MATHEMATICAL FOUNDATIONS FOR COMPUTING

Part A:3 marks each Part B: 6 marks each

MODULE 1

PART A

1. Let A = {1,2,3,4} and B = {p, q, r, s} and if R = {(1,p), (1,q), (1,r), (2,q), (2,r), (2,s)} is a relation from A to B. Write the matrix representation of R.
2. Show that (A ∪ B)’ = A’ ∩ B’
3. Verify De-Morgan’s laws for the following sets.
   A = {1, 2, 3}, B = {2, 3, 4} and U = {1, 2, 3, 4, 5, 6}.
4. Define the following and give one example for each.
   i) one-one function
   ii) on to function
5. Prove that $(A \cup B)' = A' \cap B'$
6. If A = {1,2,3,4,5} and B = {1,2,3,8,9} and the functions f : A → B and g : A → A defined by f = {(1,8), (3,9), (4,3), (2,1), (5,2)}and g = {(1,2), (3,1), (2,2), (4,3), (5,2)}
   Find:
   (1) f ∘ g
   (2) g ∘ g
7. If A = {1,2,3,4} give an example of a relation R on A that is
   a) Reflexive and symmetric, but not transitive.
   b) Reflexive and transitive, but not symmetric.
   c) Symmetric and transitive, but not reflexive
8. Consider a relation R on A = {1,2,3} whose matrix representation is given.
   Determine its inverse.
   $$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
9. Let A = {1,2,3,4} and
   R = {(1,2), (1,3), (2,4), (3,2)} be a relation on A.
   Find the relation matrix.
10. Show that the function f : R → R defined by
    f(x) = 3x + 7 is injective.

PART B

1. Define Equivalence relation. Prove that for x, y ∈ Z the relation defined by
   R = {(x, y) ; 5 divides x − y} is an equivalence relation.

2. Using Warshall's algorithm to find the transitive closure of the relation $\{(1,2), (2,3), (3,4), (2,1)\}$ on $\{1,2,3,4\}$.
3. a) Let $f, g : R \rightarrow R$ be defined by $f(x) = x^2 - 2$, $g(x) = x + 4$,
   Find $f \circ g$ and $g \circ f$. Is $f \circ g = g \circ f$?
   b) Let R be a relation on the set of integers defined by
   $R = \{(x, y) \mid x - y \text{ is divisible by } 6\}$.
4. a) Define a partial ordering relation. Show that the inclusion relation $\subseteq$ is a partial ordering relation on the power set P(S) of a given set S.
   b) Using Warshall's algorithm, find the transitive closure of the relation $\{(1,3), (3,2), (2,4), (3,1), (4,1)\}$ on $\{1, 2, 3, 4\}$.
5. Define Equivalence Relation.
   Prove that the relation R on the set of integers Z defined by
   $R = \{(x, y) \mid x - y \text{ is divisible by } 6\}$
   is an equivalence relation.
6. Explain closure of relations. Using Warshall's Algorithm find the transitive closure of the relation
   $R = \{(1,2), (2,3), (3,3)\}$ on the set $A = \{1,2,3\}$.
7. Let $A = \{1,2,3,4,5\} \times \{1,2,3,4,5\}$ and define R on A by
   $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
   Verify that R is an equivalence relation on A.
8. Using Warshall's algorithm, find the transitive closure of the relation
   $R = \{(1,2), (2,3), (3,3)\}$ on the set $A = \{1,2,3\}$.
9. a) For any sets A, B and C prove that
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
   b) Prove that for $x, y \in Z$ the relation
   $R = \{(x, y) : 5 \text{ divides } x - y\}$
   is an equivalence relation.
10. a) Let $f, g : R \rightarrow R$ defined by
    $f(x) = x + 1$,
    $g(x) = 2x^2 + 3$.
    Find $f \circ g$ and $g \circ f$. Is $f \circ g = g \circ f$?
    b) Show that the function $f : R \rightarrow R$ defined by
    $f(x) = x^2$ is not invertible.

MODULE 2

PART A

1. Use Euclidean algorithm to obtain x and y satisfying gcd (752, 1000) = 752x + 1000y.
2. Solve the recurrence relation $6a_n - 7a_{n-1} = 0$, $n \geq 1$, $a_3 = 343$.
3. Using division algorithm, find the gcd of 86 and 58.
4. If $a \equiv b \pmod{m}$ show that $ac \equiv bc \pmod{m}$.
5. Define GCD. Find GCD (2406, 654).
6. Solve the recurrence relation
   $a_{n+2} = 4a_{n+1} - 4a_n$,
   $a_0 = 1$, $a_1 = 3$.
7. Use the Euclidean algorithm to obtain integers x, y satisfying
   gcd(1769, 2378) = 1769x + 2378y.
8. Solve the recurrence relation
   $a_n = 5a_{n-1} + 6a_{n-2}$.
9. Find gcd(306, 657).
10. Solve the recurrence relation
    $3a_{n+1} - 4a_n = 0$, $n \geq 0$, $a_1 = 5$.

