Hypothesis Testing

1) Step 1 : Null and Alternate hypothesis

Null Hypothesis → 5% of children had autism.

Alternate hypothesis → more than 5% of children had autism.

$H_0 \Rightarrow P = 0.05$

$H_A \Rightarrow P > 0.05$

It is one tailed test as we check only one end

Step 2: Determination of test as we perform Z-test.

step 3: Significance value is not given so. by default.

$\alpha = 5\% \Rightarrow \alpha = 0.05$

Step 4: Establish Decision rule

for Z-Critical.

if Z-critical < Z-score

we will reject null hypothesis

For P-Value

if P-value < significance value.

we will reject null hypothesis

Step 5: Data Gathering.

Examined 384 children and found 46 showed signs of autism

5% of children had autism

Step 6: Analysing the data

$P = 0.05$, $n = 384$.

$\hat{P} = \dfrac{46}{384} \Rightarrow 0.11$

$\boxed{\hat{P} = 0.11}$

$q = 1 - P$

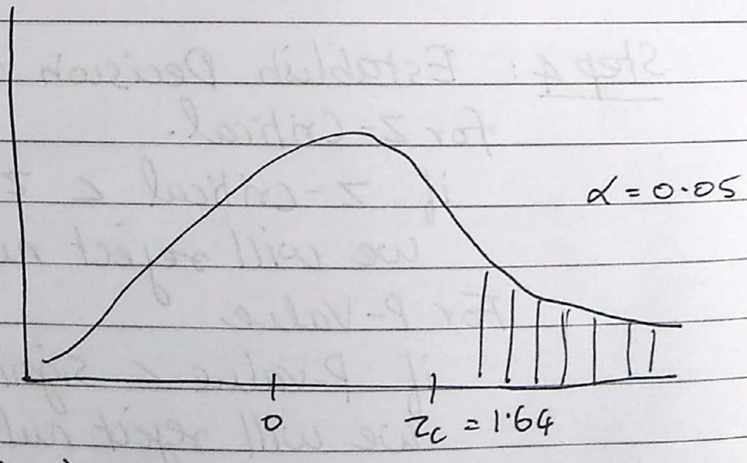
$= 1 - 0.05$

$\boxed{q = 0.95}$

$$Z\text{-score} = \frac{\hat{P} - P}{\sqrt{\dfrac{P \cdot q}{n}}} \Rightarrow \frac{0.11 - 0.05}{\sqrt{\dfrac{0.05 \times 0.95}{384}}}$$

$$= \frac{0.06}{0.011}$$

$$= 5.45$$



Step 7: Using Z table.

Z critical $= 1.64$ Z score $= 5.45$.

Z critical $<$ Z score

we reject null hypothesis

So more than 5% had autism

Hence increase in certain chemicals in environment led to increase in autism

2) Step 1: Null and Alternate hypothesis

Null Hypothesis - 20% of cars failed to meet pollution guidelines

Alternate hypothesis - more than 20% failed to meet pollution guidelines.

$H_0 \Rightarrow P = 0.20$

$H_A \Rightarrow P > 20$

It is one tailed test as we test only one side

Step 2: Determine the test

Z-test is to be performed.

Step 3: Significance value = 10%.

$\alpha = 0.10$.

Step 4: For Z-critical,

Z critical < Z score

we reject null hypothesis

For P-value

if P-value < significance level

we reject null hypothesis

Step 5: Data collecting

In 150 cars 7 out of 22 cars tested

failed to meet pollution guidelines

Step 6: Data Analysis:

For Z-Score = $\dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$ $\qquad \hat{p} = \dfrac{7}{22}$

