1.19 Programming Weekly Production in a Metallurigcal Company

The PRODA, S.A. industrial products firm has to face the problem of scheduling the weekly production of its three products (P1, P2 and P3). These products are sold to large industrial firms and PRODA, S.A. wishes to supply its products in quantities that are more profitable for it.

Each product entails three operations: smelting; mechanisation; assembly and packaging. The smelting operations for products P1 and P2 could be subcontracted, but the smelting operation for product P3 requires special equipment, thus preventing the use of subcontracts. The direct costs of all three operations and the sale prices of the respective products are provided in Table 1.28.

Each unit of product P1 requires 6 min of smelting time (if performed at PRODA, S.A.), 6 min of mechanisation time and 3 min of assembly and packaging time, respectively. For product P2, the times are 10, 3 and 2 min, respectively. One unit of product P3 needs 8 min of smelting time, 8 min of mechanisation and 2 min for assembly and packaging.

PRODA, S.A. has weekly capacities of 8,000 min of smelting time, 12,000 min of mechanisation time and 10,000 min of assembly and packaging time.

(a) Build a linear programming model which maximises PRODA, S.A.'s weekly profits.
(b) Pose the dual problem of (a) by indicating the meaning of the variables, the objective function and the dual constraints.

The solution of the proposed problem has been obtained by WinQSB® (see the sample in Table 1.29). Answer the questions below:

(c) Given the optimum product mix, should the firm follow what the solution provided by linear programming states? If your answer is no, how can this be introduced into the linear programming model?
(d) If product 3 generates more unit profits, why is it not included in the optimum product mix?
(e) PRODA, S.A. is thinking about updating its assembly and packaging line to achieve better times. Is this a suitable strategy?
(f) There are two processes available to smelt product 2 and both are employed in the optimum problem solution. How do you assess the impact of preventing subcontracting the smelting process on product 2?

Table 1.28 Cost of operations and sale prices in dollars

Direct costs and sale prices ($)	P1	P2	P3
Cost of smelting at PRODA, S.A.	0.30	0.50	0.40
Cost of subcontracted smelting	0.50	0.60	–
Cost of mechanisation	0.20	0.10	0.27
Cost of assembly and packaging	0.30	0.20	0.20
Sale price	1.50	1.80	1.97

Table 1.29 Solution obtained by WinQSB®

Decision variable	Solution value	Unit cost or profit c(j)	Total contribution	Reduced cost	Basic status	Allowable Min. c(j)	Allowable Max. c(j)
X_1	0	0.7000	0	−1.1600	At bound	−M	1.8600
X_2	0	0.5000	0	−1.3000	At bound	−M	1.8000
X_3	800.0000	1.0000	800.0000	0	Basic	0.9000	M
X_4	3200.0000	0.9000	2880.0000	0	Basic	0.2500	1.0000
X_5	0	1.1000	0	−1.3800	At bound	−M	2.4800
Objective function	Max=	3680.0000					

Constraint	Left-hand side	Direction	Right-hand side	Slack or Surplus	Shadow price	Allowable Min. RHS.	Allowable Max. RHS.
C1	8000.0000	<=	8000.0000	0	0.0100	0	40000.0000
C2	12000.0000	<=	12000.0000	0	0.3000	2400.0000	15000.0000
C3	8000.0000	<=	10000.0000	2000.0000	0	8000.0000	M

Solution

(a) Design a linear programming model that maximises PRODA, S.A.'s weekly profits

Decision variables:

X_1 = units of product P1 with smelting performed at PRODA, S.A.
X_2 = units of product P1 with a subcontracted smelting process.
X_3 = units of product P2 with smelting performed at PRODA, S.A.
X_4 = units of product P2 with a subcontracted smelting process.
X_5 = units of product P3.

Objective function:

$$\text{Max } z = 0.7X_1 + 0.5X_2 + X_3 + 0.9X_4 + 1.1X_5$$

Constraints:

$6X_1 + 10X_3 + 8X_5 \leq 8000$ (Capacity available for smelting)
$6X_1 + 6X_2 + 3X_3 + 3X_4 + 8X_5 \leq 12000$ (Capacity available for mechanisation)
$3X_1 + 3X_2 + 2X_3 + 2X_4 + 2X_5 \leq 10000$ (Capacity available for assembly and packaging)
$X_1, X_2, X_3, X_4, X_5 \geq 0$ (Non-negativity constraint)

(b) Pose the dual problem of (a) by indicating the meaning of the variables, the objective function and the dual constraints.

Decision variables:

U_1 = Shadow price, marginal value or opportunity cost of the minutes of smelting time at PRODA, S.A.

U_2 = Shadow price, marginal value or opportunity cost of the minutes of mechanisation time at PRODA, S.A.
U_3 = Shadow price, marginal value or opportunity cost of the minutes of assembly and packaging time at PRODA, S.A.

Objective function:

$$\text{Min } z = 8000U_1 + 12000U_2 + 10000U_3$$

(To minimise what is no longer earned due to not having more resources or the total value of the resources employed)
Constraints:
$6U_1 + 6U_2 + 3U_3 \geq 0.7$ (The value of the resources invested in product 1 must be at least equal to the profit generated)

$$6U_2 + 3U_3 \geq 0.5$$

$$10U_1 + 3U_2 + 2U_3 \geq 1$$

$$3U_2 + 2U_3 \geq 0.9$$

$$8U_1 + 8U_2 + 2U_3 \geq 1.1$$

(c) Given the optimum product mix, should the firm follow what the solution provided by linear programming states? If your answer is no, how can this be introduced into the linear programming model?
The optimum product mix is: 800 units of product 2 (smelted at PRODA, S.A.) and 3,200 units of product 2 (by a subcontracted smelting process). This implies that the firm becomes a producer of a single product; in other words, it should change strategy. Should it wish to offer a wider variety of products (2 or 3), this should be reflected with additional minimum productions constraints.
(d) If product 3 generates more unit profits, why is it not included in the optimum product mix?
This is because product P3 employs scarcer resources more intensively. For example, P3 takes 8 min mechanisation time, while P2 takes only 3.
(e) PRODA, S.A. is thinking about updating its assembly and packaging line to achieve better times. Is this a suitable strategy?
No because there is currently a slack variable of 2,000 min, so increasing the time available would not improve the current situation.
(f) There are two processes available to smelt product 2 and both are employed in the optimum problem solution. How do you assess the impact of preventing subcontracting the smelting process on product 2?
The corresponding variable (X_4) would be removed from the problem structure and the problem would need to be solved again.