K. J. SOMAIYA COLLEGE OF SCIENCE AND COMMERCE

ASSIGNMENT

TIME SERIES

ANALYSIS

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INTRODUCTION

In this project, the goal is to enhance the analysis of sales data by applying smoothing techniques. Smoothing methods help reduce noise and highlight underlying trends in sales data, providing a clearer picture of overall patterns. By employing statistical algorithms or mathematical models, we aim to create more accurate forecasts, identify seasonality, and improve decision-making processes for businesses relying on sales insights. This project explores various smoothing techniques to refine the representation of sales data, ultimately aiding in better-informed strategies and predictions.

WHAT IS SMOOTHING?

Smoothing is usually done to help us better to see patterns, for example trends in time series. Generally smooth out the irregular roughness to see a clearer signal. For seasonal data, we might smooth out the seasonality so that we can identify the trend. Smoothing doesn't provide us with a model, but it can be a good first step in describing various components of the series. − The term filter is sometimes used to describe a smoothing procedure. For instance, if the smoothed value for a particular time is calculated as a linear combination of observations for surrounding times, it might be said that we've applied a linear filter to the data (not the same as saying the result is a straight line, by the way). − We can often think of a data set as consisting of two distinct components: signal and noise. Signal represents any pattern caused by the intrinsic dynamics of the process from which the data is collected. These patterns can take various forms from a simple constant process to a more complicated structure that cannot be extracted visually or with any basic statistical tools. − Smoothing can be seen as a technique to separate the signal and the noise as much as possible and in that smoother acts as a filter to obtain an "estimate" for the signal.

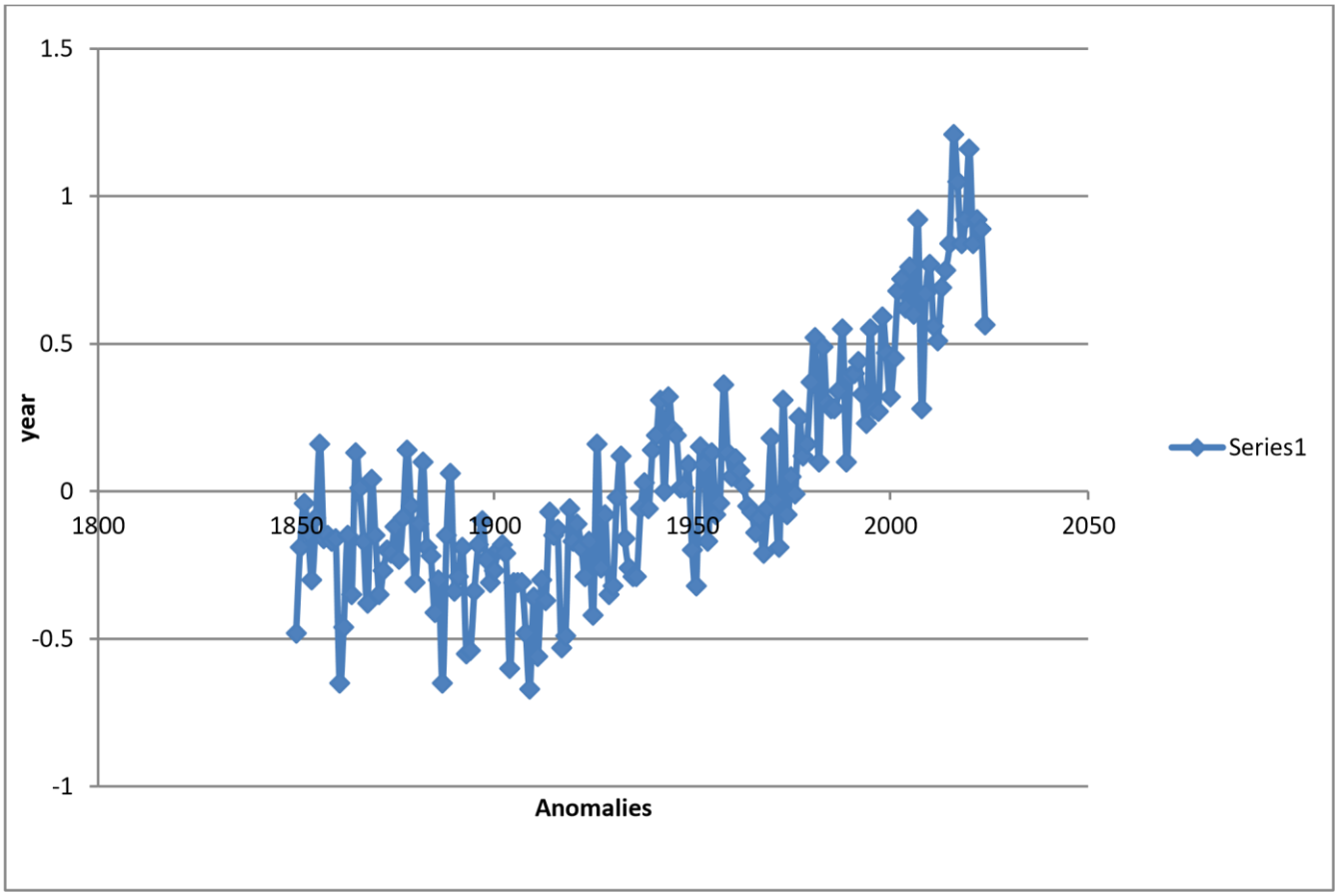
DATA

Global Land and Ocean January Temperature Anomalies

Units: Degrees Celsius

Base Period: 1801-2000

- Plot



The graph indicates that there is a increasing trend in the data.

- Moving Average, Centered Moving Average, Weighted Moving Average
- Formulae used

Moving average : $\frac{-0.48+(-0.19)+(-0.04)}{3} = -0.23667$

Centered Moving Average : $\frac{-.023367+(-0.12333)}{2} = -0.18$

Weighted Moving Average : $(-0.48)x(0.2)+(-0.19)x(0.3)+(-0.04)x(0.5) = -0.173$

Year	Anomaly	MA (3 months)	centered MA	Weighted MA (0.2, 0.3, 0.5)
1850	-0.48			
1851	-0.19	-0.23667		-0.173
1852	-0.04	-0.12333	-0.18	-0.12
1853	-0.14	-0.16	-0.14167	-0.2

1854	-0.3	-0.17667		-0.16833		-0.163		
1855	-0.09	-0.07667		-0.12667		-0.007		
1856	0.16	-0.03		-0.05333		-0.05		
1857	-0.16	-0.05		-0.04		-0.091		
1858	-0.15	-0.16		-0.105		-0.162		
1859	-0.17	-0.16		-0.16		-0.161		
1860	-0.16	-0.32667		-0.24333		-0.407		
1861	-0.65	-0.42333		-0.375		-0.457		
1862	-0.46	-0.42		-0.42167		-0.343		
1863	-0.15	-0.32		-0.37		-0.312		
1864	-0.35	-0.12333		-0.22167		-0.07		
1865	0.13	-0.07		-0.09667		-0.026		
1866	0.01	-0.01		-0.04		-0.056		
1867	-0.17	-0.18		-0.095		-0.239		
1868	-0.38	-0.17		-0.175		-0.128		
1869	0.04	-0.16333		-0.16667		-0.139		
1870	-0.15	-0.15333		-0.15833		-0.212		
1871	-0.35	-0.25667		-0.205		-0.27		
1872	-0.27	-0.27333		-0.265		-0.251		
1873	-0.2	-0.22667		-0.25		-0.219		
1874	-0.21	-0.17667		-0.20167		-0.163		
1875	-0.12	-0.18667		-0.18167		-0.193		

1876	-0.23	-0.14667		-0.16667		-0.138		
1877	-0.09	-0.06		-0.10333		-0.003		
1878	0.14	4.63E-18		-0.03		-0.001		
1879	-0.05	-0.07333		-0.03667		-0.142		
1880	-0.31	-0.15667		-0.115		-0.158		
1881	-0.11	-0.10667		-0.13167		-0.045		
1882	0.1	-0.06667		-0.08667		-0.087		
1883	-0.19	-0.10333		-0.085		-0.147		
1884	-0.22	-0.27333		-0.18833		-0.309		

1885	-0.41	-0.31		-0.29167		-0.317		
1886	-0.3	-0.45333		-0.38167		-0.497		
1887	-0.65	-0.36667		-0.41		-0.33		
1888	-0.15	-0.24667		-0.30667		-0.145		
1889	0.06	-0.14333		-0.195		-0.182		
1890	-0.34	-0.19		-0.16667		-0.235		
1891	-0.29	-0.27333		-0.23167		-0.25		
1892	-0.19	-0.34333		-0.30833		-0.39		
1893	-0.55	-0.42667		-0.385		-0.473		
1894	-0.54	-0.47667		-0.45167		-0.442		
1895	-0.34	-0.35333		-0.415		-0.3		
1896	-0.18	-0.20667		-0.28		-0.172		
1897	-0.1	-0.17		-0.18833		-0.181		
1898	-0.23	-0.21333		-0.19167		-0.244		
1899	-0.31	-0.27		-0.24167		-0.274		
1900	-0.27	-0.26		-0.265		-0.243		
1901	-0.2	-0.21667		-0.23833		-0.204		
1902	-0.18	-0.19667		-0.20667		-0.199		
1903	-0.21	-0.33		-0.26333		-0.399		
1904	-0.6	-0.37333		-0.35167		-0.377		
1905	-0.31	-0.40667		-0.39		-0.368		
1906	-0.31	-0.31		-0.35833		-0.31		
1907	-0.31	-0.36667		-0.33833		-0.395		
1908	-0.48	-0.48667		-0.42667		-0.541		
1909	-0.67	-0.50333		-0.495		-0.477		
1910	-0.36	-0.53		-0.51667		-0.522		
1911	-0.56	-0.40667		-0.46833		-0.39		
1912	-0.3	-0.41		-0.40833		-0.387		
1913	-0.37	-0.24667		-0.32833		-0.206		
1914	-0.07	-0.19667		-0.22167		-0.17		
1915	-0.15	-0.11667		-0.15667		-0.124		
1916	-0.13	-0.27		-0.19333		-0.334		