$\boxed{\hat{p} = 0.31}$

$p = 0.20 \quad n = 22$

$q = 1 - p$

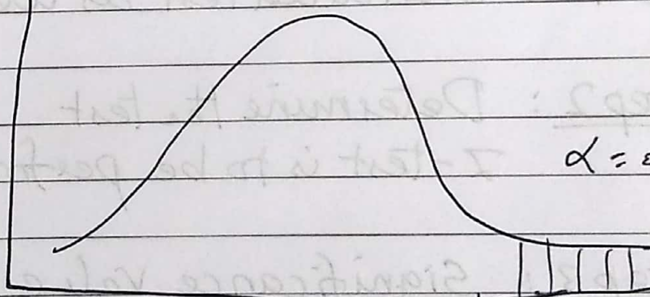
$= 1 - 0.20$

$\boxed{q = 0.80}$

Z score = $\dfrac{0.31 - 0.20}{\sqrt{\dfrac{0.20 \times 0.80}{22}}}$

$= \dfrac{0.11}{0.084}$

$\boxed{Z \text{ score} = 1.28}$



$\alpha = 0.10$

$Z_c = 1.3$

β values = 1.003 by Z-table.

Step 7: statistical analysis

For Z critical.

Z critical > Z score

$1.3 > 1.28$

we will not reject null hypothesis

we accept null hypothesis

For Pvalues

P value > Significant value.

1.003 > 0.10.

we accept null hypothesis.

So, 20% of cars failed to meet population guidelines.

Step 8: So we need to improve the cars to meet the population guidelines

2) For $\alpha = 5\%$.

$\boxed{\alpha = 0.05}$

Z score = 1.28

Using Z table Z-critical = 1.64.

On basis of decision rule

Z critical > Z-score

1.64 > 1.28

we accept null hypothesis

3) For $\alpha = 1\%$.

$\boxed{\alpha = 0.01}$

Z score = 1.28.

Using Z table Z critical = 2.33.

on basis of decision rule.

Z critical > Z score

2.33 > 1.28

We accept null hypothesis

Thus 20% of cars failed to meet population guidelines.

3) Step 1: Null hypothesis and alternate hypothesis

Null hypothesis : 44% of adults have never smoked.

Alternate hypothesis: more than 44% of adults never smoked

$H_0 = P = 0.44$

$H_A = P > 0.44$

It is one tailed test as we test only one end

Step 2: Significance level.
confidence level = 98%.
so then

$\alpha = 2\%$

$\boxed{\alpha = 0.02\%}$

Step 3: We need to perform Z-test.

Step 4: For Z-critical,
if Z-critical < Z-test.
we reject null hypothesis

For P-values
If P-value < significance value.
we reject null hypothesis

Step 5: Data gathering
with 891 adults interview 463 states
they never smoked.

Scanned by CamScanner

Step 6: Data Analysis

for Z-score $= \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$

$p = 0.44$ $\quad n = 891$ $\quad \hat{p} = \dfrac{463}{891}$

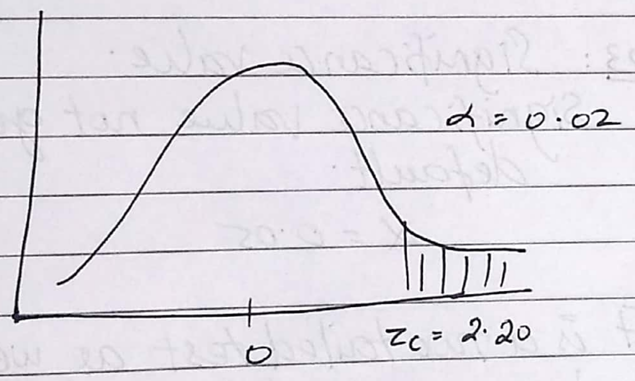
$q = 1 - p$

$\boxed{q = 0.56}$ $\quad \boxed{\hat{p} = 0.51}$

Z-score $= \dfrac{0.51 - 0.44}{\sqrt{\dfrac{0.44 \times 0.56}{891}}}$

Z Score $= \dfrac{0.07}{0.016}$

Z score $= 4.375$



By Z-table
Z critical (Z_c) $= 2.20$.

Step 7:

Z score $= 4.37$
Z critical $= 2.20$.

Step 8:-

Based on decision rule
Z critical < Z score. So more than 44% of adults never smoked.

4) Step 1: Null and Hypothesis Alternate.

Null hypothesis: Distance from lens to object and distance from lens to real image is same.

