breadth.
7. Area of a square is $s\times s$, where s is the side of the square.

M7W3H7

CHAPTER 12

Data Handling

Learning Outcomes:-

The Students are able to

- arrange raw data into classified data
- understand the usage of bargraph, pictograph in daily life
- represent the data into pictographs and bargraphs
- interprete the tabular data and converts into verbal form

Content Items

F2D2G4

12.0 Introduction

Siri's father wants to buy a mobile phone. He asks his friends about the different types of models available in the market and writes their prices and features.

He prepares the following table:

Features	**Brand-1 Mobile**	**Brand-2 Mobile**	**Brand-3 Mobile**
Price	₹7500	₹6000	₹ 9000
MP3	✓	✓	✓
Camera	×	×	✓
Bluetooth	×	×	✓
Alarm	✓	✓	✓
FM	✓	×	✓
Guarantee Period	1 year	3 months	6 months

Siri asked her father, why he prepared the table? Her father replied, "I want to buy a mobile. To find a model that suits my needs, I have to compare the features of the different models. So, I have collected all the information and then organised in the form of a table."

Siri liked the idea, that for taking the right decision it is often necessary to **collect information** and **organise** it.

Information either in the form of numbers or words, which helps us to take decisions is called ***data***. In the above example, the price of the mobile phones, the presence or absence of a camera in cell phone, the presence and absence of FM in cellphones etc., is all data. In our daily life we come across several situations where we collect information to take decisions.

12.1 Recording of data

Laxmi is preparing to go for a picnic with her friends. She has to take fruits for everybody in the picnic. Laxmi's mother asked her to find the required number of fruits each type. Laxmi prepared a list like this:

Person	Like to have
Laxmi	Orange
Preeti	Guava
Radha	Orange
Uma	Custard apple
Reshma	Guava
Mary	Orange
Latha	Orange
Gouri	Banana
Salma	Custard Apple
Rita	Guava

LET'S EXPLORE

- Give two examples of ***data in numerical figures.***
- Give two examples of ***data in words.***

She gave the list to her mother. Her mother read the list. To find the number of fruits required for each type, first counted the number of oranges by going over all the names in the list. She then repeated this process for the guavas, then the bananas and then the custard apples.

She finally wrote as

Oranges - 4, Guava - 3, Banana - 1, Custard apple - 2

Here Oranges came 4 times. So 4 is called the ***frequency*** of an orange. Similarly frequency of Guava is 3 ....

Would it have been so easy for Laxmi's mother to count, if the number of children in class had been 50?

Laxmi's mother needs a way in which she can count all the fruits simultaniously.

12.2 Organisation of Data

In Census 2011, an enumerator collected information about the family size of 55 families in a habitation.

Sarala made tally marks by crossing every four tally marks with a fifth tally mark.

Family size	Tally Marks	Number of families
2	卌 |	6
3	卌 卌 卌 ||||	19
4	卌 卌 卌 卌 |||	23
5	卌	5
6	||	2

The manner in which Sarala has made the tally marks is generally used to obtain the frequency or the count of the data items. ***A table showing the frequency or count of various items is called a frequency distribution table.***

Example-1. 25 students in a class got the following marks in an assignment- 5, 6, 7, 5, 4, 2, 2, 9, 10, 2, 4, 7, 4, 6, 9, 5, 5, 4, 3, 7, 9, 5, 2, 4, 5, 7. The assignment was for 10 marks.

(i) Organise the data and represent in the form of a frequency distribution table using tally marks.

(ii) Find out the marks obtained by maximum number of students.

(iii) Find out how many students received least marks.

(iv) How many students got 8 marks?

***Solution*:**

(i)

Marks obtained	Tally Marks	Number of Students
2	||||	4
4	卌	5
5	卌 |	6
6	||	2
7	||||	4
9	|||	3
10	|	1

(ii) Maximum number of students (6) got 5 marks

(iii) Least mark (2) was obtained by 4 students.

(iv) No student in the class got 8 marks.

Exercise - 12.1

1. The favourite colours of 25 students in a class are given below:

   Blue, Red, Green, White, Blue, Green, White, Red, Orange, Green, Blue, White, Blue, Orange, Blue, Blue, White, Red, White, White, Red, Green, Blue, Blue, White.
   Write a frequency distribution table using tally marks for the data. Which is the least favourite colour for the students?

2. A TV channel invited a SMS poll on 'Ban of Liquor' giving options :

   A - Complete ban B - Partial ban C - Continue sales

   They received the following SMS, in the first hour-

A	A	B	C	A	B	B	C	A	A
A	A	C	C	B	A	A	C	B	A
A	A	A	B	B	C	C	A	A	C
C	B	B	B	A	A	A	A	A	C

   Represent the data in a frequency distribution table using tally marks.

3. Vehicles that crossed a checkpost between 10 AM and 11 AM are as follows:
   car, lorry, bus, lorry, auto, lorry, lorry, bus, auto, bike, bus, lorry, lorry, zeep, lorry, bus, zeep, car, bike, bus, car, lorry, bus, lorry, bus, bike, car, zeep, bus, lorry, lorry, bus, car, car, bike, auto.
   Represent the data in a frequency distribution table using tally marks.

LET'S DO ACTIVITY

Take a die. Throw it and record the number. Repeat the activity 40 times and record the numbers. Represent the data in a frequency distribution table using tally marks.

12.3 Representation of Data

Data that has been organised and presented in frequency distribution tables can also be presented using pictographs and bar graphs.

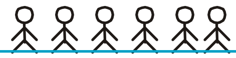
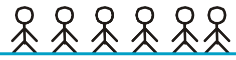
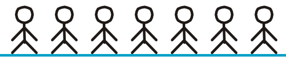
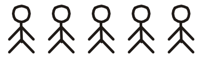
Let us represent the strength of a school in the form of a pictograph.

Class	VI	VII	VIII	IX	X
Number of Students	30	30	35	25	20

Is it reasonable to represent 35 students using 35 symbols? To draw the pictograph conveniently, in such situations we can assume that 5 students can be represented by one symbol. This is called ***scaling***. Generally the scale must be the Greatest Common Divisor of all the frequencies.

