Indian Institute of Technology (ISM), Dhanbad
Department of Computer Science and Engineering

Theory of Computation (CSC208)

Mid Semester, Date: 25 February 2024

Timing: 2 PM - 4 PM | Winter Semester 2023-24 | Max mark: 60

Attempt all questions. Write full justifications for your answers to be evaluated.

1. Consider the following language over the alphabet $\Sigma = \{0, 1\}$

$$L = \{w \in \Sigma^* | w \text{ starts with 10 but does not end with 10}\}$$

   (a) Write a regular expression for L **(5)**

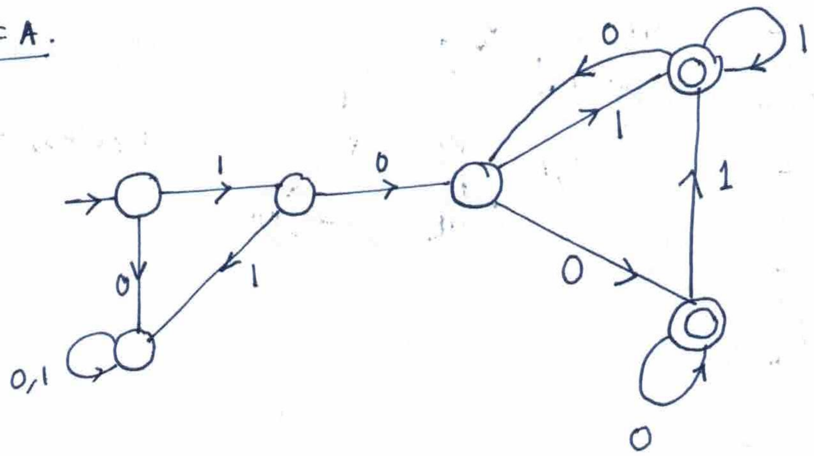
   (b) Design a DFA for L **(7)**

(a) Regular expression

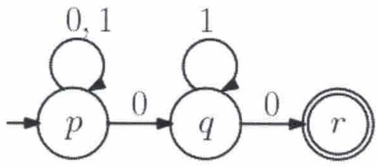
$$101 + 100 + 10(1+0)^*(11+01+00)$$

$$= 10(1 + 0 + (1+0)^*(11+01+00))$$

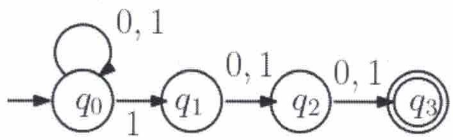
(b) DFA.



2. (a) Convert the following NFA to an equivalent DFA using the **subset-construction procedure** (*No marks shall be awarded for using any other method*). Clearly describe all the steps (partial reasoning is not acceptable). Draw the state-transition diagram of the DFA with *all the possible states.* Write all unreachable states (that is, states inaccessible from the start state) in your DFA. Consider $\{0, 1\}$ as your input alphabet. **(8)**

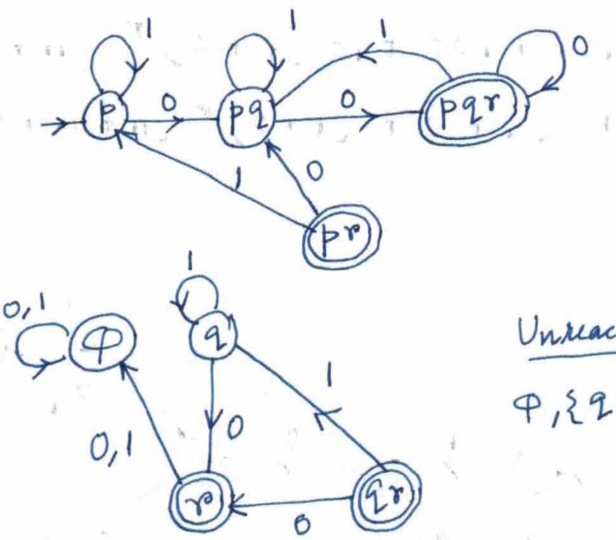


(b) Show all the possible state sequences if we run the NFA M on the string **0110110**. From the state sequences decide whether the string is in $L(M)$. **(4)**



(a)

	0	1
ϕ	ϕ	ϕ
$\{p\}$	$\{pq\}$	$\{p\}$
$\{q\}$	$\{r\}$	$\{q\}$
$\{r\}$	ϕ	ϕ
$\{pq\}$	$\{pqr\}$	$\{pq\}$
$\{qr\}$ ~~$\{pr\}$~~	$\{r\}$	$\{q\}$
$\{pr\}$	$\{pq\}$	$\{p\}$
$\{pqr\}$	$\{pqr\}$	$\{pq\}$



Unreachable states

ϕ, $\{q\}$, $\{r\}$, $\{p, r\}$, $\{q, r\}$

(b) Refer to slides. Solution given in lecture notes.

3. Consider the following grammars. Describe the languages denoted by them. If a language is not regular then prove it using *pumping lemma* otherwise write the regular expression for the language (*Marks will be deducted for fallacious and incomplete reasoning*).

(a) $S \to 0|1|2S2$ **(6)**

(b) $S \to 0|1|S2S$ **(6)**

(a) $L_1 = \{2^k 0 2^k \mid k \geq 0\} \cup \{2^k 1 2^k \mid k \geq 0\}$

(1) Let m be the pumping length proposed by the adversary.

(2) We can put forth the string $2^m 0 2^m$ or $2^m 1 2^m$ as string.

(3) By the conditions of pumping lemma, the decomposition xyz should be such that $|xy| \leq m$ and $|y| \geq 1$

The opponent is bound to choose the decomposition such that xy is completely in 2^m.