Alternate hypothesis: Distance from lens to object and distance from lens to real image is not same

$H_0 = \mu_A = \mu_B$

$H_A = \mu_A \neq \mu_B$

Step 2: Determine the test.
perform Z-test.

Step 3: Significance value.
Significance value not given hence as default.

$\alpha = 0.05$

It is a two tailed test as we check the two right and left end.

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025.$$

Step 4: With Decision Rule
For Critical value
if Z critical < Z score
we reject Null hypothesis.

For p-values:
If p-values < significance value
we reject Null hypothesis

Scanned by CamScanner

Step 5: Data gathering

Sample mean $\bar{S}_1 = 26.6$ cm

$\bar{S}_2 = 13.8$ cm.

Standard deviation for $S_1 = 0.1$ cm

Standard deviation for $S_2 = 0.5$ cm.

Step 6: Data analysis

For two sample Z-test to be performed.

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

For Distance from lens to Object S_1

$\bar{S}_1 = 26.6$ cm $\sigma_1 = 0.1$ cm $n_1 = 25$

For Distance from lens to Real image S_2.

$\bar{S}_2 = 13.8$ c $\sigma_2 = 0.5$ cm $n_2 = 25$

For Z score

$$Z\,score = \frac{\bar{S}_1 - \bar{S}_2}{\sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}}}$$

$$= \frac{26.6 - 13.8}{\sqrt{\frac{(0.1)^2}{25} + \frac{(0.5)^2}{25}}}$$

$$= \frac{12.8}{\sqrt{0.0004 + 0.01}}$$

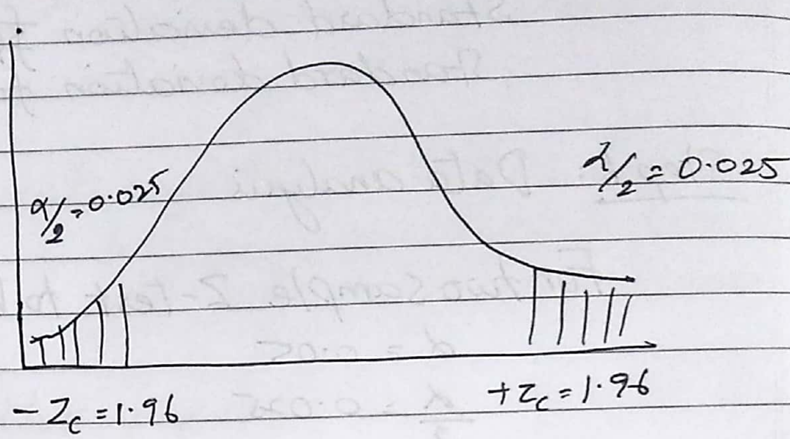
$$= \frac{12.8}{0.102}$$

$$\boxed{Z\,score = 125.51}$$

Using Z table.

Z critical = 1.96.

P-value = 0.0000



Step 7: Taking Statistics.

Using Decision Rule.

For critical Value,

if Z-critical < Z-score.

1.96 < 125.51

We reject null hypothesis

For critical Value

If P-value < significance Value

0.0000 < 0.05

We reject null hypothesis

Step 8: By testing we say that.

Distance from lens to object and distance from lens to real image are not same.

$\mu_A \neq \mu_B$

5. **Step 1**: Null and Alternate hypothesis.

Null hypothesis - Mean body temperature is 98.6.

Alternate hypothesis - Mean body temperature is not equal to 98.6.

$H_0 : \mu = 98.6$

$H_A : \mu \neq 98.6$

It is two tailed test as we test it on two ends.

$$\frac{\alpha}{2} = \frac{0.02}{2} = 0.01.$$

Step 2: Determine the test

perform t-test

Step 3: Significance value.