Scale  represents 5 students

Now, let us construct a pictograph for the data given above-

Class	Number of Students
VI	
VII	
VIII	
IX	
X	

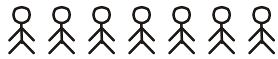
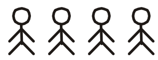
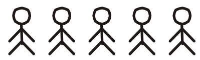
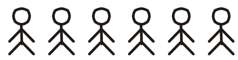
Example-2: In a class of 25, students like various games. The details are shown in the following pictograph. (No student plays more than one game).

(i) How many students play badminton?

(ii) Which game is played by most number of students?

(iii) What is the game in which least number of students are interested?

(iv) How many students do not play any game?

Game	Number of Students
Kabaddi	
Tennikoit	
Badminton	
Cricket	

Scale  represents 1 student

Solution: i. 5 students play badminton.

ii. Kabaddi is played by most number of students i.e. 7.

iii. Tennikoit is played by least number of students i.e. 4.

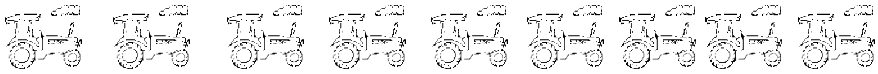
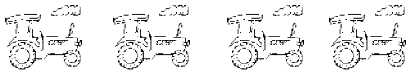
iv. Total number of players = 7 + 4 + 5 + 6 = 22

Number of students in the classroom = 25

Thus, number of student who do not play any game = 25 - 22 = 3

Example-3:The following pictograph shows the number of tractors in five different villages.

Scale :  = 2 Tractors

Village	Number
A	
B	
C	
D	
E	

(i) Which village has the minimum number of tractors?

(ii) Which village has the maximum number of tractors?

(iii) How many more tractors does village C compared to village B.

(iv) What is the total number of tractors in all the five villages?

***Solution*:** (i) Villages B and E have the minimum number of tractors, 8 tractors each.

(ii) Village D has the maximum number of tractors, 20 tractors.

(iii) Village C has 10 tractors more than B.

(iv) There are 66 tractors in all in the village.

Exercise - 12.2

1. The number of wrist watches manufactured by a factory in a week are as follows:

Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**
300	350	250	400	300	275

Represent the data using a pictograph. Choose a suitable scale.

2. Votes polled for various candidates in a Sarpanch election are shown below, against their symbols in the following table.

Symbol	Sun	Pot	Tree	Watch
Number of votes	400	550	350	200

Represent the data using a pictograph. Choose a suitable scale.

Answer the following questions:

(i) Which symbol got least votes?

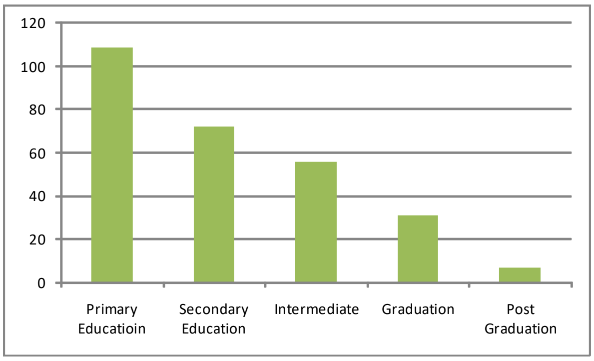
(ii) Which symbol candidate won in the election?

• Collect as many pictographs as possible from news papers and magazines and study them carefully and prepare questions on your own

12.4 Bar Graph

Akash collected data about the educational qualifications of 275 people in his locality. He organised the data into a frequency distribution table:

Education Level	Number of People
Primary Education	109
Secondary Education	72
Intermediate	56
Graduation	31
Post graduation	7



He tried to represent the data using a pictograph. But he found that this is not only time consuming but also difficult. So he decided to use bar graph, which is shown in the above figure.

Generally, bar graphs are used to represent independent observations with frequencies.

In bar graph, bars of uniform width are drawn horizontally or vertically with equal spacing between them. The length of the bars represents the frequency of the data items

From the above bar graph we can observe that most people have not studied beyond school. It also shows that a few people hold post graduate degrees.

• *In what way is the bar graph better than the pictograph?*

Construction of a bar graph

The professions of people living in a colony are given in the following table:

Profession	Farmers	Businessmen	Private Employees	Govt. Employees	Labourers
No. of persons	40	10	15	35	5

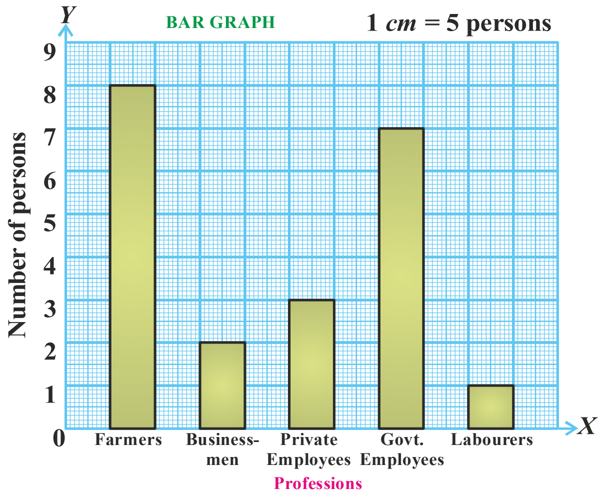
To represent the above data in the form of a vertical bar diagram, the steps are given below:

(i) Draw two perpendicular lines-one horizontal (X-axis) and one vertical (Y-axis).

(ii) Along the Y-axis mark 'number of people' and along the X-axis mark 'professions'.

(iii) Select a suitable scale on the X-axis, say 1 cm = 5 persons.

(iv) Calculate the heights of the bars by dividing the frequencies with the scale:



Farmer $40 \div 5 = 8$ Businessman $10 \div 5 = 2$

Private Employees $15 \div 5 = 3$ Govt. Employees $35 \div 5 = 7$

Labourers $5 \div 5 = 1$

(v) Draw rectangular, vertical bars of same width on the X-axis with heights calculated above. Similarly let us represent the above data as horizantal bar diagram.