1917	-0.53	-0.38333		-0.32667		-0.43		
1918	-0.49	-0.36		-0.37167		-0.283		
1919	-0.06	-0.24		-0.3		-0.201		
1920	-0.17	-0.11333		-0.17667		-0.118		
1921	-0.11	-0.15667		-0.135		-0.162		
1922	-0.19	-0.19667		-0.17667		-0.224		
1923	-0.29	-0.21667		-0.20667		-0.21		
1924	-0.17	-0.29333		-0.255		-0.319		
1925	-0.42	-0.14333		-0.21833		-0.08		
1926	0.16	-0.17333		-0.15833		-0.166		
1927	-0.26	-0.06		-0.11667		-0.086		
1928	-0.08	-0.23		-0.145		-0.251		
1929	-0.35	-0.25		-0.24		-0.281		
1930	-0.32	-0.23		-0.24		-0.176		
1931	-0.02	-0.07333		-0.15167		-0.01		
1932	0.12	-0.02		-0.04667		-0.048		
1933	-0.16	-0.1		-0.06		-0.154		
1934	-0.26	-0.23667		-0.16833		-0.255		
1935	-0.29	-0.28		-0.25833		-0.284		
1936	-0.29	-0.21333		-0.24667		-0.175		
1937	-0.06	-0.10667		-0.16		-0.061		
1938	0.03	-0.03		-0.06833		-0.033		
1939	-0.06	0.036667		0.003333		0.058		
1940	0.14	0.09		0.063333		0.125		
1941	0.19	0.213333		0.151667		0.24		
1942	0.31	0.166667		0.19		0.131		
1943	0	0.21		0.188333		0.222		
1944	0.32	0.176667		0.193333		0.201		
1945	0.21	0.24		0.208333		0.222		
1946	0.19	0.136667		0.188333		0.104		
1947	0.01	0.07		0.103333		0.046		

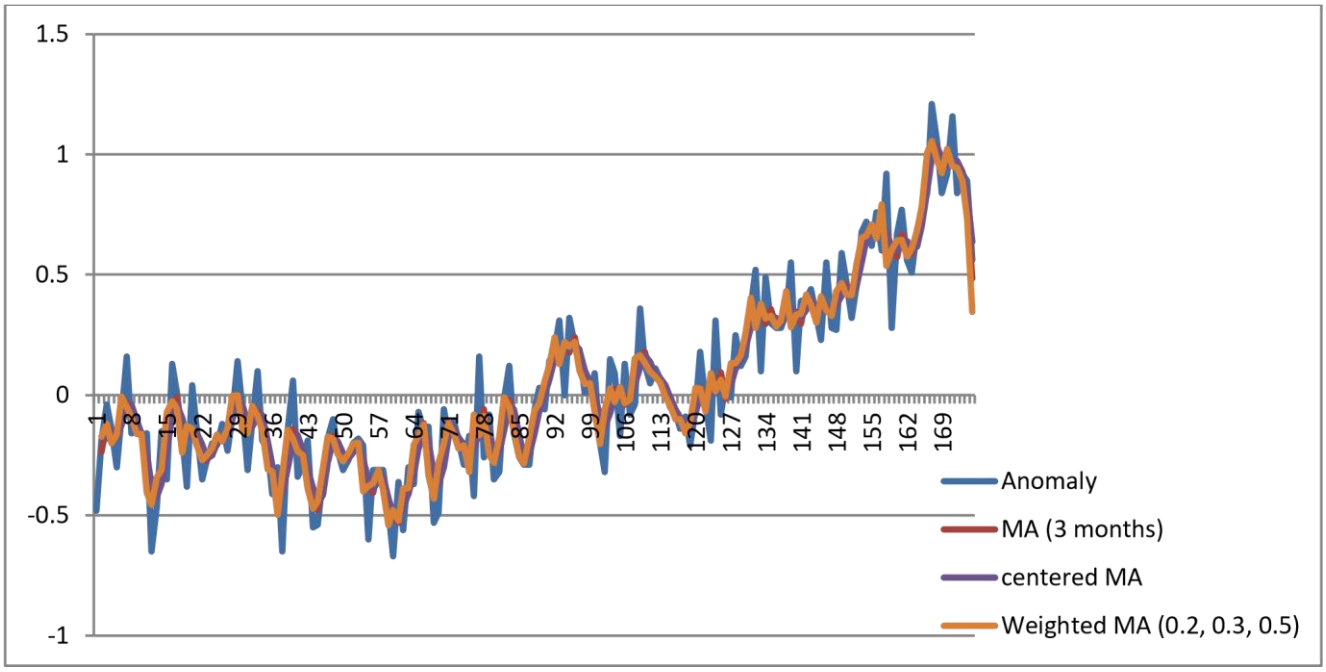
1948	0.01	0.036667		0.053333		0.05		
1949	0.09	-0.03333		0.001667		-0.071		
1950	-0.2	-0.14333		-0.08833		-0.202		
1951	-0.32	-0.12333		-0.13333		-0.061		
1952	0.15	-0.02667		-0.075		0.026		
1953	0.09	0.023333		-0.00167		-0.028		
1954	-0.17	0.016667		0.02		0.032		
1955	0.13	-0.04		-0.01167		-0.035		
1956	-0.08	0.003333		-0.01833		-0.018		
1957	-0.04	0.08		0.041667		0.152		

1958	0.36	0.15		0.115		0.165		
1959	0.13	0.18		0.165		0.136		
1960	0.05	0.096667		0.138333		0.096		
1961	0.11	0.076667		0.086667		0.078		
1962	0.07	0.066667		0.071667		0.053		
1963	0.02	0.013333		0.04		-0.005		
1964	-0.05	-0.03333		-0.01		-0.046		
1965	-0.07	-0.08667		-0.06		-0.101		
1966	-0.14	-0.1		-0.09333		-0.101		
1967	-0.09	-0.14667		-0.12333		-0.16		
1968	-0.21	-0.12		-0.13333		-0.111		
1969	-0.06	-0.03		-0.075		0.03		
1970	0.18	0.03		0		0.027		
1971	-0.03	-0.01333		0.008333		-0.068		
1972	-0.19	0.03		0.008333		0.092		
1973	0.31	0.013333		0.021667		0.015		
1974	-0.08	0.093333		0.053333		0.063		
1975	0.05	-0.01333		0.04		-0.006		
1976	-0.01	0.096667		0.041667		0.132		
1977	0.25	0.12		0.108333		0.133		
1978	0.12	0.176667		0.148333		0.166		
1979	0.16	0.216667		0.196667		0.257		
1980	0.37	0.35		0.283333		0.403		

1981	0.52	0.33		0.34		0.28		
1982	0.1	0.37		0.35		0.379		
1983	0.49	0.296667		0.333333		0.317		
1984	0.3	0.356667		0.326667		0.328		
1985	0.28	0.286667		0.321667		0.284		
1986	0.28	0.3		0.293333		0.31		
1987	0.34	0.39		0.345		0.433		
1988	0.55	0.33		0.36		0.283		
1989	0.1	0.346667		0.338333		0.335		
1990	0.39	0.296667		0.321667		0.337		
1991	0.4	0.41		0.353333		0.418		
1992	0.44	0.39		0.4		0.377		
1993	0.33	0.333333		0.361667		0.302		
1994	0.23	0.37		0.351667		0.41		
1995	0.55	0.353333		0.361667		0.351		
1996	0.28	0.366667		0.36		0.329		
1997	0.27	0.38		0.373333		0.432		
1998	0.59	0.443333		0.411667		0.466		
1999	0.47	0.46		0.451667		0.419		
2000	0.32	0.413333		0.436667		0.415		
2001	0.45	0.483333		0.448333		0.539		
2002	0.68	0.616667		0.55		0.654		
2003	0.72	0.673333		0.645		0.662		
2004	0.62	0.7		0.686667		0.71		
2005	0.76	0.66		0.68		0.652		
2006	0.6	0.76		0.71		0.792		
2007	0.92	0.6		0.68		0.536		
2008	0.28	0.623333		0.611667		0.603		
2009	0.67	0.573333		0.598333		0.642		
2010	0.77	0.666667		0.62		0.645		
2011	0.56	0.613333		0.64		0.577		
2012	0.51	0.586667		0.6		0.61		
2013	0.69	0.65		0.618333		0.684		
2014	0.75	0.76		0.705		0.783		

2015	0.84	0.933333		0.846667		1.007		
2016	1.21	1.033333		0.983333		1.056		
2017	1.05	1.033333		1.033333		0.977		
2018	0.84	0.936667		0.985		0.922		
2019	0.92	0.973333		0.955		1.024		
2020	1.16	0.973333		0.973333		0.952		
2021	0.84	0.973333		0.973333		0.944		
2022	0.92	0.883333		0.928333		0.889		
2023	0.89	0.790984		0.837159		0.732476		
2024	0.562952	0.484317		0.637651		0.346886		

Plot



From the graph we can see that MA, Centered MA, and Weighted MA fits the original data.

- **FIRST ORDER EXPONENTIAL SMOOTHING AND SECOND ORDER EXPONENTIAL SMOOTHING**

1st order expo smoothing						2nd order	
0.2		0.3		0.4		Y^t^(2)	Y^t
-0.48	0.00	-0.48	0	-0.48	0	-0.48	-0.48
-0.42	0.23	-0.393	0.203	-0.364	0.174	-0.364	-0.364
-0.35	0.31	-0.2871	0.2471	-0.2344	0.1944	-0.2344	-0.2344
-0.30	0.16	-0.24297	0.10297	-0.19664	0.05664	-0.19664	-0.19664
-0.30	0.00	-0.26008	-0.03992	-0.23798	-0.06202	-0.23798	-0.23798
-0.26	0.17	-0.20906	0.119055	-0.17879	0.08879	-0.17879	-0.17879
-0.18	0.34	-0.09834	0.258339	-0.04327	0.203274	-0.04327	-0.04327
-0.17	0.01	-0.11684	-0.04316	-0.08996	-0.07004	-0.08996	-0.08996
-0.17	0.02	-0.12679	-0.02321	-0.11398	-0.03602	-0.11398	-0.11398
-0.17	0.00	-0.13975	-0.03025	-0.13639	-0.03361	-0.13639	-0.13639
-0.17	0.01	-0.14583	-0.01417	-0.14583	-0.01417	-0.14583	-0.14583
-0.26	-0.39	-0.29708	-0.35292	-0.3475	-0.3025	-0.3475	-0.3475
-0.30	-0.16	-0.34595	-0.11405	-0.3925	-0.0675	-0.3925	-0.3925
-0.27	0.12	-0.28717	0.137168	-0.2955	0.1455	-0.2955	-0.2955
-0.29	-0.06	-0.30602	-0.04398	-0.3173	-0.0327	-0.3173	-0.3173
-0.20	0.33	-0.17521	0.305212	-0.13838	0.26838	-0.13838	-0.13838
-0.16	0.17	-0.11965	0.129649	-0.07903	0.089028	-0.07903	-0.07903
-0.16	-0.01	-0.13475	-0.03525	-0.11542	-0.05458	-0.11542	-0.11542
-0.21	-0.17	-0.20833	-0.17167	-0.22125	-0.15875	-0.22125	-0.22125
-0.16	0.20	-0.13383	0.173829	-0.11675	0.15675	-0.11675	-0.11675
-0.16	0.01	-0.13868	-0.01132	-0.13005	-0.01995	-0.13005	-0.13005
-0.19	-0.16	-0.20208	-0.14792	-0.21803	-0.13197	-0.21803	-0.21803
-0.21	-0.06	-0.22245	-0.04755	-0.23882	-0.03118	-0.23882	-0.23882
-0.21	0.01	-0.21572	0.015717	-0.22329	0.023291	-0.22329	-0.22329
-0.21	0.00	-0.214	0.004002	-0.21797	0.007974	-0.21797	-0.21797
-0.19	0.07	-0.1858	0.065802	-0.17878	0.058785	-0.17878	-0.17878
-0.20	-0.03	-0.19906	-0.03094	-0.19927	-0.03073	-0.19927	-0.19927
-0.18	0.09	-0.16634	0.076343	-0.15556	0.065562	-0.15556	-0.15556
-0.11	0.25	-0.07444	0.21444	-0.03734	0.177337	-0.03734	-0.03734
-0.10	0.05	-0.06711	0.017108	-0.0424	-0.0076	-0.0424	-0.0424