PART B

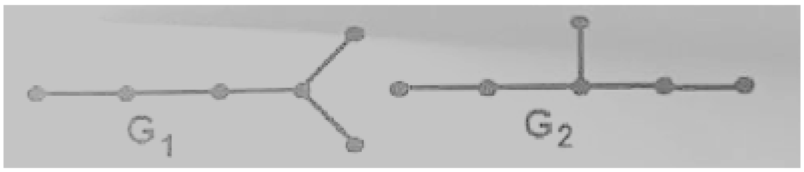
1. Solve the linear Diophantine equation 24x + 138y = 18
2. Solve the recurrence relation $a_n = 7a_{n-1} - 12a_{n-2}$, with $a_0 = 3$, $a_1 = 11$.
3. a) Solve the set of simultaneous congruences:
   $x \equiv 3 \pmod{5}$;
   $x \equiv 4 \pmod{7}$;
   $x \equiv 6 \pmod{9}$.
4. a) Solve the recurrence relation
   $a_n - 5a_{n-1} + 6a_{n-2} = 3^n + n$,
   given $a_0 = 0$ and $a_1 = 1$.
5. Solve the linear Diophantine Equation 60x + 33y = 9.
6. Solve the recurrence equation
   $a_{n+2} - 4a_{n+1} + 3a_n = -200$,
   $a_0 = 0$, $a_1 = 1$.
7. a) Show that the square of any integer is of the form 3k or 3k + 1.
   b) Find the remainders when 2^{50} and 41^{65} are divided by 7.
8. Find the solution for the following set of congruent equations using Chinese Remainder Theorem:
   $x \equiv 2 \pmod{3}$

$x \equiv 3 \pmod 5$
$x \equiv 2 \pmod 7$

9. Solve the linear Diophantine equation
   $54x + 21y = 906$.
10. Solve the recurrence relation
    $2a_n = 7a_{n-1} - 3a_{n-2}$,
    $a_0 = 2$, $a_1 = 5$.

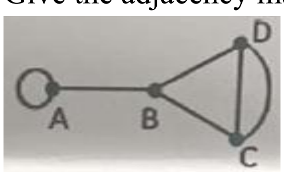
MODULE 3

PART A

1. Define planar and non-planar graphs.
2. A connected planar graph has 5 vertices having degrees 4, 3, 3, 2, 2.
3. Define
   i) complete graph and
   ii) bipartite graph
   and give one example for each.
4. Define Hamiltonian cycle and Euler circuit with examples.
5. Check whether the following graphs are isomorphic.
   

6. Define complete bipartite graph. Draw the graph $K_{2,4}$.
7. Prove that the sum of the degrees of all vertices of a graph G is twice the number of edges in it.
8. Define a planar graph with examples.
9. Show that $K_{3,3}$ is non-planar.
10. Let G be an undirected graph, prove that the sum of the degrees of vertices in G is equal to twice the number of edges in G.

PART B

1. Prove that a connected graph G is a Euler graph if all vertices of G are of even degree.
2. Prove that for a planar graph $v - e + r = 2$, where $|V| = v$, $|E| = e$, r = number of regions.
3. a) Show that a connected graph G is Euler if and only if all the vertices of G are of even degree.
4. a) Show that the maximum number of edges in a simple graph with n vertices is nC_2.
5. Give the adjacency matrix and incidence matrix for the following graph.
   

6. Define Hamiltonian cycle and Euler circuit with example.
7. a) Prove that a non-empty connected graph G is Eulerian if and only if its vertices are all of even degree.
   b) Represent the Königsberg bridge problem by means of a graph. Does it have a solution? Justify.

8. Give an example of a connected graph that has
   a) Neither an Euler circuit nor a Hamiltonian cycle.
   b) Both a Hamiltonian cycle and Euler circuit.
9. Use Fleury's algorithm to find an Euler circuit for the given graph.
10. Define adjacency matrix and incidence matrix. Find the adjacency matrix and incidence matrix of the given graph.