Steps of construction:

(i) Draw two perpendicular lines on a graph sheet -one horizontal (X-axis) and one vertical (Y-axis).

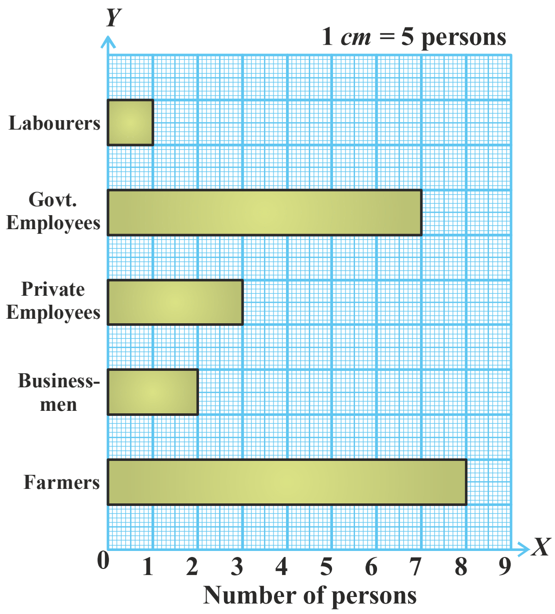
(ii) Along the X-axis mark 'number of persons' and along the Y-axis mark 'professions'.

(iii) Select a suitable scale on the Y-axis, say 1 cm = 5 persons.

(iv) Calculate the lenghts of the bars by dividing the frequencies with the scale:

Farmers	$40 \div 5 = 8$
Businessman	$10 \div 5 = 2$
Private Employees	$15 \div 5 = 3$
Govt. Employees	$35 \div 5 = 7$
Labourers	$5 \div 5 = 1$

(v) Draw rectangular, horizontal bars of same width on the Y-axis with lengths calculated above.



Exercise - 12.3

1. The life spans of some animals are given below:

   Bear - 40 years, Camel - 50 years, Cat - 25 years, Donkey - 45 years, Goat - 15 years, Horse - 10 years, Elephant - 70 years.

   Draw a horizontal bar graph to represent the data.

2. Travelling time from Hyderabad to Tirupati by different means of transport are-

   Car - 8 hours, Bus - 15 hours, Train - 12 hours, Aeroplane - 1 hour. Represent the information using a bar diagram.

3. A survey of 120 school students was conducted to find which activity they prefer to do in their free time.

Preferred activity	Playing	Reading story books	Watching TV	Listening to music	Painting
Number of students	25	10	40	10	15

Draw a bar graph to illustrate the above data.

PROJECT

1. Collect different kinds of bar graphs from news papers, magazines etc. and make an album. Try to interpret each of the bar graphs.

2. Go round your colony. Note how many houses of different kinds i.e. thached houses, tiled housed, RCC slab houses, appartments are there. Tabulate the findings and represent the data as a bar graph.

Unit Exercise

1. Given below are the ages of 20 Students of Class VI in a School.

   i) Organise the data and represent in the form of a freequency distribution table using tally marks.

   ii) Find out the age having more number of students.

   iii) How many students are there in 10 Year age?

   iv) Find out No. of Students who are having more age.

   13, 10, 11, 12, 10, 11, 11, 13, 12, 11

   10, 11, 12, 11, 13, 11, 10, 13, 10, 12

2. A dice was thrown 30 times and following scores were obtained

5	3	4	6	2	3	6	2	2	3	1	5
2	5	4	6	2	1	4	5	1	6	2	1
3	1	3	3	4	6						

I1I9M3

i) Prepare a frequency table of the scores.
ii) Which number obtained more times?
iii) How many times was a score greater than 4 obtained.
iv) Find the total number of times an odd number obtained.

3. Following is the data regarding pass percentage of students in different calsses.

Classes	VI	VII	VIII	IX	X
% of Passing	65	75	85	60	80

Draw a vertical Bar graph to represent the above data.

4. The number of Mathematics books sold by a shopkeeper by six consecutive days is shown below.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of Books Sold	65	40	30	50	70	20

Draw a Horizontal Bar graph to represent the above data.

P. C. Mahalanobis (India)
1893 - 1972
He is known as Father of Indian Statastics.
He is the founder of Indian Statastical Research Institute in Kolkatta. His 'National sample surveys' gained international recognition.

Points to Remember

1. Data is a collection of numbers gathered to give some information.
2. To get a particular information from the given data quickly, the data can be arranged in a tabular form using tally marks.
3. We learnt how a pictograph represents data in the form of pictures, objects or parts of objects. We have also seen how to interpret a pictograph and answer the related questions. We have drawn pictographs using symbols to represent a certain number of items or things. For example,  = 100 books.
4. We have discussed about representation of data by using a bar diagram or a bar graph. In a bar graph, bars of uniform width are drawn horizontally or vertically with equal spacing between them. The length of each bar represents the respective frequency.

ANSWERS

Chapter 1

Exercise -1.1

1) Greatest Smallest
   i) 76547 15476
   ii) 274347 64567
2) i) 24625, 75645, 77845, 85690
   Descending order: 85690, 77845, 75645, 24625
   ii) 6790, 16176, 27895, 50000
   Descending order: 50000, 27895, 16176, 6790
3) i) Seventy three thousand and sixty two
   ii) One lakh eighty thousand five hundred and sixty five
   iii) Twenty five lakhs forty five thousand five hundred and five
4) i) 60,066 ii) 78,414 iii) 9,96,090
5) ii) Greatest 8752 Smallest 2578 Difference 6174
   iii) Greatest 98640 Smallest 40689 Difference 57951
   iv) Greatest 98743 Smallest 34789 Difference 93951
6) 5670, 5607, 5067, 5076, 5760, 5706, 6057, 6075, 6750, 6705, 6507, 6570, 7065, 7056, 7605, 7650, 7506, 7560