-0.14	-0.17	-0.13998	-0.17002	-0.14944	-0.16056	-0.14944	-0.14944
-0.14	0.03	-0.13098	0.020983	-0.13366	0.023665	-0.13366	-0.13366
-0.09	0.19	-0.06169	0.161688	-0.0402	0.140199	-0.0402	-0.0402
-0.11	-0.08	-0.10018	-0.08982	-0.10012	-0.08988	-0.10012	-0.10012
-0.13	-0.09	-0.13613	-0.08387	-0.14807	-0.07193	-0.14807	-0.14807
-0.19	-0.22	-0.21829	-0.19171	-0.25284	-0.15716	-0.25284	-0.25284
-0.21	-0.09	-0.2428	-0.0572	-0.27171	-0.02829	-0.27171	-0.27171
-0.30	-0.35	-0.36496	-0.28504	-0.42302	-0.22698	-0.42302	-0.42302
-0.27	0.12	-0.30047	0.150473	-0.31381	0.163814	-0.31381	-0.31381
-0.20	0.26	-0.19233	0.252331	-0.16429	0.224288	-0.16429	-0.16429
-0.23	-0.11	-0.23663	-0.10337	-0.23457	-0.10543	-0.23457	-0.23457
-0.24	-0.05	-0.25264	-0.03736	-0.25674	-0.03326	-0.25674	-0.25674
-0.23	0.04	-0.23385	0.04385	-0.23005	0.040046	-0.23005	-0.23005
-0.30	-0.25	-0.32869	-0.22131	-0.35803	-0.19197	-0.35803	-0.35803
-0.34	-0.20	-0.39209	-0.14791	-0.43082	-0.10918	-0.43082	-0.43082
-0.34	0.00	-0.37646	0.03646	-0.39449	0.05449	-0.39449	-0.39449
-0.31	0.13	-0.31752	0.137522	-0.30869	0.128694	-0.30869	-0.30869
-0.27	0.17	-0.25227	0.152266	-0.22522	0.125216	-0.22522	-0.22522
-0.26	0.03	-0.24559	0.015586	-0.22713	-0.00287	-0.22713	-0.22713

-0.27	-0.04	-0.26491	-0.04509	-0.26028	-0.04972	-0.26028	-0.26028
-0.27	0.00	-0.26644	-0.00356	-0.26417	-0.00583	-0.26417	-0.26417
-0.26	0.06	-0.24651	0.046506	-0.2385	0.0385	-0.2385	-0.2385
-0.24	0.06	-0.22655	0.046554	-0.2151	0.0351	-0.2151	-0.2151
-0.23	0.02	-0.22159	0.011588	-0.21306	0.00306	-0.21306	-0.21306
-0.31	-0.29	-0.33511	-0.26489	-0.36784	-0.23216	-0.36784	-0.36784
-0.31	0.00	-0.32758	0.017578	-0.3447	0.034702	-0.3447	-0.3447
-0.31	0.00	-0.3223	0.012305	-0.33082	0.020821	-0.33082	-0.33082
-0.31	0.00	-0.31861	0.008613	-0.32249	0.012493	-0.32249	-0.32249
-0.34	-0.14	-0.36703	-0.11297	-0.3855	-0.0945	-0.3855	-0.3855
-0.41	-0.26	-0.45792	-0.21208	-0.4993	-0.1707	-0.4993	-0.4993
-0.40	0.04	-0.42854	0.068544	-0.44358	0.083578	-0.44358	-0.44358

-0.43	-0.13	-0.46798	-0.09202	-0.49015	-0.06985	-0.49015	-0.49015
-0.40	0.10	-0.41759	0.117587	-0.41409	0.114088	-0.41409	-0.41409
-0.40	0.03	-0.40331	0.033311	-0.39645	0.026453	-0.39645	-0.39645
-0.33	0.26	-0.30332	0.233317	-0.26587	0.195872	-0.26587	-0.26587
-0.30	0.15	-0.25732	0.107322	-0.21952	0.069523	-0.21952	-0.21952
-0.26	0.13	-0.21913	0.089126	-0.18371	0.053714	-0.18371	-0.18371
-0.32	-0.21	-0.31239	-0.21761	-0.32223	-0.20777	-0.32223	-0.32223
-0.35	-0.14	-0.36567	-0.12433	-0.38934	-0.10066	-0.38934	-0.38934
-0.29	0.23	-0.27397	0.21397	-0.2576	0.197602	-0.2576	-0.2576
-0.27	0.10	-0.24278	0.072779	-0.22256	0.052561	-0.22256	-0.22256
-0.24	0.13	-0.20295	0.092945	-0.17754	0.067537	-0.17754	-0.17754
-0.23	0.04	-0.19906	0.009062	-0.18252	-0.00748	-0.18252	-0.18252
-0.24	-0.05	-0.22634	-0.06366	-0.22551	-0.06449	-0.22551	-0.22551
-0.23	0.06	-0.20944	0.03944	-0.20331	0.033308	-0.20331	-0.20331
-0.26	-0.16	-0.27261	-0.14739	-0.28998	-0.13002	-0.28998	-0.28998
-0.18	0.34	-0.14283	0.302826	-0.10999	0.269991	-0.10999	-0.10999
-0.20	-0.06	-0.17798	-0.08202	-0.16999	-0.09001	-0.16999	-0.16999
-0.17	0.09	-0.14858	0.068585	-0.134	0.053997	-0.134	-0.134
-0.21	-0.14	-0.20901	-0.14099	-0.2204	-0.1296	-0.2204	-0.2204
-0.23	-0.09	-0.24231	-0.07769	-0.26024	-0.05976	-0.26024	-0.26024
-0.19	0.17	-0.17561	0.155615	-0.16414	0.144143	-0.16414	-0.16414
-0.13	0.25	-0.08693	0.20693	-0.05049	0.170486	-0.05049	-0.05049
-0.13	-0.03	-0.10885	-0.05115	-0.09429	-0.06571	-0.09429	-0.09429
-0.16	-0.10	-0.1542	-0.1058	-0.16057	-0.09943	-0.16057	-0.16057
-0.18	-0.11	-0.19494	-0.09506	-0.21234	-0.07766	-0.21234	-0.21234
-0.21	-0.08	-0.22346	-0.06654	-0.24341	-0.04659	-0.24341	-0.24341
-0.18	0.12	-0.17442	0.114419	-0.17004	0.110044	-0.17004	-0.17004
-0.14	0.17	-0.11309	0.143093	-0.09003	0.120027	-0.09003	-0.09003
-0.12	0.06	-0.09717	0.037165	-0.07802	0.018016	-0.07802	-0.07802

-0.07	0.21	-0.02602	0.166016	0.00919	0.13081	0.00919	0.00919
-0.02	0.21	0.038789	0.151211	0.081514	0.108486	0.081514	0.081514

0.05	0.26	0.120152	0.189848	0.172909	0.137091	0.172909	0.172909
0.04	-0.04	0.084107	-0.08411	0.103745	-0.10375	0.103745	0.103745
0.10	0.22	0.154875	0.165125	0.190247	0.129753	0.190247	0.190247
0.12	0.09	0.171412	0.038588	0.198148	0.011852	0.198148	0.198148
0.13	0.06	0.176989	0.013011	0.194889	-0.00489	0.194889	0.194889
0.11	-0.10	0.126892	-0.11689	0.120933	-0.11093	0.120933	0.120933
0.09	-0.08	0.091824	-0.08182	0.07656	-0.06656	0.07656	0.07656
0.09	0.00	0.091277	-0.00128	0.081936	0.008064	0.081936	0.081936
0.03	-0.23	0.003894	-0.20389	-0.03084	-0.16916	-0.03084	-0.03084
-0.04	-0.28	-0.09327	-0.22673	-0.1465	-0.1735	-0.1465	-0.1465
0.00	0.15	-0.02029	0.170292	-0.0279	0.177902	-0.0279	-0.0279
0.02	0.07	0.012796	0.077204	0.019259	0.070741	0.019259	0.019259
-0.02	-0.15	-0.04204	-0.12796	-0.05644	-0.11356	-0.05644	-0.05644
0.01	0.12	0.00957	0.12043	0.018133	0.111867	0.018133	0.018133
-0.01	-0.07	-0.0173	-0.0627	-0.02112	-0.05888	-0.02112	-0.02112
-0.01	-0.03	-0.02411	-0.01589	-0.02867	-0.01133	-0.02867	-0.02867
0.06	0.30	0.091122	0.268878	0.126797	0.233203	0.126797	0.126797
0.07	0.06	0.102786	0.027214	0.128078	0.001922	0.128078	0.128078
0.07	-0.02	0.08695	-0.03695	0.096847	-0.04685	0.096847	0.096847
0.08	0.03	0.093865	0.016135	0.102108	0.007892	0.102108	0.102108
0.08	-0.01	0.086706	-0.01671	0.089265	-0.01926	0.089265	0.089265
0.06	-0.04	0.066694	-0.04669	0.061559	-0.04156	0.061559	0.061559
0.04	-0.09	0.031686	-0.08169	0.016935	-0.06694	0.016935	0.016935
0.02	-0.09	0.00118	-0.07118	-0.01784	-0.05216	-0.01784	-0.01784
-0.01	-0.13	-0.04117	-0.09883	-0.0667	-0.0733	-0.0667	-0.0667
-0.03	-0.06	-0.05582	-0.03418	-0.07602	-0.01398	-0.07602	-0.07602
-0.06	-0.15	-0.10208	-0.10792	-0.12961	-0.08039	-0.12961	-0.12961
-0.06	0.00	-0.08945	0.029453	-0.10177	0.041768	-0.10177	-0.10177
-0.01	0.19	-0.00862	0.188617	0.010939	0.169061	0.010939	0.010939
-0.02	-0.01	-0.01503	-0.01497	-0.00544	-0.02456	-0.00544	-0.00544
-0.05	-0.14	-0.06752	-0.12248	-0.07926	-0.11074	-0.07926	-0.07926
0.02	0.29	0.045734	0.264266	0.076443	0.233557	0.076443	0.076443

