State-of-the-art algorithms for computing approximate maximum flows in undirected graphs

A Project Report

submitted by

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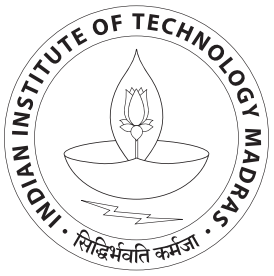
for the award of the degree of

BACHELOR OF TECHNOLOGY



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THESIS CERTIFICATE

This is to certify that the thesis entitled **State-of-the-art algorithms for computing approximate maximum flows in undirected graphs**, submitted by **Karthik Abinav S**, to the Indian Institute of Technology, Madras, for the award of the degree of **Bachelor of Technology**, is a bona fide record of the research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Graph Theory, Maximum Flow, Laplacian solvers

Maximum flow problem has been a very important optimization problem in computer science and mathematics. This problem has a lot of practical relevance. Some of the age-old applications involving maximum flow have been in electrical circuits, water supply networks, etc. With the advent of social media and social networks, this problem has found a newer practical relevance. And since the graphs in these networks are typically very large, researchers are sought after creating faster and more efficient algorithms for this problem.

Some of the classical algorithms to solve this problem are the Ford-Fulkerson's augmenting path algorithm and the Dinic's Algorithm. These algorithms compute the exact value of the maximum flow. It is also fairly straightforward to obtain the optimal flow vector after the termination of the algorithm. The main drawback with this algorithm is that the running time, though polynomial, is very high and is expensive to use in many practical situations. Following these algorithms a series of push-relabel algorithms were devised which had a slightly better running time as compared to the classical algorithms. The series of algorithms terminated with the algorithm by Goldberg-Rao which gave a $O(m^{\frac{3}{2}})$-time algorithm. For extremely large graphs, as in the case of social networks, this algorithm is still far from being practical.

In 2008, the breakthrough result by Spielman and Teng gave an algorithm to solve the Symetric Diagonally Dominant(SDD) system of equations in near-linear time. This work was immediately extended by Koutis-Miller-Peng which gave an efficient algorithm to produce an incremental graph sparsifier. Developement of these techniquies led to the developement of an almost linear time algorithm to the maximum flow problem by Cristiano-Kelner-Madry-Spielman. This involved looking at the graph as a resistive network, approximating the electrical flow and producing an approximate s-t flow from this approximate electrical flow. This algorithm broke the running time barrier of Golderg-Rao and gave the first almost-linear time algorithm.

In this thesis, we give a survey of the above algorithms for the maximum flow problem. We first start off by giving a brief description of the classcial algorithms. We then present the required tools from spectral graph theory and linear algebra that is required to understand the almost linear time algorithm. Finally, we present the Cristiano-Kelner-Madry-Spielman algorithm in detail. We will also present some of the key theorems from the Koutis-Miller-Peng SDD solver. Finally, we give some insights obtained by us and a recursive implementation of a push-relabel algorithm.

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ABBREVIATIONS

IITM Indian Institute of Technology, Madras

RTFM Read the Fine Manual

NOTATION

r	Radius, m
α	Angle of thesis in degrees
β	Flight path in degrees

CHAPTER 1

INTRODUCTION

This document provides a simple template of how the provided `iitmdiss.cls` LaTeX class is to be used. Also provided are several useful tips to do various things that might be of use when you write your thesis.

Before reading any further please note that you are strongly advised against changing any of the formatting options used in the class provided in this directory, unless you are absolutely sure that it does not violate the IITM formatting guidelines. *Please do not change the margins or the spacing.* If you do change the formatting you are on your own (don't blame me if you need to reprint your entire thesis). In the case that you do change the formatting despite these warnings, the least I ask is that you do not redistribute your style files to your friends (or enemies).

It is also a good idea to take a quick look at the formatting guidelines. Your office or advisor should have a copy. If they don't, pester them, they really should have the formatting guidelines readily available somewhere.

To compile your sources run the following from the command line:

```
% latex thesis.tex
% bibtex thesis
% latex thesis.tex
% latex thesis.tex
```

Modify this suitably for your sources.

To generate PDF's with the links from the `hyperref` package use the following command:

```
% dvipdfm -o thesis.pdf thesis.dvi
```

1.1 Package Options

Use this thesis as a basic template to format your thesis. The `iitmdiss` class can be used by simply using something like this:

```
\documentclass[PhD]{iitmdiss}
```

To change the title page for different degrees just change the option from `PhD` to one of `MS`, `MTech` or `BTech`. The dual degree pages are not supported yet but should be quite easy to add. The title page formatting really depends on how large or small your thesis title is. Consequently it might require some hand tuning. Edit your version of `iitmdiss.cls` suitably to do this. I recommend that this be done once your title is final.