Exercise-1.2

1) i) 60,75,92,502 ii) 944,60,55,486 iii) 10,00,10,010
2) i) 57657560- Five crore Seventy six lakh fifty seven thousand five hundred and sixty
   ii) 70560762- Seven crore Five lakh fifty Sixty thousand Seven hundred and Sixty two
   iii) 97256775613-Nine thousand seven hundred twenty five crores Sixty seven lakh Seventy five thousand Six hundred and thirteen
3) i) 756723 - 7,00,000+56,000+700+20+3
   ii) 6056724 - 6,00,00,000+5,00,000+60,000+7000 +200+30+4
   iii) 8500756762 - 800,00,00,000+50,00,00,000+ 7,00,000+ 50,000+6000+700+60+2
4) 5,94,000

Exercise-1.3

1) i) 9,700,605 - 9,000,000+700,000+600+5
   ii) 700,872,407- 700,000,000+800,000+70,000 +2000+400+7
2) i) 717,858- Seven Hundred and seventeen thousand eight hundred and fifty eight
   ii)3,250,672- Three million two hundred fifty thousand and six hundred and seventy two
   iii)75,623,562- Seventy five millions six hundred twenty three thousand five hundred and sixty two
   iv) 956,237,676- Nine hundred fifty six millions two hundred thirty seven thousand six hundred and seventy six
3) i) 67,56,327 - Sixty seven lakhs fifty six thousand three hundred twenty seven
   6,756,327-Six million seven hundred fifty six thousand three hundred and twenty seven
   ii) 4,56,07,087- Four crore fifty six lakh seven thousand and eighty seven
   45,607,087 – Forty five million six hundred and seven thousands and eighty seven
   iii) 856,07,07,236 - Eight hundred fifty six crores seven lakh seven thousand two hundred and thirty six
   8,560,707,236 - Eight billion five hundred sixty millions seven hundred seven thousand two hundred and thirty six
5) i) Two hundred ninty three millions five hundred fifty six thousand seven hundred and fifty three
   ii) Ten Billions nine million nine hundred thousands fifty three
6 i) Nine hundred two crore forty lakhs fifty thousand seventy two
   ii) seventy thousand crore sixty lakhs four thousand seven hundred and five

Exercise-1.4

2) 4500 gms - 4.5 kg
3) 14670 kg - 146.7 quintal
4) 25,13,22,872
5) 21246 runs
6) 90110 votes
7) 987640-406789 = 58085
8) ₹. 9976

Unit Exercise

1) ii) 20,497,096,472
2) i) Indian – Eight hundred twenty seven crores fifty six lakh seventy eight thousand nine hundred and sixty
   International- Eight billion two hundred seventy five million six hundred seventy eight thousand nine hundred sixty
   ii) Indian- Five hundred seventy two crore fifty five lakhs three thousand and twenty seven
   International- Five billion seven hundred twenty four millions five hundred thousand three hundred and twenty seven
   iii)Indian- One hundred twenty three crores forty five lakhs sixty seven thousand eight hundred and ninety
   International-One billion two hundred and thirty four millions five hundred and sixty seven thousand eight hundred ninety
3) 79, 92,000
4) 900000
5) Thousand thousands
7) 5 lakhs

Chapter- 2

Exercise-2.1

1) 18
3) i) 895 ii) 10001 iii) 15678

Exercise-2.2

1) i) 1095 ii) 600
2) i) 196300 ii) 1530000
3) i) 407745 ii) 200955
4) ₹ 5000

Exercise-2.3

1) $123456 \times 8+6= 987654$

   $1234567 \times 8+6= 9876543$

   $12345678 \times 8+6= 98765432$

   $123456789 \times 8+6= 987654321$

Unit Exercise

1) i) $>$ ii) $>$ iii) $<$ iv) $>$
3) i) true ii) true iii) false
4) i) 532 ii) c iii) 85
5) i) 11040 ii) 388710
6) ₹ 330
7) i) C ii) E iii) B iv) A v) D
8) 91
9) 91, 91, 91, 91

Chapter - 3

Exercise - 3.1

1) I,II,III,V
2) 250,1250,45880
3) All 3 digits numbers framed by 2,3,4 are divisible by 9
4) i) 56- it is divisible by 2
   ii) 67- not divisible by 2,3,5,6,9
   iii)75-divisible by 3,5
5) 2
6) 6
7) 102,108,114,120,126,132,138,144,150,156, 162,168,174,180,186,192,198
8) 9999 - it is divisible by both 3 and 9
9) ii,iii and iv
10) 12344

Exercise - 3.2

1) i,ii,and iv are divisible by 11
2) 2002,2013,2024,2035,2046, 2057,2068,2079,2090
3) 1232

Exercise - 3.3

1) i) 24-1,2,3,4,6,8,12,24
   ii) 56- 1,2,4,7,8,14,28,56
   iii) 80-1,2,4,5,8,10,20,40,80,
   iv) 98-1,2,7,14,49,98
2) 97
3) (17,71), (37,73)
4) i) 18=7+11 ii) 24= 5+19 iii) 36= 5+31
   iv) 44= 7+37
5) 90,91,92,93,94,95,96
6) 29-19=10 , 23-10=13
7) (2,3), (3,7),(11,19)

Exercise - 3.4

5) 210

Exercise - 3.5

1) i) 16 ii) 18 iii) 4 iv) 5
2) 15
3) 31

Exercise - 3.6

1) i) 60 ii) 75 iii) 42
2) i) 2352 ii)2142 iii) 1980
3) 247
4) 900
5) 12

Exercise - 3.7

1) i) LCM- 120 HCF- 3
   ii) LCM- 200 HCF- 1
   iii) LCM- 48 HCF- 12
   iv) LCM- 240 HCF- 6
2) 25 3) 546 4) 18 5) yes 6) no

Unit Exercise

1) Divisible by 10 - 5500,14560
   Divisible by 9 - 972,45813
   Divisible by 8 - 14560,1790184
   Divisible by 6 - 912,179084
   Divisible by 2 - 912,5500,14560,1790184
3) i)1 ii) 2 iii) 1
4) 7 5) 9 6) 24 7) 8 8) 27