MODULE 4

PART A

1. Find the Eigen values of the matrix
   $$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
2. Show that the vectors $(1, -1, 0)$, $(1, 3, -1)$, $(5, 3, -2)$ are linearly dependent.
3. Find the rank of matrix
   $$A=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$$
4. Find the matrix C such that $Q = X^T C X$ where $Q = -3x^2 + 4xy - y^2 + 2xz - 5z^2$.
5. Determine the rank of
   $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$$
6. Solve using Gauss elimination method:
   $x_1 + x_3 = 0$,
   $x_2 + x_3 = 0$,
   $x_1 + x_2 + x_3 = 0$.
7. State the fundamental theorem for linear systems.
8. How to convert a matrix into row echelon form? What are the information obtained from this form?
9. Find the rank of the matrix
   $$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$$
10. What is meant by diagonalization of a matrix? Write the steps for diagonalization of a matrix.

PART B

1. Find the values of λ and μ for which the system of equations
   $2x + 3y + 5z = 9$
   $7x + 3y - 2z = 8$
   $2x + 3y + \lambda z = \mu$
   has (i) no solution (ii) a unique solution (iii) infinite solutions
2. Find the eigen values and eigen vectors of

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

3. a) Solve the following system of equations using Gauss – Elimination method:
   $x + 2y - z = 3$
   $3x - y + 2z = 1$
   $2x - 2y + 3z = 2$
   $x - y + z = -1$
4. a) Find the eigen values and eigen vectors of the matrix
   $$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$
5. Find the eigenvalues and eigenvectors of
   $$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$
6. What kind of conic section is given by the quadratic form
   $4x_1^2 + 6x_1x_2 - 4x_2^2 = 10$.
7. Let
   $$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$
   Find eigenvalues and eigenvectors of A.
8. Solve the following system of linear equations by Gauss Elimination method:
   $x_1 - x_2 + x_3 = 0$
   $-x_1 + x_2 - x_3 = 0$
   $10x_2 + 25x_3 = 90$
   $20x_1 + 10x_2 = 80$
9. Solve the following linear system of equations using Gauss elimination method:

   $x - y + z = 0$
   $-x + y - z = 0$
   $10y + 25z = 90$
   $20x + 10y = 80$

10. Find the eigen values and eigen vectors of the matrix
   $$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

MODULE 5

PART A

1. Define scatter diagram. Describe the various types of correlation using scatter diagram.
2. State the principle of least squares.
3. State the principle of least square.
4. Explain the terms correlation and regression.
5. Explain principle of least square.
6. Fit a straight line $y = a + bx$ to the following data by the principle of least squares:

x	0	1	3	6	8
y	1	3	2	5	4

7. In a partially destroyed laboratory record, only lines of regression y on x and x on y are available as
   $4x - 5y + 33 = 0$,
   $20x - 9y = 107$ respectively.
   Calculate the coefficient of correlation between x and y.
8. Form the normal equations for fitting a straight line in least squares method with n-data.
9. Write the normal equations for fitting the straight-line $y = ax + b$.
10. Show that the coefficient of correlation lies between -1 and 1.

PART B

1. Compute the correlation coefficient from the following data:

X	9	8	7	6	5	4	3	2	1
Y	15	16	14	13	11	12	10	8	9

2. Obtain the two regression equations from the following data:

x	3	5	6	7	10	11
y	8	12	11	14	16	17

3. a) Calculate the correlation coefficient for the following heights (in inches) of father (x) and their son (y).

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

4. a) Fit a straight line to the following set of data:

x	5	10	15	20	25
y	16	19	23	26	30

5. Fit a parabola to the following data:

X	1.0	1.5	2.0	2.5	3.1	4.0
Y	1.1	1.3	1.6	2.0	3.4	4.2

6. The marks secured by 9 students in Mathematics and English are as given below. Calculate the rank correlation coefficient.

X	10	15	12	17	13	16	24	14	22
Y	30	42	45	46	33	34	40	35	39

7. Employ the method of least squares to fit a parabola
   $y = a + bx + cx^2$
   to the following data:
   $(x, y) = (-1,2), (0,0), (0,1), (1,2)$
8. Compute Spearman's rank correlation coefficient r for the following data:

Person	A	B	C	D	E	F	G	H	I	J
Rank in Statistics	9	10	6	5	7	2	4	8	1	3
Rank in Income	1	2	3	4	5	6	7	8	9	10

9. Fit a second-degree parabola to the following data:

X	1	2	3	4	5
Y	5	12	26	60	97

10. Find the correlation coefficient between x and y from the given data:

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9