Geometric Transformation

What is Geometric Transformation

- A set of image transformations where the geometry of image is changed without altering its actual pixel values are commonly referred to as **Geometric transformation**.

- In general, multiple operations are applied on it, pixel values are not changed, the positions of pixel values are changed.
- There are two basic steps in geometric transformations:
  - Spatial transformation of the physical re-arrangement of pixels in the image
  - Interpolation, which assigns pixel intensity to the missing pixels of transformed image
- Such transformations are frequently used as pre-processing steps in different applications such as
  - document understanding, where the scanned image may be mis-aligned. •

Basic Geometric Image Transformations:

- Translation, Scaling, Shear, Rotation

Translation

- Pixel coordinates (��, ��)

undergo

geometric translation to produce an image with coordinates ��,��

- ��$_{��}$ is translation along x-axis and ��$_{��}$ is translation along y-axis

Translation mapping function:

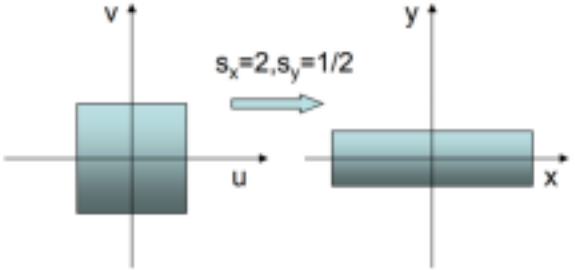
$$\text{??} = \text{??} + \text{??}_{\text{??}}$$
$$\text{??} = \text{??} + \text{??}_{\text{??}}$$

Matrix notation:

$$\begin{matrix}\text{??} \\ \text{??}\end{matrix} = \text{??} + \begin{matrix}\text{??}_{\text{??}} \\ \text{??}_{\text{??}}\end{matrix}$$

Input Translated Image

Scaling



- Pixel coordinates (��, ��) undergo image scaling to produce an image with coordinates ��, ��
  and ��$_{��}$is scale factor along x-axis and ��$_{��}$ is along y axis
  - If ��$_{��}$ < 1 and ��$_{��}$ < 1, this represents minification and shrinking.
- If ��$_{��}$ > 1 and ��$_{��}$ > 1, this represents magnification and zoom.

Scaling function:

$$
\begin{aligned}
\text{\textcircled{\scriptsize x}} &= \text{\textcircled{\scriptsize x}}_{\text{\textcircled{\scriptsize x}}}\text{\textcircled{\scriptsize x}} \\
\text{\textcircled{\scriptsize x}} &= \text{\textcircled{\scriptsize x}}_{\text{\textcircled{\scriptsize x}}}\text{\textcircled{\scriptsize x}}
\end{aligned}
$$

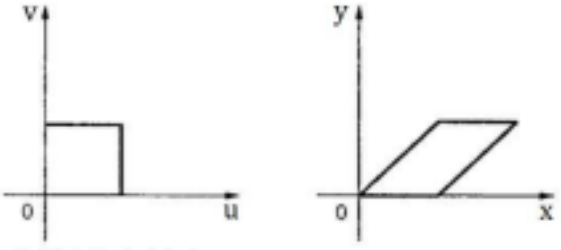
Matrix notation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_s & 0 \\ 0 & y_s \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

u, v Input Scaled Image

Shear

- Pixel coordinates (u, v) undergo shear transformation to produce an image with coordinates x, y
- shear stress, f



Shear functional representation:

$$x = u + fv$$
$$y = v$$

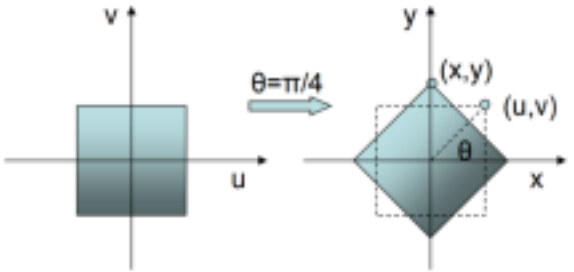
Matrix notation:

$$\begin{bmatrix} \square\square \\ \square\square \end{bmatrix} = \begin{bmatrix} \square\square & \square\square \\ 0 & \square\square \end{bmatrix} \begin{bmatrix} \square\square \\ \square\square \end{bmatrix}$$

Input Shear with different factor

Rotation

- Pixel coordinates (□□, □□) undergo geometric rotation to produce an image with coordinates □□,□□
- rotation angle is □□



Rotation function:

$$\square\square = \square\square \cos\square\square - \square\square \sin \square\square$$

$$x = x \sin x + x \cos x$$



Matrix notation:

$$x \quad x$$

$$x = \cos x \quad -\sin x$$

$$\sin x \quad \cos x$$

Input Rotation without Interpolation

Rotation after Interpolation

Kharagpur Interpolation

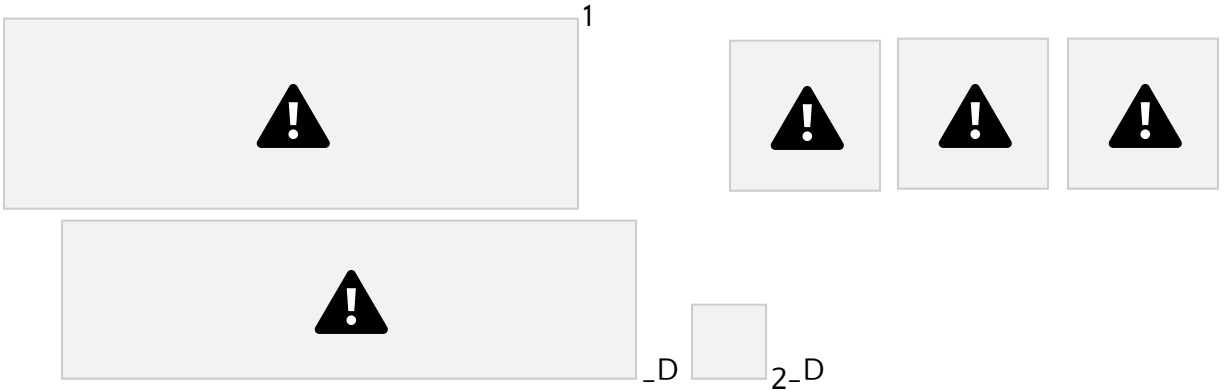
- Geometric transformation equations will produce values x and y that are not integers.
- A set of color intensity levels for non integer positions in the image are used to determine the color intensity levels that should be assigned to the integer pixel locations in the output image.
- Types of Interpolation:
  - Nearest Neighbor: Color intensity for ◆◆(◆◆, ◆◆) to the pixel having closest integer coordinates to ◆◆(◆◆, ◆◆).
  - Bilinear Interpolation: ◆◆(◆◆, ◆◆) = ◆◆◆◆ + ◆◆◆◆ + ◆◆◆◆◆◆ + ◆◆
  - Bicubic Interpolation: $^{3}\sigma_{◆◆=0}$ $^{3}◆◆_{◆◆,◆◆}◆◆^{◆◆}◆◆^{◆◆}$

◆◆ ◆◆,◆◆ = $\sigma_{◆◆=0}$

  - ◆◆(◆◆, ◆◆) is the grey value at position (◆◆, ◆◆). Grey levels of the 4 surrounding integer pixel positions are used to determine the correct grey

level of the position when the output pixel came from.

Interpolation



Nearest Bilinear Bicubic Nearest Bilinear Bicubic


Input
Nearest
Bilinear
Bicubic

Thank You