Chapter-4

Exercise - 4.1

1) i) true ii) false iii) true iv) true v) true
2) i) 1 ii) 5 iii) 9 iv) -3,-4,-5 or if any
   v) -1,0,1 or if any

Exercise - 4.2

1) i) $<$ ii) $>$ iii) $<$
2) i) increasing order: -7,3.5
   decreasing order: 5,-3,-7
   ii) increasing order: -1,0,3
   decreasing order: 3,0,-1
   iii) increasing order: -5,-3,-1
   decreasing order: -1,-3,-5
3) i) true ii) false iii) true iv) false
4) i) 0 ii) -4,-3,-2,-1 iii) -7 iv) -1,-2
5) Kufri

Exercise - 4.3

1) i) 1 ii) -10 iii) -9

2) i) 7 ii) 6 iii) 0
3) i) -154 ii) -40
4) i) 6 ii) -78

Exercise - 4.4

1) i) 18 ii) -14 iii) -33 iv) -33 v) 44 v) 19
2) i) < ii) > iii) >
3) i) 13 ii) 0 iii) -9 iv) -8

Unit Exercise

1) i) 225 ii) -1250 iii) -12°C iv) -3800
2) i) true ii) true iii) true iv) true v)true

Chapter - 5

Exercise - 5.1

1) Proper : $\frac{3}{4}, \frac{2}{3}, \frac{1}{4}, \frac{1}{4}$

Improper : $\frac{6}{5}, \frac{3}{2}, \frac{4}{1}, -\frac{18}{13}$

Mixed : $1\frac{5}{7}, 11\frac{1}{2}$

2) i) $\frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{3}{2}, \frac{8}{7}$

ii) $\frac{4}{9}, \frac{2}{7}, \frac{3}{4}, \frac{3}{8}, \frac{5}{7}$

3) $2\frac{1}{2}$ 4) $\frac{8}{15}$

5) i) $\frac{11}{15}$ ii) $\frac{29}{21}$ iii) $\frac{1}{4}$ iv) $\frac{32}{100}$

Exercise - 5.2

1) i) $1\frac{1}{4}$ ii) $22\frac{1}{2}$ iii) 1 iv) $1\frac{13}{35}$

2) i) $\frac{1}{2}$ of $\frac{6}{7}$ ii) $\frac{3}{5}$ of $\frac{5}{8}$

3) i) 210 ii) 60 iii) $\frac{32}{7}$ iv) $\frac{3}{70}$

4) ₹ 387 5) $144\frac{3}{8}$ km

Exercise - 5.3

1) i) $\frac{9}{5}$ ii) $\frac{7}{12}$ iii) $\frac{5}{11}$ iv) 8 v) $\frac{11}{13}$ vi) $\frac{3}{8}$

2) i) 20 ii) 3 and $\frac{9}{11}$ iii) $\frac{9}{7}$ iv $\frac{4}{135}$ v) $\frac{31}{98}$

3) i) $\frac{2}{3}$ ii) $\frac{1}{2}$ iii) $\frac{35}{9}$ iv) $\frac{30}{7}$

4) $\frac{31}{8}$ 5) $\frac{68}{117}$ 6) 23 7) 32 and $\frac{2}{5}$ 8) 5 and $\frac{1}{3}$

Exercise - 5.4

1) i) and ii)
2) i) 0.802, 54.320, 873.274
ii) 4.780, 9.193, 11.300
iii) 16.003, 16.900, 16.190
3) a) 7.2, 7.26, 7.62
b) 0.446, 0.464, 0.644, 0.664
c) 30.000, 30.060, 30.30
4) 16.99, 16.96, 16.42, 16.3, 16.1, 16.03, 16.01
5) i) > ii) > iii) =

Exercise - 5.5

1) i) 19.242 ii) 129.296
2) i) 141.844 ii) 1.991 iii) 1.797
3) m 4.50
4) i) ₹ 90.75 ii) m 49.20 iii) kg 12.450 iv) 150.500
5) i) kg 118.450 ii) m 152.45
6) i) kg 2.024 ii) 11.08

Unit Exercise

1) 2 and $\frac{7}{12}$ 2) 46 and $\frac{1}{3}$ 3) 3and $\frac{5}{6}$ 4) 3

5) 22 6) 14.9 7) 4.650 8) 6.387

Chapter- 6

Exercise - 6.1

1) i) 5:1 ii) 2:1
2) i) 8:3 ii) 12:1 iii) 31:1 iv) 45:34
3) Antecedent Consequent

	Antecedent	Consequent
i)	36	73
ii)	65	84
iii)	58	97
iv)	69	137

4) i) 5:11 ii) 1:40 iii) 1:2 iv) 20:3 v) 7:1 vi) 3:2
5) i) 28:75 ii) 75:47 iii) 28:47

Exercise - 6.2

1) a) yes b) No, not in proportion
c) No, not in proportion d) No, not in proportion
2) a) true b) true c) false d) false
3) a) yes, middle terms 1m, ₹45 ; extremes 15 cm, ₹ 300
b) yes, middle terms 2l, ₹ 100 ; extremes 20ml, ₹ 10000
4) a) 32 b) 14 c) 81

Exercise-6.3

1) ₹ 140 2) ₹ 75 3) 2
4) 7 hours 5) 300 litres

Exercise-6.4

1) a) $\frac{3}{20}$ b) $\frac{7}{20}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