$\alpha = 0.02$

Step 4: Establish Descion rule.

for critical value

if critical value < t-score

we reject null hypothesis

For P-value:

if p-value < significance level

we will reject the null hypothesis

Step 5: Data Collecting

$s = 0.6824$ $\bar{x} = 98.2846$ $n = 52$

Step 6: Data Analysis

$$t\text{-score} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

$n = 52$

$\bar{x} = 98.2846$

$S = 0.6824$

$DF = n - 1$

$= 52 - 1$

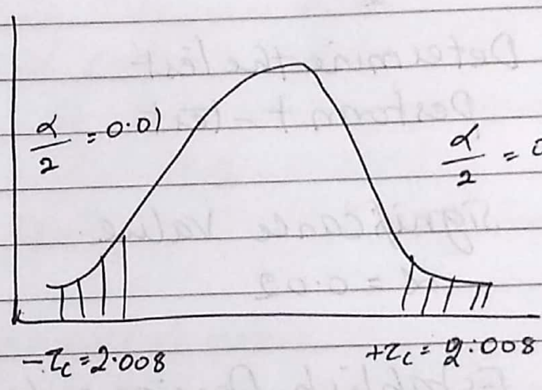
$\boxed{DF = 51}$

$$t\text{-score} = \frac{98.2846 - 98.6}{\frac{0.6824}{\sqrt{52}}}$$

Using t-table

t-critical = 2.008

p-value = 0.0016



Step 7: Statistical Action

Based on Decision rule

For critical Values

if

t-critical < t-score

2.008 < 3.333

We reject null hypothesis

For P-Values:

If p-values < significance level

0.0016 < 0.02

We reject null hypothesis

So. Mean body temp is not equal to 98.6°

6. Step 1: Null and Alternate hypothesis.

Null Hypothesis - no difference between premium and Regular gas in mileage

Alternate hypothesis - There is a difference between mileage of regular and premium gas tank.

$$H_0 \Rightarrow \mu_A = \mu_B$$
$$H_A \Rightarrow \mu_A \neq \mu_B$$

It is a two tailed test as we check both ends.

Step 2 : Determine the test.
perform T-test.

Step 3 : Set significance value
Significance value not given hence by default
$\boxed{\alpha = 0.05}$

$$\alpha = \frac{0.05}{2} = 0.025.$$

Step 4: Establish Decision rule.
For critical value
if t-critical < t-test
we reject null hypothesis.

For p-values.
if p-value < significance value.
we reject null hypothesis

Date
Page

Step 5: Data collecting

10 random chosen cars

Step 6: Data analysing.
It is two sample Variable

$$DF = \frac{\left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right]^2}{\left[\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{S_2}{n_2}\right)^2}{n_2-1}\right]}$$

$$t\text{-test} = \frac{\overline{X}_1 - \overline{X}_2}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

For Regular tank.
$\mu_A = \overline{X}_1 = 23.1$
$n_1 = 10$
$S_1 = 3.72$

For premium tank.
$\overline{X}_2 = 25.1$
$n_2 = 10$
$S_2 = 3.44$

$$DF = \frac{\left[\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right]^2}{\frac{\left(\frac{S_1}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{S_2}{n_2}\right)^2}{n_2-1}}$$

$$= \frac{(1.38 + 1.18)^2}{\frac{(1.38)^2}{9} + \frac{(1.18)^2}{9}}$$

$$= \frac{6.55}{\frac{1.90}{9} + \frac{1.39}{9}}$$

$$= \frac{6.55}{0.36}$$

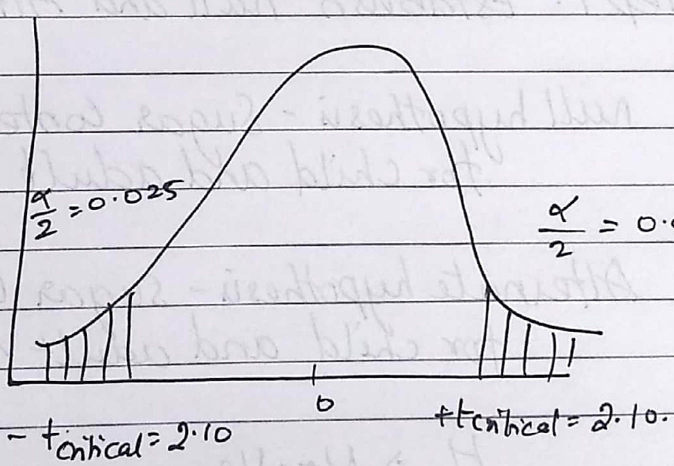
$$\Rightarrow 18$$

$$\boxed{DF = 18}$$

$$t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$= \frac{23.1 - 25.1}{\frac{(3.72)^2}{10} + \frac{(3.44)^2}{10}}$$

$$\boxed{t\text{-test} = 1.24}$$



By Using t-table,

t-critical $= 2.10$

$P = 0.22$.