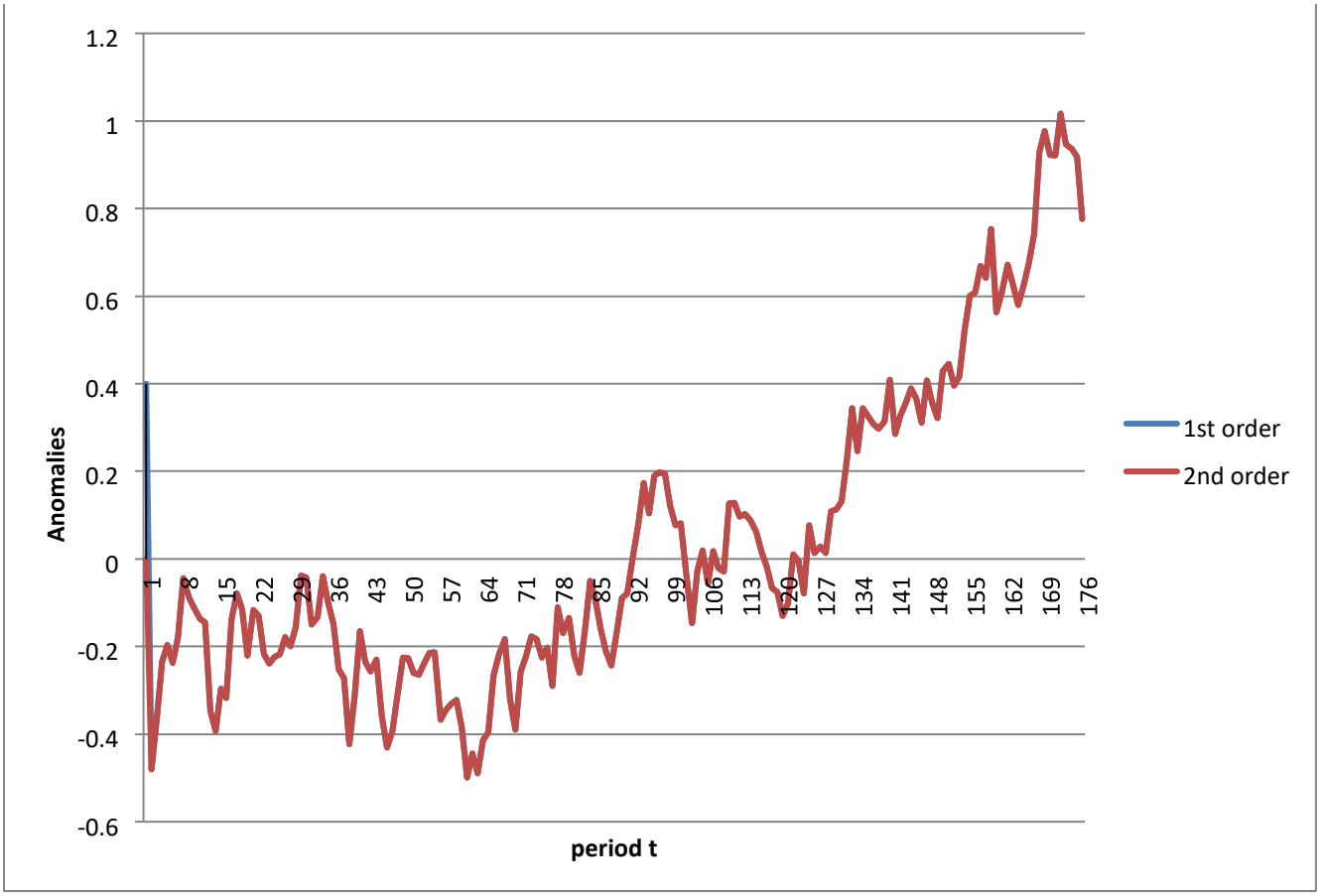
0.00	-0.08	0.008014	-0.08801	0.013866	-0.09387	0.013866	0.013866
0.01	0.04	0.02061	0.02939	0.028319	0.021681	0.028319	0.028319
0.01	-0.02	0.011427	-0.02143	0.012992	-0.02299	0.012992	0.012992
0.05	0.20	0.082999	0.167001	0.107795	0.142205	0.107795	0.107795
0.07	0.05	0.094099	0.025901	0.112677	0.007323	0.112677	0.112677
0.09	0.07	0.113869	0.046131	0.131606	0.028394	0.131606	0.131606
0.14	0.23	0.190709	0.179291	0.226964	0.143036	0.226964	0.226964

0.22	0.30	0.289496	0.230504	0.344178	0.175822	0.344178	0.344178
0.19	-0.09	0.232647	-0.13265	0.246507	-0.14651	0.246507	0.246507
0.25	0.24	0.309853	0.180147	0.343904	0.146096	0.343904	0.343904
0.26	0.04	0.306897	-0.0069	0.326342	-0.02634	0.326342	0.326342
0.27	0.01	0.298828	-0.01883	0.307805	-0.02781	0.307805	0.307805
0.27	0.01	0.29318	-0.01318	0.296683	-0.01668	0.296683	0.296683
0.28	0.06	0.307226	0.032774	0.31401	0.02599	0.31401	0.31401
0.34	0.21	0.380058	0.169942	0.408406	0.141594	0.408406	0.408406
0.29	-0.19	0.296041	-0.19604	0.285044	-0.18504	0.285044	0.285044
0.31	0.08	0.324228	0.065772	0.327026	0.062974	0.327026	0.327026
0.33	0.07	0.34696	0.05304	0.356216	0.043784	0.356216	0.356216
0.35	0.09	0.374872	0.065128	0.389729	0.050271	0.389729	0.389729
0.35	-0.02	0.36141	-0.03141	0.365838	-0.03584	0.365838	0.365838
0.32	-0.09	0.321987	-0.09199	0.311503	-0.0815	0.311503	0.311503
0.37	0.18	0.390391	0.159609	0.406902	0.143098	0.406902	0.406902
0.35	-0.07	0.357274	-0.07727	0.356141	-0.07614	0.356141	0.356141
0.33	-0.06	0.331092	-0.06109	0.321685	-0.05168	0.321685	0.321685
0.39	0.20	0.408764	0.181236	0.429011	0.160989	0.429011	0.429011
0.40	0.07	0.427135	0.042865	0.445406	0.024594	0.445406	0.445406
0.39	-0.07	0.394994	-0.07499	0.395244	-0.07524	0.395244	0.395244
0.40	0.05	0.411496	0.038504	0.417146	0.032854	0.417146	0.417146
0.46	0.22	0.492047	0.187953	0.522288	0.157712	0.522288	0.522288
0.51	0.21	0.560433	0.159567	0.601373	0.118627	0.601373	0.601373
0.53	0.09	0.578303	0.041697	0.608824	0.011176	0.608824	0.608824

0.58	0.18	0.632812	0.127188	0.669294	0.090706	0.669294	0.669294
0.58	0.02	0.622969	-0.02297	0.641576	-0.04158	0.641576	0.641576
0.65	0.27	0.712078	0.207922	0.752946	0.167054	0.752946	0.752946
0.58	-0.30	0.582455	-0.30245	0.563768	-0.28377	0.563768	0.563768
0.59	0.08	0.608718	0.061282	0.606261	0.063739	0.606261	0.606261
0.63	0.14	0.657103	0.112897	0.671756	0.098244	0.671756	0.671756
0.62	-0.06	0.627972	-0.06797	0.627054	-0.06705	0.627054	0.627054
0.59	-0.08	0.59258	-0.08258	0.580232	-0.07023	0.580232	0.580232
0.61	0.08	0.621806	0.068194	0.624139	0.065861	0.624139	0.624139
0.64	0.11	0.660264	0.089736	0.674484	0.075516	0.674484	0.674484
0.68	0.16	0.714185	0.125815	0.74069	0.09931	0.74069	0.74069
0.79	0.42	0.86293	0.34707	0.928414	0.281586	0.928414	0.928414
0.84	0.21	0.919051	0.130949	0.977048	0.072952	0.977048	0.977048
0.84	0.00	0.895335	-0.05534	0.922229	-0.08223	0.922229	0.922229
0.86	0.06	0.902735	0.017265	0.921337	-0.00134	0.921337	0.921337
0.92	0.24	0.979914	0.180086	1.016802	0.143198	1.016802	1.016802
0.90	-0.06	0.93794	-0.09794	0.946081	-0.10608	0.946081	0.946081
0.90	0.02	0.932558	-0.01256	0.935649	-0.01565	0.935649	0.935649
0.90	-0.01	0.919791	-0.02979	0.917389	-0.02739	0.917389	0.917389
0.83	-0.27	0.812739	-0.24979	0.775614	-0.21266	0.775614	0.775614
MSSE	0.023422		0.017482		0.012984		

We choose the value of alpha which gives the least MSSE, There fore, alpha = 0.4 has the least MSSE.

Plot



t	Yt	3 month MA	centered	weighted(0.2,0.3,0.5)
0				
1	112			
2	118	120.666667		123.8
3	132	126.333333	123.5	127.7
4	129	127.333333	126.8333	125.6
5	121	128.333333	127.8333	129.6
6	135	134.666667	131.5	138.7
7	148	143.666667	139.1667	145.4
8	148	144	143.8333	142
9	136	134.333333	139.1667	129.9

10	119	119.666667	127	114.9
11	104	113.666667	116.6667	114
12	118	112.333333	113	113.7
13	115	119.666667	116	121.1
14	126	127.333333	123.5	131.3
15	141	134	130.6667	135
16	135	133.666667	133.8333	131.2
17	125	136.333333	135	139
18	149	148	142.1667	154.7
19	170	163	155.5	165.8
20	170	166	164.5	164
21	158	153.666667	159.8333	147.9
22	133	135	144.3333	128.5
23	114	129	132	130.8
24	140	133	131	137.3
25	145	145	139	146.5
26	150	157.666667	151.3333	163
27	178	163.666667	160.6667	164.9
28	163	171	167.3333	170.5
29	172	171	171	173.2
30	178	183	177	187.3
31	199	192	187.5	194.8
32	199	194	193	191.5
33	184	181.666667	187.8333	176
34	162	164	172.8333	158.4
35	146	158	161	159.2
36	166	161	159.5	164.5
37	171	172.333333	166.6667	174.5
38	180	181.333333	176.8333	184.7