2) a) 750% b) 825% c) 575% d) $333\frac{1}{3}$ %

3) a) 60% b) 62.5% c) $4\frac{6}{11}$ % d) $11\frac{1}{9}$ %

4) a) 3:25 b) 1:4 c) 9:20 d) 21:25

5) a) 0.01 b) 0.06 c) 0.19 d) 00.67

6) a) 4% b) 52% c) 12.5% d) 0.06%

7) 600 8) 68 9) ₹ 35000

Unit Exercise

2) 2:3

3) Kesav - ₹ 40,000, David - ₹ 32,000

4) a) ₹ 25,000 b) 1 year 7 months

5) 22 chairs 6) 20% 7) C 8) 100

Chapter - 7

Exercise - 7.1

1) i) 3m ii) 4m iii) 3m

2) 3n

3) 7n

4) ₹ 25

5) 2y+5

Exercise - 7.2

1) i) 3z+5 ii) 9n+10 iii) 2y-16 iv) 10y+x

2) 3p 3) S=3+G 4) 5n

Exercise - 7.3

1) i, iv, v

2) i) LHS-X-5 RHS-6

ii) LHS=4Y RHS=12

iii) LHS=2z+3 RHS=7

3) i) X=2 ii) Y=9 iii) a=5

Unit Exercise

1) 1500n

2) 4p

3) x+5

4) i, iii, iv, v, vii

5) i) L.H.S=7x+8 R.H.S=22

ii) L.H.S=9Y-3 R.H.S=6

iii) L.H.S =3k-10 R.H.S=2

iv) L.H.S=3p-4q R.H.S-19

6) i) x=8 ii) y=9 iii)m=-2 iv) k=2

Chapter - 8

Exercise - 8.1

1) A, B, C, D, E, F (use any capital letter for each point)

2) i) $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}, \overline{AC}, \overline{BD}$

ii) $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EF}, \overline{FA}$

$\overline{AC}, \overline{AD}, \overline{AE}, \overline{AF}$

$\overline{BD}, \overline{BE}, \overline{BF}$

$\overline{CE}, \overline{CF}$

$\overline{DF}$

3) i) A, B, C, D, E, F, G, H, I, J (any six)

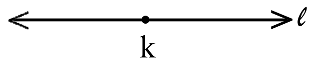
ii) $\overline{GH}, \overline{GD}, \overline{GE}, \overline{GF}, \overline{GA}, \overline{GC}$

iii) $\overrightarrow{IA}, \overrightarrow{IC}, \overrightarrow{IG}, \overrightarrow{IB}$

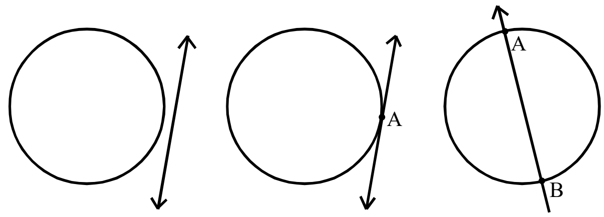
iv) $\overleftrightarrow{AC}, \overleftrightarrow{EC}, \overleftrightarrow{BE}, \overleftrightarrow{BD}, \overleftrightarrow{AD}$

4) i) False ii) True iii) True iv) False

5) i)



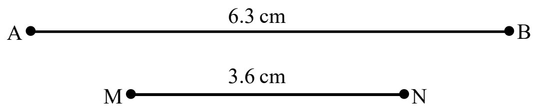
ii)



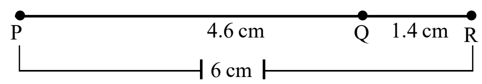
Exercise - 8.2

1)

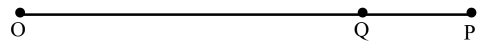
2)



3)



4)



Yes = $\overline{OP} - \overline{PQ} = \overline{OQ}$ is True.

Exercise - 8.3

1) ℓ,m are perpendicular lines, $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ are parallel lines.

2) i) $\overline{AQ} \| \overline{PC}$, $AD \perp CD$, $AB \perp BC$, $BC \perp CD$, $DA \perp AB$

ii) $\overline{QR} \perp \overline{PX}$ iii) $\overline{LP} \| \overline{ON}$, $OM \perp ON$, $KP \perp PL$

3) Intersecting lines: ℓ and m, m and n, n and o, o and ℓ

Concurent lines: P, Q and S.

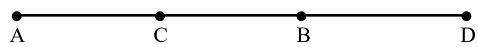
Exercise - 8.4

1) No 2) A, F, H, I, K, N, Y, Z

Unit Exercise

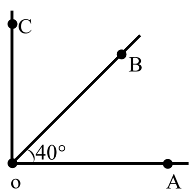
1)

2)



No $\overline{AC} \neq \overline{BD}$

3)

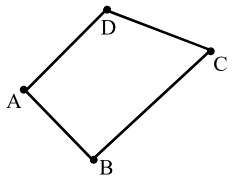


Yes $m\lfloor \overline{AOB} + m\lfloor \overline{BOC} = m\lfloor \overline{AOC}$

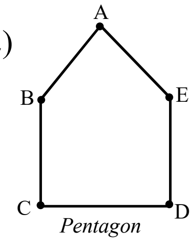
5) 1 - C | 2 - A | 3 - B 6) E, F, H, I, L, T

Chapter - 9

Exercise - 9.1

1) 

Quadrilateral

2) 

Pentagon

3) $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EF}, \overline{FA}$

4) $\angle PQR, \angle QRS, \angle RST, \angle STP, \angle TPQ$

Exercise - 9.2

1) i) X, Y, Z ii) A, I, B, J, C iii) O, L, K

2) i) 3 Sides $\overline{PQ}, \overline{QR}, \overline{RP}$ ii) 3 Verticles P, Q, R

iii) $\overline{QR}$ iv) Q

3) i) 3 angles $\angle MNO, \angle NOM, \angle OMN$

ii) $\angle NOM$ iii) $\wedge$tVertex 'O'

Exercise - 9.3

1) i) $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ ii) $\overline{CD}$ iii) D iv) $\angle A$

v) $\angle A$ and $\angle B$; $\angle B$ and $\angle C$; $\angle C$ and $\angle D$; $\angle D$ and $\angle A$

4 pars

vi) $\angle B$ and $\angle D$; $\angle C$ and $\angle A$

2) Square: 4, Circle: Infinite, Equilateral Triangle: 3

Exercise - 9.3

1) i) Cuboid ii) Cylinder iii) Sphere iv) Cone

2) i) Cone ii) Cube iii) Sphere iv) Cylinder

3) a $\rightarrow$ 5, b $\rightarrow$ 3, c $\rightarrow$ 4, d $\rightarrow$ 1, e $\rightarrow$ 2