Let $x = 2^i$ $y = 2^j$ $j \geq 1$. $i+j \leq m$, $z = 02^m$

(4) If we set $l=0$, in $xy^l z$ where, then $xy^0z = xz$

The number of 2s in the first half ~~resulting string~~ portion of the resulting string will have less twos than the number of 2s in z. So pumping lemma fails to hold.

Hence, L_1 is not regular.

(b) It can be converted to a right-linear grammar. So L_2 is regular.

$S \longrightarrow 0A \mid 1A$

$A \longrightarrow 20A \mid 21A \mid \epsilon$

It will result in strings.

$(0+1)\,2\,(0+1)\,2 \ldots 2(0+1)$

Regular expression

$(0+1)\,(2(0+1))^*$

4. (a) Convert the CFG $G = (\{S, X, Y\}, \{a, b, c\}, S, P)$ to Chomsky normal form. Do not introduce any new start symbol. **(8)**

$$S \to aXbX$$
$$X \to aY|bY|\epsilon$$
$$Y \to X|c$$

(b) Draw the NFA corresponding to the *right linear* grammar G. **(4)**
$V_0 \to aV_1$
$V_1 \to abV_0|b$

(a)

$S \to aXbX$
$X \to aY \mid bY \mid \epsilon$
$Y \to X \mid c$

$\{X, Y\}$ are nullable

Remove ϵ-transition

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to aY \mid bY$
$Y \to X \mid c \mid \epsilon$

$\Rightarrow$

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to aY \mid bY \mid a \mid b$
$Y \to X \mid c$

Remove unit production

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to aY \mid bY \mid a \mid b$
$Y \to aY \mid bY \mid a \mid b \mid c$

Convert to CNF

step 1

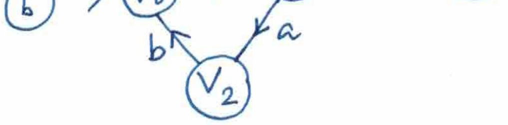
$S \to ABX \mid AXB \mid AB \mid AXBX$
$X \to AY \mid BY \mid a \mid b$
$Y \to AY \mid BY \mid a \mid b \mid c$
$A \to a$
$B \to b$

$\Downarrow$

$S \to AB_1 \mid AB_2 \mid AB \mid AB_3$
$B_1 \to BX$
$B_2 \to XB$
$B_3 \to XB_1$
$X \to AY \mid BY \mid a \mid b$
$Y \to AY \mid BY \mid a \mid b \mid c$
$A \to a$
$B \to b$

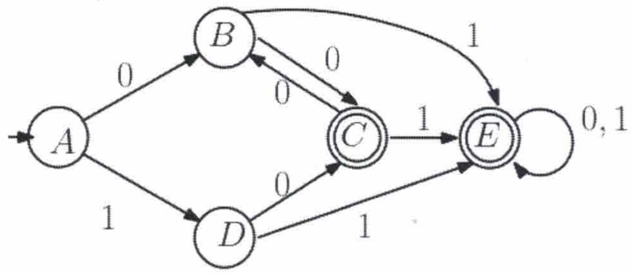
There can be other possibilities. ~~If~~ Full marks will be awarded if answer is correct.

(b)

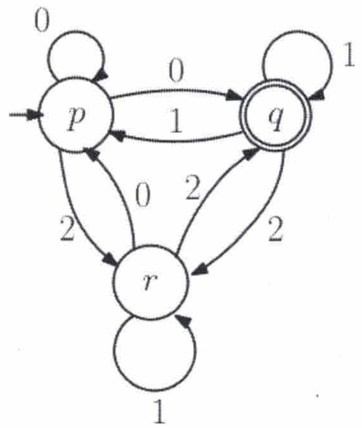


5. Answer the following questions

(a) Use the *slot-filling* algorithm to minimize the DFA. Show the table construction (*Use of any other algorithm will induce zero marks*). For each *distinguishable state-pair* clearly state why you find them *distinguishable*. Draw the minimized DFA. **(6)**



(b) Find the regular expression for the DFA using the *state elimination* algorithm. *No marks shall be awarded for using any other method.* **(6)**



(a)

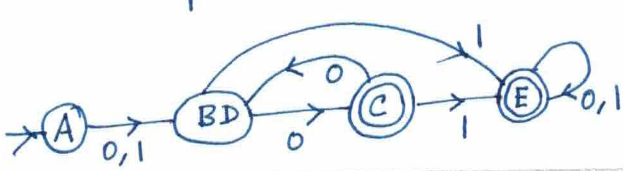
D	X			
C	X	X		
B	X		X	
A	X	X	X	X
	E	D	C	B

① A, B and D (Non-final) states are distinguishable from C & E (Final) States.

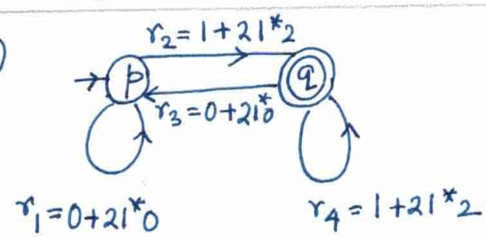
② $\delta(C,0) = B$ and $\delta(E,0) = E$ [B & E are dist. So C & E are dist.]

③ $\delta(A,0) \notin F$ and $\delta(B,0) \in F$

④ $\delta(A,1) \notin F$ and $\delta(D,1) \in F$



(b)



$(r_1 + r_2 r_4^* r_3)^* r_2 r_4^*$

& $r_1^* r_2 (r_4 + r_3 r_1^* r_2)^*$

Either one form is correct.