39	193	184.666667	183	184.4
40	181	185.666667	185.1667	184.4

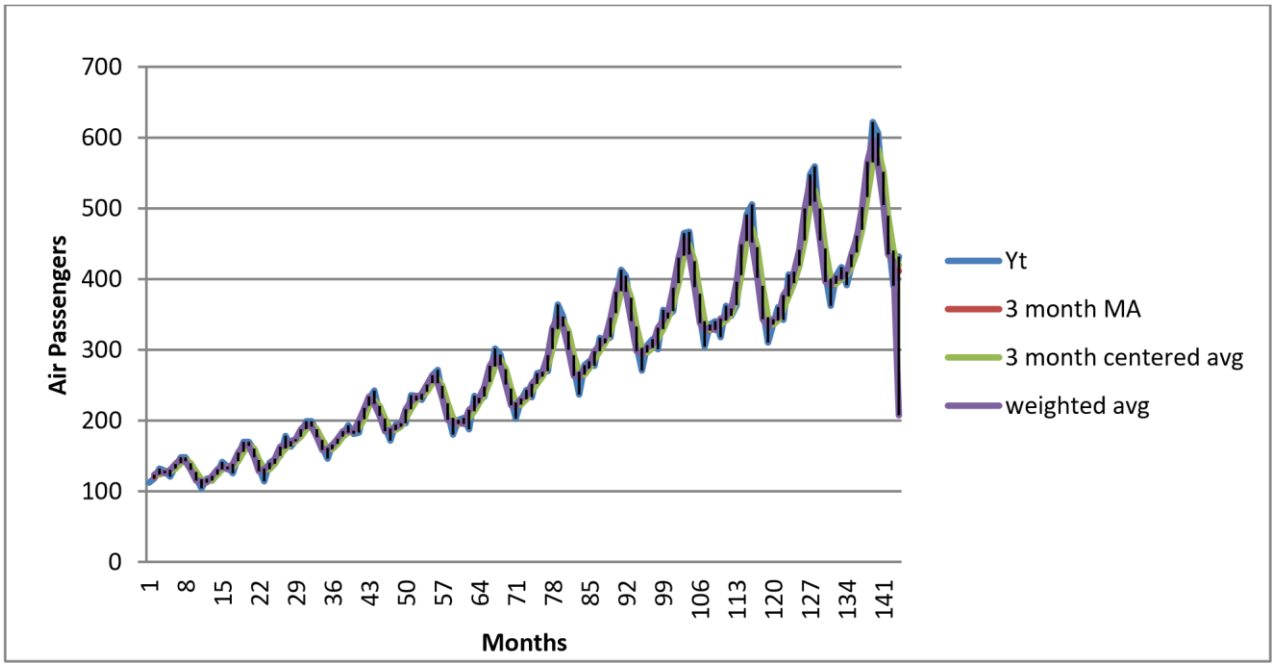
41	183	194	189.8333	200.1
42	218	210.333333	202.1667	217
43	230	230	220.1667	233.6
44	242	227	228.5	223.1
45	209	214	220.5	206.6
46	191	190.666667	202.3333	185.1
47	172	185.666667	188.1667	186.8
48	194	187.333333	186.5	190.6
49	196	195.333333	191.3333	195.6
50	196	209.333333	202.3333	216
51	236	222.333333	215.8333	227.5
52	235	233.333333	227.8333	232.2
53	229	235.666667	234.5	237.2
54	243	245.333333	240.5	250.7
55	264	259.666667	252.5	263.8
56	272	257.666667	258.6667	252.9
57	237	240	248.8333	231
58	211	209.333333	224.6667	200.7
59	180	197.333333	203.3333	196.7
60	201	195	196.1667	198.3
61	204	197.666667	196.3333	195.4
62	188	209	203.3333	214.7
63	235	216.666667	212.8333	221.6
64	227	232	224.3333	232.1
65	234	241.666667	236.8333	247.6
66	264	266.666667	254.1667	277
67	302	286.333333	276.5	289.9
68	293	284.666667	285.5	277.8
69	259	260.333333	272.5	250.8
70	229	230.333333	245.3333	222
71	203	220.333333	225.3333	221.2
72	229	225	222.6667	230.8

73	243	235	230	235.2
74	233	247.666667	241.3333	252
75	267	256.333333	252	261.2
76	269	268.666667	262.5	269.1
77	270	284.666667	276.6667	292.3
78	315	316.333333	300.5	330.5
79	364	342	329.1667	345.7

80	347	341	341.5	332.9
81	312	311	326	300
82	274	274.333333	292.6667	263.1
83	237	263	268.6667	264.9
84	278	266.333333	264.6667	272.8
85	284	279.666667	273	279.3
86	277	292.666667	286.1667	298.4
87	317	302.333333	297.5	307
88	313	316	309.1667	316.3
89	318	335	325.5	345
90	374	368.333333	351.6667	382.3
91	413	397.333333	382.8333	401.2
92	405	391	394.1667	381.6
93	355	355.333333	373.1667	340.5
94	306	310.666667	333	298.3
95	271	294.333333	302.5	295.5
96	306	297.333333	295.8333	303.5
97	315	307.333333	302.3333	306.2
98	301	324	315.6667	331.3
99	356	335	329.5	341
100	348	353	344	353.1
101	355	375	364	387.1
102	422	414	394.5	430.1
103	465	451.333333	432.6667	457.4

104	467	445.333333	448.3333	435.1
105	404	406	425.6667	388.1
106	347	352	379	337.4
107	305	329.333333	340.6667	328.9
108	336	327	328.1667	331.8
109	340	331.333333	329.1667	328.2
110	318	340	335.6667	344.4
111	362	342.666667	341.3333	346.2
112	348	357.666667	350.1667	358.3
113	363	382	369.8333	396
114	435	429.666667	405.8333	448.6
115	491	477	453.3333	486.8
116	505	466.666667	471.8333	451.7
117	404	422.666667	444.6667	401.7
118	359	357.666667	390.1667	343.5
119	310	335.333333	346.5	333.3
120	337	335.666667	335.5	343.1
121	360	346.333333	341	346.4
122	342	369.333333	357.8333	377.6
123	406	381.333333	375.3333	388.2
124	396	407.333333	394.3333	410
125	420	429.333333	418.3333	441.2
126	472	480	454.6667	499.6
127	548	526.333333	503.1667	538.3
128	559	523.333333	524.8333	508.8
129	463	476.333333	499.8333	454.2
130	407	410.666667	443.5	395.7
131	362	391.333333	401	392.5
132	405	394.666667	393	402.4
133	417	404.333333	399.5	401.6
134	391	409	406.6667	410.2
135	419	423.666667	416.3333	434.4

136	461	450.666667	437.1667	458.1
137	472	489.333333	470	501.3
138	535	543	516.1667	565.9
139	622	587.666667	565.3333	596.6
140	606	578.666667	583.1667	560.2
141	508	525	551.8333	504.1
142	461	453	489	434.9
143	390	427.666667	440.3333	425.2
144	432	411	419.3333	207.6



- First Order Exponential Smoothing and Second Order Exponential Smoothing

t	Yt	Y^t^(1)	sse		Y^t^(2)		Y^t
0		112			112		
1	112	112	0		112		112
2	118	114.4	3.6		112.96		115.84
3	132	121.44	10.56		116.352		126.528
4	129	124.464	4.536		119.5968		129.3312
5	121	123.0784	-2.0784		120.9894		125.1674

6	135	127.847	7.15296		123.7325		131.9616
7	148	135.9082	12.09178		128.6028		143.2137
8	148	140.7449	7.255066		133.4596		148.0302
9	136	138.847	-2.84696		135.6146		142.0794
10	119	130.9082	-11.9082		133.732		128.0843
11	104	120.1449	-16.1449		128.2972		111.9926
12	118	119.2869	-1.28694		124.6931		113.8808
13	115	117.5722	-2.57217		121.8447		113.2996
14	126	120.9433	5.0567		121.4841		120.4025
15	141	128.966	12.03402		124.4769		133.4551
16	135	131.3796	3.620412		127.238		135.5212
17	125	128.8278	-3.82775		127.8739		129.7816
18	149	136.8967	12.10335		131.483		142.3103
19	170	150.138	19.86201		138.945		161.331
20	170	158.0828	11.91721		146.6001		169.5655
21	158	158.0497	-0.04968		151.1799		164.9194
22	133	148.0298	-15.0298		149.9199		146.1397
23	114	134.4179	-20.4179		143.7191		125.1167
24	140	136.6507	3.34927		140.8917		132.4097
25	145	139.9904	5.009562		140.5312		139.4497
26	150	143.9943	6.005737		141.9164		146.0721
27	178	157.5966	20.40344		148.1885		167.0046
28	163	159.7579	3.242065		152.8163		166.6996
29	172	164.6548	7.345239		157.5517		171.7579
30	178	169.9929	8.007144		162.5281		177.4576
31	199	181.5957	17.40429		170.1552		193.0363
32	199	188.5574	10.44257		177.5161		199.5988
33	184	186.7345	-2.73446		181.2034		192.2655
34	162	176.8407	-14.8407		179.4583		174.223
35	146	164.5044	-18.5044		173.4768		155.5321
36	166	165.1026	0.897357		170.1271		160.0782

37	171	167.4616	3.538414		169.0609		165.8623
38	180	172.477	7.523049		170.4273		174.5266
39	193	180.6862	12.31383		174.5309		186.8415

40	181	180.8117	0.188298		177.0432		184.5802
41	183	181.687	1.312979		178.9007		184.4733
42	218	196.2122	21.78779		185.8253		206.5991
43	230	209.7273	20.27267		195.3861		224.0685
44	242	222.6364	19.3636		206.2862		238.9866
45	209	217.1818	-8.18184		210.6445		223.7192
46	191	206.7091	-15.7091		209.0703		204.3479
47	172	192.8255	-20.8255		202.5724		183.0785
48	194	193.2953	0.704723		198.8615		187.729
49	196	194.3772	1.622834		197.0678		191.6865
50	196	195.0263	0.9737		196.2512		193.8014
51	236	211.4158	24.58422		202.317		220.5145
52	235	220.8495	14.15053		209.73		231.9689
53	229	224.1097	4.890319		215.4819		232.7375
54	243	231.6658	11.33419		221.9554		241.3762
55	264	244.5995	19.40051		231.0131		258.1859
56	272	255.5597	16.44031		240.8317		270.2877
57	237	248.1358	-11.1358		243.7534		252.5183
58	211	233.2815	-22.2815		239.5646		226.9984
59	180	211.9689	-31.9689		228.5263		195.4115
60	201	207.5813	-6.58134		220.1483		195.0143
61	204	206.1488	-2.1488		214.5485		197.7491
62	188	198.8893	-10.8893		208.2848		189.4937
63	235	213.3336	21.66643		210.3043		216.3628
64	227	218.8001	8.199859		213.7026		223.8976
65	234	224.8801	9.119915		218.1736		231.5865
66	264	240.5281	23.47195		227.1154		253.9407
67	302	265.1168	36.88317		242.316		287.9177
68	293	276.2701	16.7299		255.8976		296.6426
69	259	269.3621	-10.3621		261.2834		277.4407
70	229	253.2172	-24.2172		258.0569		248.3775
71	203	233.1303	-30.1303		248.0863		218.1744