4)

6	12	8
5	9	6
5	8	5
6	12	8

Unit Exercise

1)

2) i) $\triangle$ PQR, $\overline{PQ}, \overline{QR}, \overline{RP}$, $\angle PQR, \angle QRP, \angle RPQ$

P, Q, R

3) i) Quadrilateral

ii) $\overline{EF}$ and $\overline{FG}$; $\overline{FG}$ and $\overline{GH}$; $\overline{GH}$ and $\overline{HE}$; $\overline{HE}$ and $\overline{EF}$; $\angle E$ and $\angle F$; $\angle F$ and $\angle G$; $\angle G$ and $\angle H$; $\angle H$ and $\angle F$

iii) E, F, G, H

iv) $\overline{EF}$ and $\overline{GH}$; $\overline{FG}$ and $\overline{EH}$;

$\angle F$ and $\angle H$; $\angle E$ and $\angle G$

4) i) True ii) False iii) False

Chapter - 11

Exercise - 11.1

1) i) Yes 3×2cm = 6cm

ii) Yes 4×3cm = 12cm

iii) Yes 4×2cm = 8cm

2) 160 m ; 200 m

Yes Perimeter of Rectangle = $2 \times$ length + $2 \times$ breadth

3) a) 3×3.5cm = 10.5cm

b) 4×4.8cm = 19.2 cm

4) 500cm

5) 1

6) 22cm, 18cm

7) B

Exercise - 11.2

1) a) 44cm b)22cm c) 88cm

2) a) 0.7m b) 28cm c) 0.245cm

3) 70

4) 11cm

5) 300 times

6) 3.5cm

7) 3100 cm

Exercise - 11.3

1) 120 sq.cm

2) 256 sq.cm

3) 112 sq.cm

4) a) Perimeter: 4.8cm

Area : 128 sq.cm

b) Perimeter: 48cm

Area : 140 sq.cm

c) i) A and B

ii) No

Unit Exercise

1) 144 sq.cm

2) 98 sq.cm

3) 44 sq.cm

4) 168 sq.cm, $8 \times 18 = 144$ sq.cm, Difference = 24 sq.cm

5) 40cm, 40cm, 40cm

6) 32cm

256 cm

sum of perimeter of all 64 small squares

= 8 $\times$ perimeter of all squares

***

TABLES

2 x 1 = 2	3 x 1 = 3	4 x 1 = 4	5 x 1 = 5
2 x 2 = 4	3 x 2 = 6	4 x 2 = 8	5 x 2 = 10
2 x 3 = 6	3 x 3 = 9	4 x 3 = 12	5 x 3 = 15
2 x 4 = 8	3 x 4 = 12	4 x 4 = 16	5 x 4 = 20
2 x 5 = 10	3 x 5 = 15	4 x 5 = 20	5 x 5 = 25
2 x 6 = 12	3 x 6 = 18	4 x 6 = 24	5 x 6 = 30
2 x 7 = 14	3 x 7 = 21	4 x 7 = 28	5 x 7 = 35
2 x 8 = 16	3 x 8 = 24	4 x 8 = 32	5 x 8 = 40
2 x 9 = 18	3 x 9 = 27	4 x 9 = 36	5 x 9 = 45
2 x 10 = 20	3 x 10 = 30	4 x 10 = 40	5 x 10 = 50
2 x 11 = 22	3 x 11 = 33	4 x 11 = 44	5 x 11 = 55
2 x 12 = 24	3 x 12 = 36	4 x 12 = 48	5 x 12 = 60
2 x 13 = 26	3 x 13 = 39	4 x 13 = 52	5 x 13 = 65
2 x 14 = 28	3 x 14 = 42	4 x 14 = 56	5 x 14 = 70
2 x 15 = 30	3 x 15 = 45	4 x 15 = 60	5 x 15 = 75
2 x 16 = 32	3 x 16 = 48	4 x 16 = 64	5 x 16 = 80
2 x 17 = 34	3 x 17 = 51	4 x 17 = 68	5 x 17 = 85
2 x 18 = 36	3 x 18 = 54	4 x 18 = 72	5 x 18 = 90
2 x 19 = 38	3 x 19 = 57	4 x 19 = 76	5 x 19 = 95
2 x 20 = 40	3 x 20 = 60	4 x 20 = 80	5 x 20 = 100

6 x 1 = 6	7 x 1 = 7	8 x 1 = 8	9 x 1 = 9
6 x 2 = 12	7 x 2 = 14	8 x 2 = 16	9 x 2 = 18
6 x 3 = 18	7 x 3 = 21	8 x 3 = 24	9 x 3 = 27
6 x 4 = 24	7 x 4 = 28	8 x 4 = 32	9 x 4 = 36
6 x 5 = 30	7 x 5 = 35	8 x 5 = 40	9 x 5 = 45
6 x 6 = 36	7 x 6 = 42	8 x 6 = 48	9 x 6 = 54
6 x 7 = 42	7 x 7 = 49	8 x 7 = 56	9 x 7 = 63
6 x 8 = 48	7 x 8 = 56	8 x 8 = 64	9 x 8 = 72
6 x 9 = 54	7 x 9 = 63	8 x 9 = 72	9 x 9 = 81
6 x 10 = 60	7 x 10 = 70	8 x 10 = 80	9 x 10 = 90
6 x 11 = 66	7 x 11 = 77	8 x 11 = 88	9 x 11 = 99
6 x 12 = 72	7 x 12 = 84	8 x 12 = 96	9 x 12 = 108
6 x 13 = 78	7 x 13 = 91	8 x 13 = 104	9 x 13 = 117
6 x 14 = 84	7 x 14 = 98	8 x 14 = 112	9 x 14 = 126
6 x 15 = 90	7 x 15 = 105	8 x 15 = 120	9 x 15 = 135
6 x 16 = 96	7 x 16 = 112	8 x 16 = 128	9 x 16 = 144
6 x 17 = 102	7 x 17 = 119	8 x 17 = 136	9 x 17 = 153
6 x 18 = 108	7 x 18 = 126	8 x 18 = 144	9 x 18 = 162
6 x 19 = 114	7 x 19 = 133	8 x 19 = 152	9 x 19 = 171
6 x 20 = 120	7 x 20 = 140	8 x 20 = 160	9 x 20 = 180