To write a synopsis simply use the `synopsis.tex` file as a simple template. The synopsis option turns this on and can be used as shown below.

```
\documentclass[PhD,synopsis]{iitmdiss}
```

Once again the title page may require some small amount of fine tuning. This is again easily done by editing the class file.

This sample file uses the `hyperref` package that makes all labels and references clickable in both the generated DVI and PDF files. These are very useful when reading the document online and do not affect the output when the files are printed.

1.2 Example Figures and tables

Figure 1.1 shows a simple figure for illustration along with a long caption. The formatting of the caption text is automatically single spaced and indented. Table 1.1 shows a sample table with the caption placed correctly. The caption for this should always be placed before the table as shown in the example.

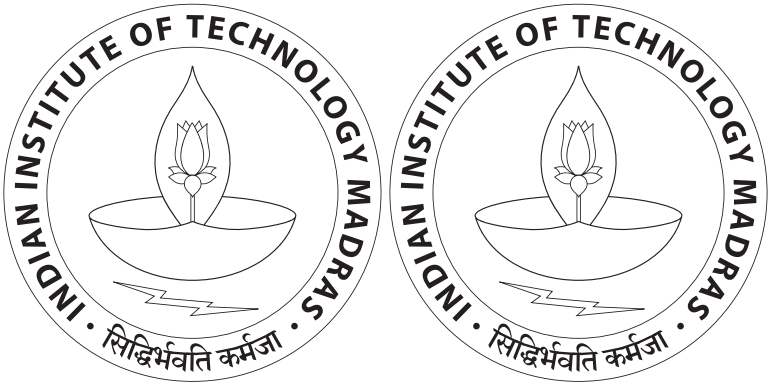

Figure 1.1: Two IITM logos in a row. This is also an illustration of a very long figure caption that wraps around two two lines. Notice that the caption is single-spaced.

1.3 Bibliography with BIBTEX

I strongly recommend that you use BIBTEX to automatically generate your bibliography. It makes managing your references much easier. It is an excellent way to

Table 1.1: A sample table with a table caption placed appropriately. This caption is also very long and is single-spaced. Also notice how the text is aligned.

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64

organize your references and reuse them. You can use one set of entries for your references and cite them in your thesis, papers and reports. If you haven't used it anytime before please invest some time learning how to use it.

I've included a simple example BIBTEX file along in this directory called `refs.bib`. The `iitmdiss.cls` class package which is used in this thesis and for the synopsis uses the `natbib` package to format the references along with a customized bibliography style provided as the `iitm.bst` file in the directory containing `thesis.tex`. Documentation for the `natbib` package should be available in your distribution of LATEX. Basically, to cite the author along with the author name and year use `\cite{key}` where `key` is the citation key for your bibliography entry. You can also use `\citet{key}` to get the same effect. To make the citation without the author name in the main text but inside the parenthesis use `\citep{key}`. The following paragraph shows how citations can be used in text effectively.

More information on BIBTEX is available in the book by Lamport [1994]. There are many references [Lamport, 1994; Kopka and Daly, 2003; Griffiths and Higham, 1997] that explain how to use BIBTEX. Read the `natbib` package documentation for more details on how to cite things differently. Different type of citations are

book Bellman [1957], one atricle in the book Amarel [1968], thesis Manning [1990], technical report Ravindran and Barto [2001], journal Barto *et al.* [1995], conference Knoblock [1990] and other like url Crawford [1992].

1.4 Other useful LaTeX packages

The following packages might be useful when writing your thesis.

- It is very useful to include line numbers in your document. That way, it is very easy for people to suggest corrections to your text. I recommend the use of the `lineno` package for this purpose. This is not a standard package but can be obtained on the internet. The directory containing this file should contain a lineno directory that includes the package along with documentation for it.
- The `listings` package should be available with your distribution of LaTeX. This package is very useful when one needs to list source code or pseudocode.
- For special figure captions the `ccaption` package may be useful. This is specially useful if one has a figure that spans more than two pages and you need to use the same figure number.
- The notation page can be entered manually or automatically generated using the `nomencl` package.

More details on how to use these specific packages are available along with the documentation of the respective packages.

APPENDIX A

A SAMPLE APPENDIX

Just put in text as you would into any chapter with sections and whatnot. Thats the end of it.

Publications

1. S. M. Narayanamurthy and B. Ravindran (2007). Efficiently Exploiting Symmetries in Real Time Dynamic Programming. *IJCAI 2007, Proceedings of the 20th International Joint Conference on Artificial Intelligence*, pages 2556–2561.

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