<u>Step 7:</u> Using statistics.

On basis of decision rule

For t critical,

if t-critical < t-score

we reject null hypothesis.

t-critical > t-score.

2.10 > 1.24.

We accept null hypothesis

For p-value.

if p-value > significance value

0.22 > 0.05

we accept null hypothesis

So. We conclude as $\mu_A = \mu_B$ there is no difference in Mileage between regular and premium tank.

7. Step 1: Establish null and Alternate hypothesis

null hypothesis - Sugar content of cerels for child and adult are same.

Alternate hypothesis - sugar content of cerels for child and adult are not same

$H_0 \Rightarrow \mu_A = \mu_B$

$H_A \Rightarrow \mu_A \neq \mu_B$

It is a two tailed test as we check both ends.

Step 2: perform t-test.

Scanned by CamScanner

Date
Page

step3 : significance value.
As confidence level is 95%.

$$\alpha = 5\%$$

$$\boxed{\alpha = 0.05}$$

$$\alpha = \frac{0.05}{2} = 0.025.$$

step4: Establish Decision Rule.

For critical value
if t-critical < t-test.
we reject Null hypothesis.

For p-values
If p-value < significance level
we reject null hypothesis

Step5 Data collection

step 6 Data Analysis.
It is t-test for two sample variable.

$$df = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

For children

$\bar{X}_1 = \mu_A = 46.8$

$n_1 = 19$

$S_1 = 6.41$

For Adult:

$\bar{X}_2 = \mu_B = 10.16$

$n_2 = 29$

$S_2 = 7.47.$

$$DF = \frac{\left[\frac{S_1^{\,2}}{n_1} + \frac{S_2^{\,2}}{n_2}\right]^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

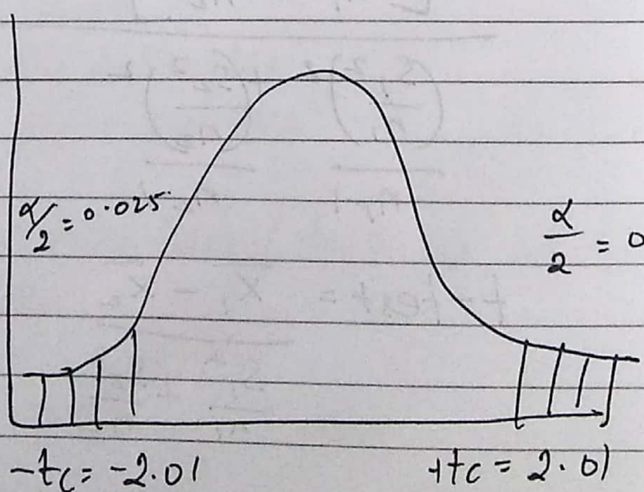
$$= \frac{2.16 + 1.92}{\frac{(2.16)^2}{18} + \frac{(1.92)^2}{28}} \Rightarrow \frac{16.64}{0.38}$$

$$\boxed{DF = 43}$$

$$t\text{-test} = \frac{\bar{X}_1 - \bar{X}_2}{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \Rightarrow \frac{6.41 - 7.47}{\sqrt{\frac{(6.41)^2}{19} + \frac{(7.47)^2}{29}}}$$

$$= \frac{36.38}{1.96}$$

$$\boxed{t\text{-test} = 18.10}$$



Using t-table.

$t_c = 2.01$

P-value = 0.0001

<u>Step 7</u> On basis of decision rule.

For critical value,

t-critical < t-test

2.01 < 18.10.

we reject null hypothesis

For p-value.

P-value < significance value

0.0001 < 0.05

we reject null hypothesis.

so Sugar Content of cereals for child and adult are not same or equal.