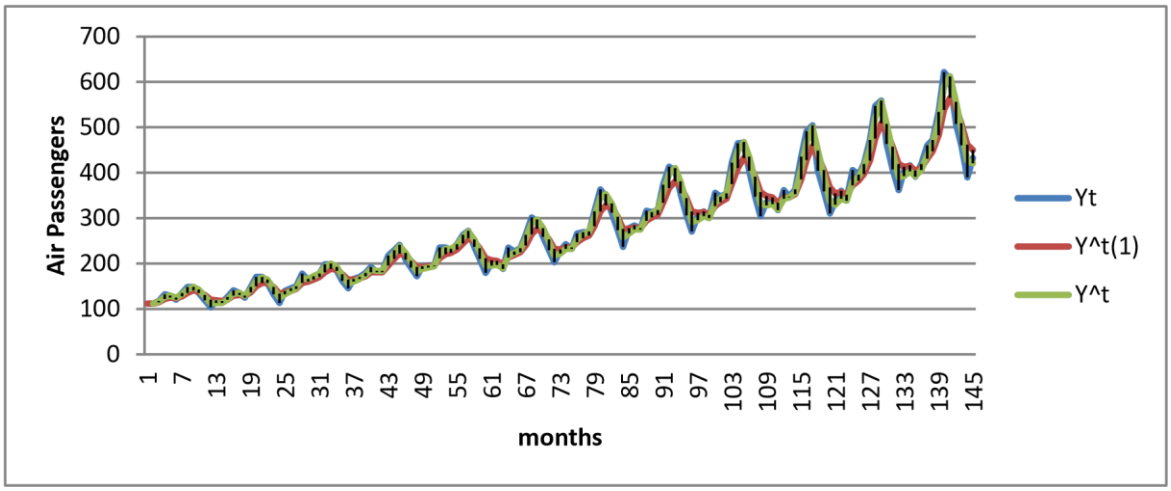
72	229	231.4782	-2.4782		241.4431		221.5134
73	243	236.0869	6.913077		239.3006		232.8732
74	233	234.8522	-1.85215		237.5212		232.1831
75	267	247.7113	19.28871		241.5973		253.8253
76	269	256.2268	12.77322		247.4491		265.0045
77	270	261.7361	8.263935		253.1639		270.3083

78	315	283.0416	31.95836		265.115		300.9683
79	364	315.425	48.57502		285.239		345.611
80	347	328.055	18.94501		302.3654		353.7446
81	312	321.633	-9.63299		310.0724		333.1936
82	274	302.5798	-28.5798		307.0754		298.0842
83	237	276.3479	-39.3479		294.7844		257.9114
84	278	277.0087	0.991273		287.6741		266.3433
85	284	279.8052	4.194764		284.5266		275.0839
86	277	278.6831	-1.68314		282.1892		275.1771
87	317	294.0099	22.99012		286.9175		301.1023
88	313	301.6059	11.39407		292.7929		310.419
89	318	308.1636	9.836441		298.9411		317.386
90	374	334.4981	39.50186		313.1639		355.8323
91	413	365.8989	47.10112		334.2579		397.5398
92	405	381.5393	23.46067		353.1705		409.9082
93	355	370.9236	-15.9236		360.2717		381.5755
94	306	344.9542	-38.9542		354.1447		335.7636
95	271	315.3725	-44.3725		338.6358		292.1092
96	306	311.6235	-5.6235		327.8309		295.4161
97	315	312.9741	2.025902		321.8882		304.06
98	301	308.1845	-7.18446		316.4067		299.9622
99	356	327.3107	28.68932		320.7683		333.8531
100	348	335.5864	12.41359		326.6955		344.4773
101	355	343.3518	11.64816		333.3581		353.3456
102	422	374.8111	47.18889		349.9393		399.6829

103	465	410.8867	54.11334		374.3182		447.4551
104	467	433.332	33.668		397.9237		468.7403
105	404	421.5992	-17.5992		407.3939		435.8045
106	347	391.7595	-44.7595		401.1402		382.3789
107	305	357.0557	-52.0557		383.5064		330.605
108	336	348.6334	-12.6334		369.5572		327.7097
109	340	345.1801	-5.18006		359.8063		330.5538
110	318	334.308	-16.308		349.607		319.009
111	362	345.3848	16.61518		347.9181		342.8515
112	348	346.4309	1.569108		347.3232		345.5385
113	363	353.0585	9.941465		349.6174		356.4997
114	435	385.8351	49.16488		364.1045		407.5658
115	491	427.9011	63.09893		389.6231		466.179
116	505	458.7406	46.25936		417.2701		500.2112
117	404	436.8444	-32.8444		425.0998		448.5889
118	359	405.7066	-46.7066		417.3425		394.0707
119	310	367.424	-57.424		397.3751		337.4728
120	337	355.2544	-18.2544		380.5268		329.9819
121	360	357.1526	2.847368		371.1771		343.1281
122	342	351.0916	-9.09158		363.1429		339.0402
123	406	373.0549	32.94505		367.1077		379.0022
124	396	382.233	13.76703		373.1578		391.3081
125	420	397.3398	22.66022		382.8306		411.849
126	472	427.2039	44.79613		400.5799		453.8278
127	548	475.5223	72.47768		430.5569		520.4878
128	559	508.9134	50.08661		461.8995		555.9273
129	463	490.548	-27.548		473.3589		507.7372
130	407	457.1288	-50.1288		466.8669		447.3908
131	362	419.0773	-57.0773		447.751		390.4035
132	405	413.4464	-8.44638		434.0292		392.8636
133	417	414.8678	2.132175		426.3646		403.371
134	391	405.3207	-14.3207		417.9471		392.6943

135	419	410.7924	8.207583		415.0852		406.4996
136	461	430.8755	30.12455		421.4013		440.3496
137	472	447.3253	24.67473		431.7709		462.8797
138	535	482.3952	52.60484		452.0206		512.7697
139	622	538.2371	83.7629		486.5072		589.967
140	606	565.3423	40.65774		518.0412		612.6433
141	508	542.4054	-34.4054		527.7869		557.0238
142	461	509.8432	-48.8432		520.6094		499.077
143	390	461.9059	-71.9059		497.128		426.6838
144	432	449.9436	-17.9436		478.2542		421.6329
		msse	3.520245				

- **Plot**



Holt's linear exponential smoothing :

The three equations used in Holt's method are:

The current level estimate $\mathbf{Lt = \alpha Yt + (1-\alpha)(Lt-1 + Tt-1)}$ ----------(1)

The Trend estimate $\mathbf{Tt = \beta (Lt - Lt-1) + (1-\beta) Tt-1}$ ----------(2)

The forecast for p periods into the future $\mathbf{Yt+p = Lt + pTt\hat{}}$----------(3)

Where,

Lt = the new smoothed value (estimate of current level) α = the smoothing constant for the level ($0 < \alpha < 1$) β = the smoothing constant for the trend estimate ($0 < \beta < 1$) Tt = the trend estimate
Yt = actual value of the series in period t
$\hat{Y}_{t+p}$= the forecast for the p periods into the future
With $\alpha = 0.4$ and $\beta = 0.1$

	y = 2.6572x + 87.66		0.4	0.1
t	Yt	Lt	Tt	Y^t
0		87.66	2.657	
1	112	95.8018	3.20548	90.32
2	118	102.7578	3.580531	99.01
3	132	112.3064	4.177335	106.34
4	129	116.4774	4.176707	116.48
5	121	115.7804	3.689337	120.65
6	135	121.2547	3.867826	119.47
7	148	129.6321	4.318788	125.12
8	148	134.388	4.362498	133.95
9	136	132.4153	3.728979	138.75
10	119	124.8118	2.595731	136.14
11	104	114.9296	1.347942	127.41
12	118	115.349	1.255086	116.28
13	115	114.4564	1.040312	116.60
14	126	118.4496	1.335608	115.50
15	141	126.6684	2.023925	119.79
16	135	128.7867	2.033361	128.69
17	125	126.052	1.556555	130.82
18	149	134.2973	2.225427	127.61
19	170	147.2431	3.297468	136.52
20	170	154.3674	3.680149	150.54
21	158	153.6123	3.23663	158.05

22	133	143.4254	1.894275	156.85
23	114	130.5187	0.414174	145.32
24	140	134.0627	0.727159	130.93
25	145	138.0013	1.048305	134.79
26	150	142.1718	1.360523	139.05
27	178	155.6868	2.575967	143.53
28	163	157.0665	2.456341	158.26
29	172	161.5661	2.660667	159.52
30	178	166.5433	2.892317	164.23
31	199	177.7906	3.727816	169.44
32	199	184.0376	3.979743	181.52
33	184	181.6347	3.341478	188.02
34	162	171.776	2.021452	184.98
35	146	160.2527	0.666981	173.80
36	166	162.1514	0.790156	160.92
37	171	165.2168	1.017674	162.94
38	180	170.5195	1.446175	166.23
39	193	178.644	2.114009	171.97
40	181	178.318	1.870009	180.76
41	183	179.0688	1.758088	180.19
42	218	193.5864	3.034043	180.83
43	230	206.3314	4.00514	196.62
44	242	218.1958	4.79106	210.34
45	209	211.6428	3.65666	222.99
46	191	201.1917	2.245881	215.30
47	172	188.1675	0.718872	203.44
48	194	190.0692	0.837153	188.89
49	196	191.9392	0.940442	190.91
50	196	192.9993	0.952403	192.88
51	236	209.6281	2.520048	193.95