TABLES

10 x 1 = 10	11 x 1 = 11	12 x 1 = 12	13 x 1 = 13
10 x 2 = 20	11 x 2 = 22	12 x 2 = 24	13 x 2 = 26
10 x 3 = 30	11 x 3 = 33	12 x 3 = 36	13 x 3 = 39
10 x 4 = 40	11 x 4 = 44	12 x 4 = 48	13 x 4 = 52
10 x 5 = 50	11 x 5 = 55	12 x 5 = 60	13 x 5 = 65
10 x 6 = 60	11 x 6 = 66	12 x 6 = 72	13 x 6 = 78
10 x 7 = 70	11 x 7 = 77	12 x 7 = 84	13 x 7 = 91
10 x 8 = 80	11 x 8 = 88	12 x 8 = 96	13 x 8 = 104
10 x 9 = 90	11 x 9 = 99	12 x 9 = 108	13 x 9 = 117
10 x 10 = 100	11 x 10 = 110	12 x 10 = 120	13 x 10 = 130
10 x 11 = 110	11 x 11 = 121	12 x 11 = 132	13 x 11 = 143
10 x 12 = 120	11 x 12 = 132	12 x 12 = 144	13 x 12 = 156
10 x 13 = 130	11 x 13 = 143	12 x 13 = 156	13 x 13 = 169
10 x 14 = 140	11 x 14 = 154	12 x 14 = 168	13 x 14 = 182
10 x 15 = 150	11 x 15 = 165	12 x 15 = 180	13 x 15 = 195
10 x 16 = 160	11 x 16 = 176	12 x 16 = 192	13 x 16 = 208
10 x 17 = 170	11 x 17 = 187	12 x 17 = 204	13 x 17 = 221
10 x 18 = 180	11 x 18 = 198	12 x 18 = 216	13 x 18 = 234
10 x 19 = 190	11 x 19 = 209	12 x 19 = 228	13 x 19 = 247
10 x 20 = 200	11 x 20 = 220	12 x 20 = 240	13 x 20 = 260

14 x 1 = 14	15 x 1 = 15	16 x 1 = 16	17 x 1 = 17
14 x 2 = 28	15 x 2 = 30	16 x 2 = 32	17 x 2 = 34
14 x 3 = 42	15 x 3 = 45	16 x 3 = 48	17 x 3 = 51
14 x 4 = 56	15 x 4 = 60	16 x 4 = 64	17 x 4 = 68
14 x 5 = 70	15 x 5 = 75	16 x 5 = 80	17 x 5 = 85
14 x 6 = 84	15 x 6 = 90	16 x 6 = 96	17 x 6 = 102
14 x 7 = 98	15 x 7 = 105	16 x 7 = 112	17 x 7 = 119
14 x 8 = 112	15 x 8 = 120	16 x 8 = 128	17 x 8 = 136
14 x 9 = 126	15 x 9 = 135	16 x 9 = 144	17 x 9 = 153
14 x 10 = 140	15 x 10 = 150	16 x 10 = 160	17 x 10 = 170
14 x 11 = 154	15 x 11 = 165	16 x 11 = 176	17 x 11 = 187
14 x 12 = 168	15 x 12 = 180	16 x 12 = 192	17 x 12 = 204
14 x 13 = 182	15 x 13 = 195	16 x 13 = 208	17 x 13 = 221
14 x 14 = 196	15 x 14 = 210	16 x 14 = 224	17 x 14 = 238
14 x 15 = 210	15 x 15 = 225	16 x 15 = 240	17 x 15 = 255
14 x 16 = 224	15 x 16 = 240	16 x 16 = 256	17 x 16 = 272
14 x 17 = 238	15 x 17 = 255	16 x 17 = 272	17 x 17 = 289
14 x 18 = 252	15 x 18 = 270	16 x 18 = 288	17 x 18 = 306
14 x 19 = 266	15 x 19 = 285	16 x 19 = 304	17 x 19 = 323
14 x 20 = 280	15 x 20 = 300	16 x 20 = 320	17 x 20 = 340

TABLES

18 x 1 = 18	19 x 1 = 19	20 x 1 = 20
18 x 2 = 36	19 x 2 = 38	20 x 2 = 40
18 x 3 = 54	19 x 3 = 57	20 x 3 = 60
18 x 4 = 72	19 x 4 = 76	20 x 4 = 80
18 x 5 = 90	19 x 5 = 95	20 x 5 = 100
18 x 6 = 108	19 x 6 = 114	20 x 6 = 120
18 x 7 = 126	19 x 7 = 133	20 x 7 = 140
18 x 8 = 144	19 x 8 = 152	20 x 8 = 160
18 x 9 = 162	19 x 9 = 171	20 x 9 = 180
18 x 10 = 180	19 x 10 = 190	20 x 10 = 200
18 x 11 = 198	19 x 11 = 209	20 x 11 = 220
18 x 12 = 216	19 x 12 = 228	20 x 12 = 240
18 x 13 = 234	19 x 13 = 247	20 x 13 = 260
18 x 14 = 252	19 x 14 = 266	20 x 14 = 280
18 x 15 = 270	19 x 15 = 285	20 x 15 = 300
18 x 16 = 288	19 x 16 = 304	20 x 16 = 320
18 x 17 = 306	19 x 17 = 323	20 x 17 = 340
18 x 18 = 324	19 x 18 = 342	20 x 18 = 360
18 x 19 = 342	19 x 19 = 361	20 x 19 = 380
18 x 20 = 360	19 x 20 = 380	20 x 20 = 400

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