52	235	218.2648	3.131716	212.15

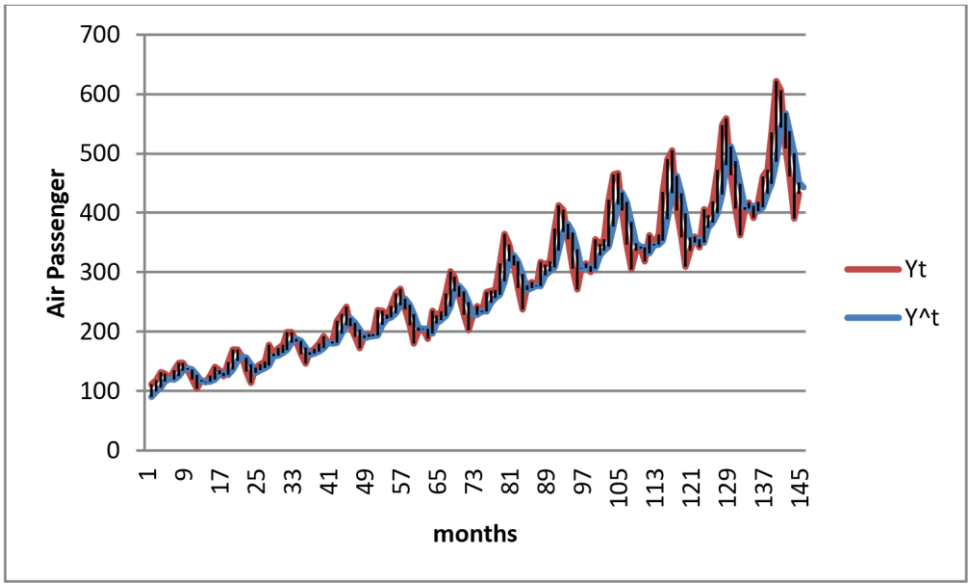
53	229	220.6799	3.060047	221.40
54	243	227.7719	3.463245	223.74
55	264	240.1852	4.35825	231.24
56	272	250.2962	4.933522	244.54
57	237	242.0176	3.612312	255.23
58	211	227.4432	1.793639	245.63
59	180	207.3897	-0.39107	229.24
60	201	205.0685	-0.58409	207.00
61	204	204.9915	-0.53337	204.48
62	188	198.5149	-1.12769	204.46
63	235	213.7856	0.512139	197.39
64	227	218.7641	0.958774	214.30
65	234	224.2832	1.414807	219.72
66	264	239.321	2.777111	225.70
67	302	262.7263	4.839932	242.10
68	293	271.9318	5.276489	267.57
69	259	263.5932	3.914977	277.21
70	229	247.4069	1.904852	267.51
71	203	228.5013	-0.1762	249.31
72	229	228.8065	-0.12806	228.33
73	243	234.5607	0.460171	228.68
74	233	233.6603	0.324115	235.02
75	267	246.8017	1.605843	233.98
76	269	254.7175	2.236839	248.41
77	270	259.4884	2.490244	256.95
78	315	280.1989	4.312268	261.98
79	364	311.132	6.974349	284.51
80	347	321.2946	7.293174	318.11
81	312	313.2008	5.754483	328.59
82	274	294.0678	3.265732	318.96
83	237	269.2813	0.460502	297.33
84	278	272.4924	0.735572	269.74

85	284	276.6541	1.078182	273.23
86	277	276.1456	0.919508	277.73
87	317	291.9356	2.406564	277.07
88	313	298.9174	2.864088	294.34
89	318	304.832	3.169137	301.78
90	374	330.5977	5.428794	308.00
91	413	360.3014	7.856278	336.03
92	405	373.467	8.387219	368.16

93	355	361.0479	6.306582	381.85
94	306	335.2448	3.095613	367.35
95	271	307.6895	0.030524	338.34
96	306	306.9954	-0.04194	307.72
97	315	310.2224	0.284954	306.95
98	301	306.3625	-0.12953	310.51
99	356	326.2952	1.876693	306.23
100	348	333.8511	2.444614	328.17
101	355	340.8439	2.899431	336.30
102	422	371.5667	5.681767	343.74
103	465	405.5309	8.510017	377.25
104	467	425.0126	9.607176	414.04
105	404	410.8432	7.229526	434.62
106	347	380.9682	3.519072	418.07
107	305	348.4695	-0.08271	384.49
108	336	343.5313	-0.56825	348.39
109	340	342.4597	-0.61859	342.96
110	318	333.047	-1.498	341.84
111	362	345.527	-0.1002	331.55
112	348	346.5763	0.01475	345.43
113	363	353.1369	0.669338	346.59
114	435	385.4806	3.836766	353.81
115	491	425.3863	7.443661	389.32

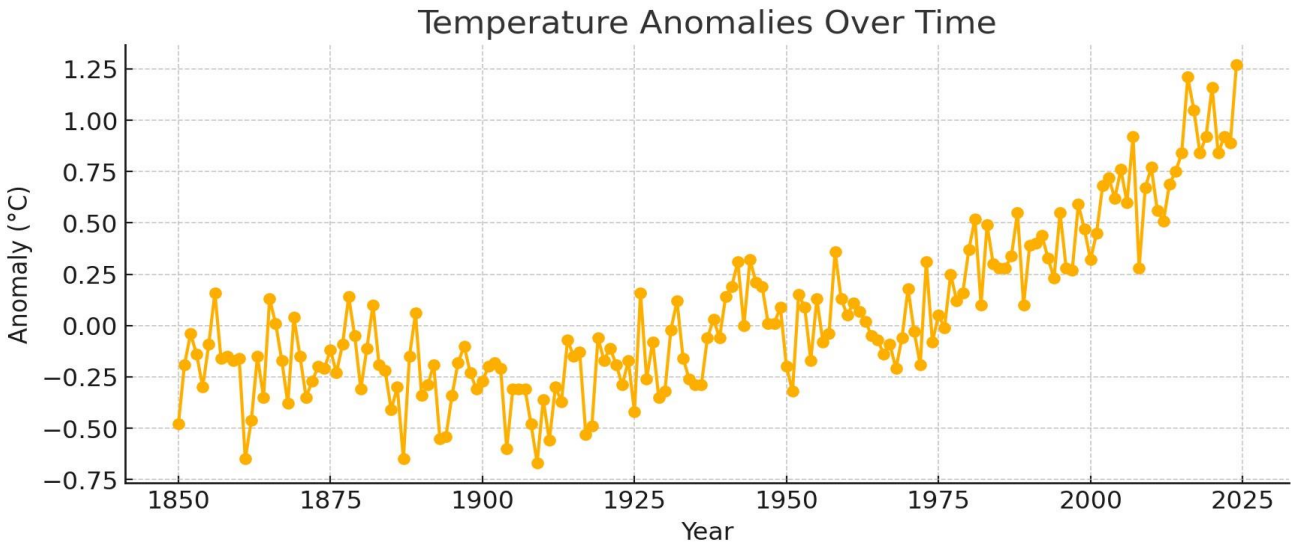
116	505	452.7656	9.437224	432.83
117	404	427.597	5.976645	462.20
118	359	396.5722	2.276502	433.57
119	310	360.5774	-1.55063	398.85
120	337	352.0768	-2.24562	359.03
121	360	356.5935	-1.5694	349.83
122	342	351.6977	-1.90203	355.02
123	406	374.5599	0.574383	349.80
124	396	382.7913	1.340088	375.13
125	420	396.8707	2.614023	384.13
126	472	425.354	5.20095	399.48
127	548	471.2918	9.274638	430.55
128	559	500.8103	11.29902	480.57
129	463	478.9068	7.978766	512.11
130	407	445.3568	3.825892	486.89
131	362	409.7185	-0.12052	449.18
132	405	407.9034	-0.28998	409.60
133	417	411.7161	0.120278	407.61
134	391	403.3575	-0.72761	411.84
135	419	410.051	0.01451	402.63
136	461	430.4219	2.050147	410.07
137	472	445.8231	3.385246	432.47
138	535	479.4627	6.410684	449.21
139	622	532.6312	11.08647	485.87
140	606	555.3268	12.24738	543.72
141	508	529.0477	8.394729	567.57
142	461	496.7918	4.329665	537.44
143	390	451.4773	-0.63475	501.12
144	432	444.0672	-1.31228	450.84
				442.75

• Plot



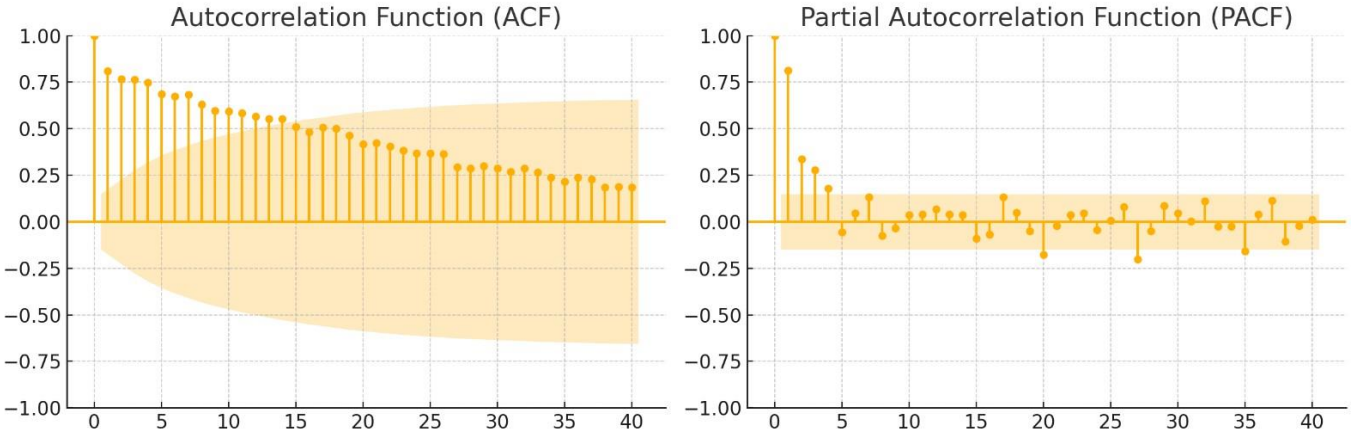
The estimated values obtained from Holt Linear Exponential Smoothing traces the original data very well.

The dataset consists of 175 years of temperature anomaly data from 1850 onward. Now, I'll check for stationarity and plot the ACF and PACF graphs.



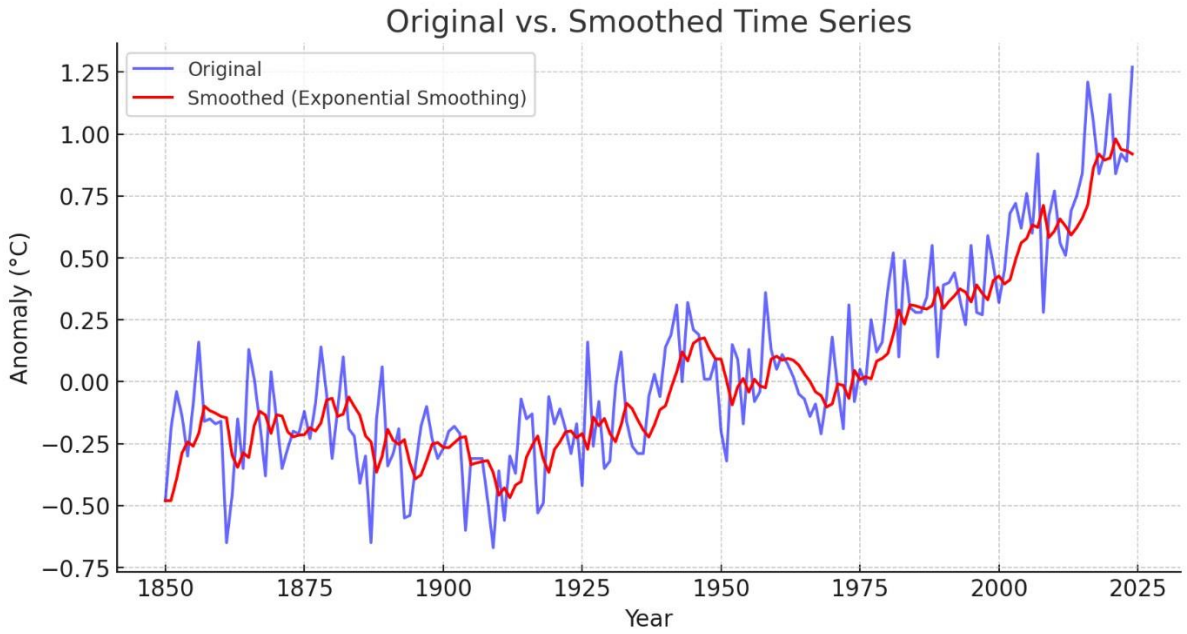
The p-value is **0.99**, which is much higher than the typical threshold of 0.05. This indicates that the time series is **not stationary**.

I'll plot the **ACF** and **PACF** to visualize the autocorrelation structure before applying any transformations.



The ACF plot shows a slow decay, and the PACF has significant spikes at multiple lags, confirming that the time series is non-stationary.

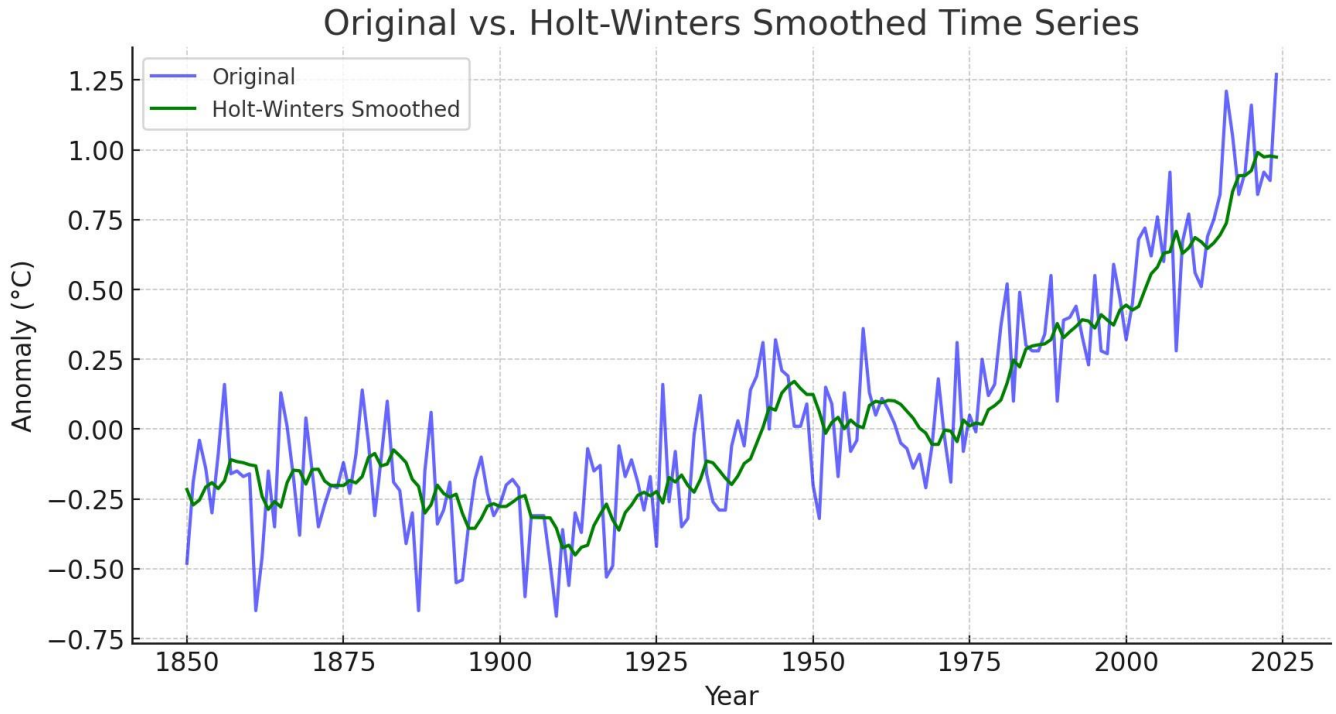
Using first order expo-smoothening:



- **p-value: 0.96**

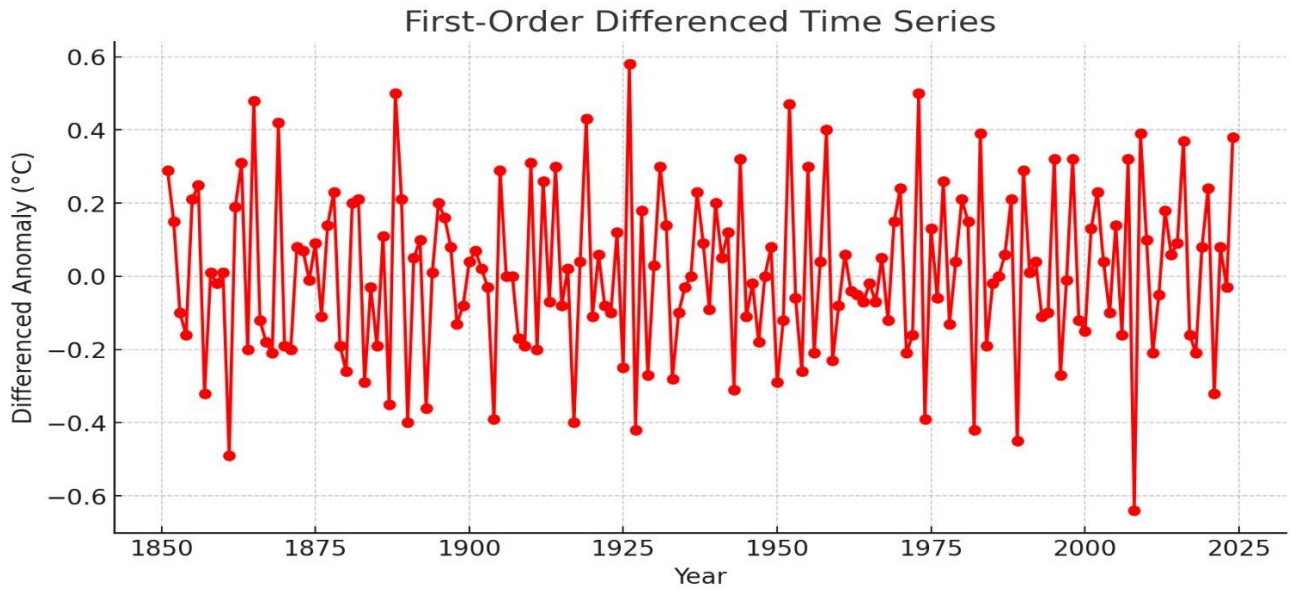
Since the p-value is still much greater than 0.05, the series remains non-stationary even after smoothing.

Applying Holt -Winter Additive model



- **p-value: 0.998**

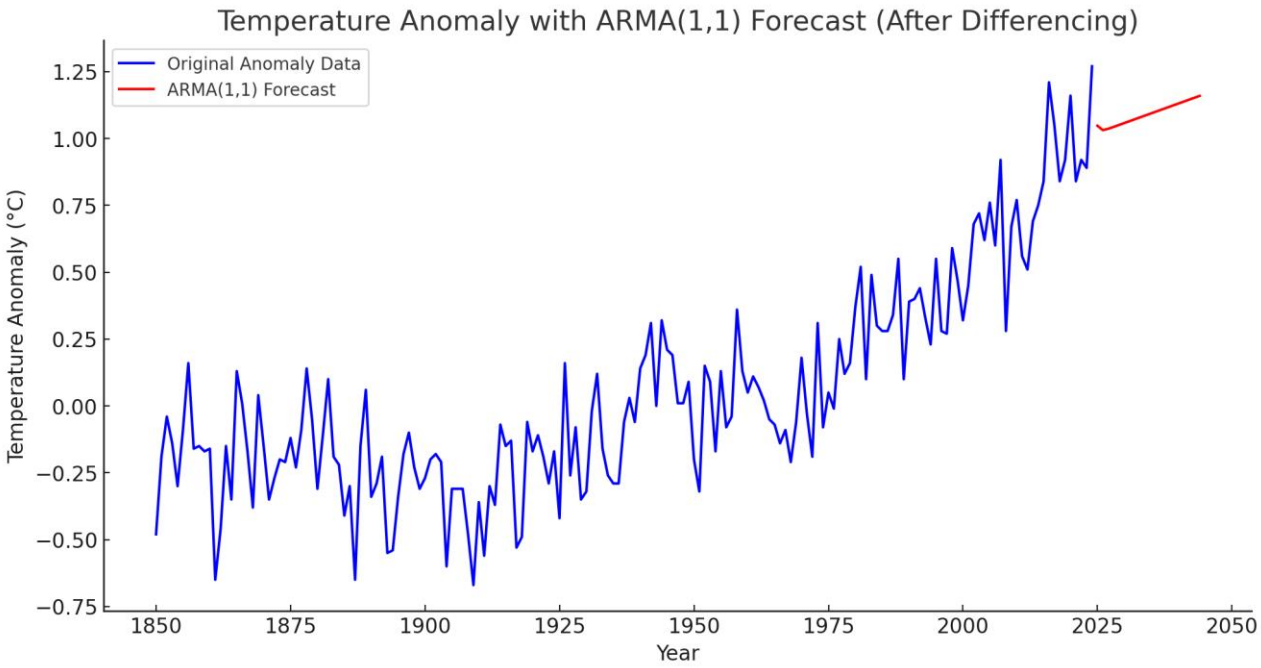
Since the p-value is still much higher than 0.05, the series remains non-stationary even after smoothing. To achieve stationarity, we will need to apply differencing.



Since the p-value is **much lower than 0.05**, the series is now **stationary**. We can now fit an appropriate ARIMA model.

- **p (AutoRegressive order)** = 1
- **d (Differencing order)** = 0 (since we already differenced the data manually before fitting)
- **q (Moving Average order)** = 1

Since the differencing was already performed on the data before model fitting, this essentially behaves like an **ARMA(1,1)** model.



Forecast Using ARMA(1,1) (Red Line)

- This is the predicted anomaly values for the next 20 years.
- Since the ARMA model is applied after first-order differencing, the forecast maintains a relatively smooth trend with moderate variation.
- The red line suggests that future temperature anomalies will likely continue following a similar pattern to the recent historical data.

Final Conclusion:

1. **Data Preparation**:
    - The temperature anomaly data was cleaned and differenced to remove trends and make the series stationary.
    - The Augmented Dickey-Fuller (ADF) test confirmed that the differenced series is stationary (p-value < 0.05).
2. **Model Fitting**:
    - An **ARIMA(1,0,1)** model was fitted, which effectively behaves as an **ARMA(1,1)** model because the data was already differenced.
    - The model successfully captured the series' dynamics, with the Moving Average (MA) term being the most significant.