Retarded Potentials due to an Oscillating Magnetic Dipole

Semester 10

PG.P.10.5 Classical Electrodynamics - II

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Introduction :
Retarded Potentials

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{1}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{3}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{4}$$

The four equations by Maxwell give rise to the formulation of a scalar electric potential V and a vector magnetic potential $\vec{A}$. These potentials are related to the respective fields by

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \tag{5}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{6}$$

Using equations 5 and 6, the four Maxwell's equations can be written into two coupled inhomogenous differential equations:

$$\nabla^2 V + \frac{\partial}{\partial t}\left(\vec{\nabla} \cdot \vec{A}\right) = -\frac{\rho}{\epsilon_0} \tag{7}$$

$$\left(\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}\right) - \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \vec{J} \tag{8}$$

The above two equations contain all the information in Maxwell's equations.

Gauge Transformations

For any scalar function $\lambda(\vec{r}, t)$, we can add $\vec{\nabla}\lambda$ to $\vec{A}$, provided we simultaneously subtract $\left(\frac{\partial \lambda}{\partial t}\right)$ from V. This will not affect the physical quantities $\vec{E}$ and $\vec{B}$. Such changes in V and $\vec{A}$ are called gauge transformations.
i.e.,

$$
\begin{aligned}
\vec{A}' &= \vec{A} + \vec{\nabla}\lambda && (9) \\
V' &= V - \frac{\partial \vec{A}}{\partial t} && (10)
\end{aligned}
$$

Lorenz Gauge

In Lorenz gauge, we pick:

$$\vec{\nabla} \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \tag{11}$$

From equation (8),

$$\left(\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} \right) - \vec{\nabla} \left(\vec{\nabla} \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} \right) = -\mu_0 \vec{J}$$

Applying the gauge condition the two coupled inhomogenous equations reduce to a much simpler *symmetric* form,

$$\begin{aligned} \nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} &= -\mu_0 \vec{J} \qquad (12) \\ \nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} &= -\frac{\rho}{\epsilon_0} \qquad (13) \end{aligned}$$

$$\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

$$\nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

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In the static case, these equations reduce to the Poisson's equations

$$\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

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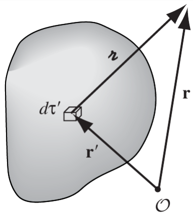

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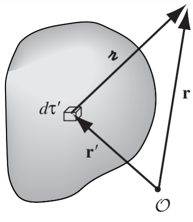


The solutions of the Poisson's equations are:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r'})}{\mathscr{r}}\, d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r'})}{\mathscr{r}}\, d\tau'$$

The same general solution holds true for non-static sources as well;

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r'},t_r)}{\scriptr}\,d\tau' \tag{14}$$

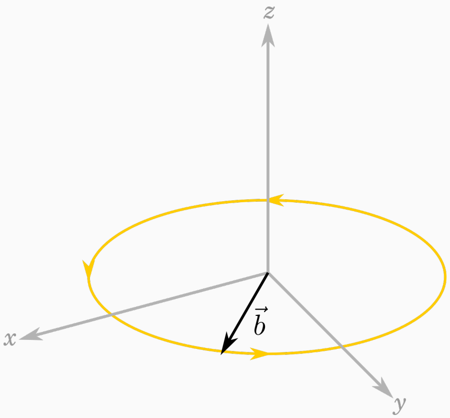
$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r'},t_r)}{\scriptr}\,d\tau' \tag{15}$$

- In the non-static case, it's not the status of the source *right now* that matters, but its condition at some *earlier time* t_r (called the retarded time) when the "message" left. Since this message must travel a distance $\scriptr$ (with a *finite* speed c), the delay is $\left(\frac{\scriptr}{c}\right)$:

$$t_r \equiv t - \frac{\scriptr}{c}$$

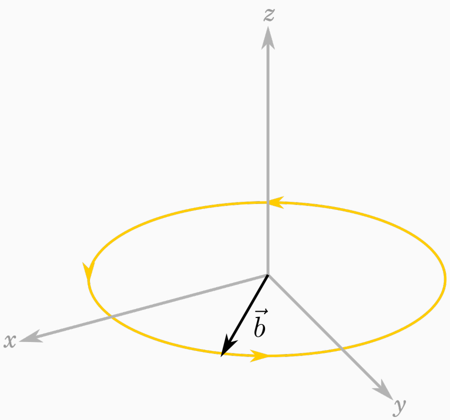
- Here $\rho(\vec{r'},t_r)$ is the charge density that prevailed at point $\vec{r'}$ at the retarded time t_r. Because the integrands are evaluated at the retarded time, these are called retarded potentials.

Retarded Potentials of a Magnetic Dipole



Consider a wire loop of radius b, around which an alternating current is flowing:

$$I(t) = I_0 \cos(\omega t)$$

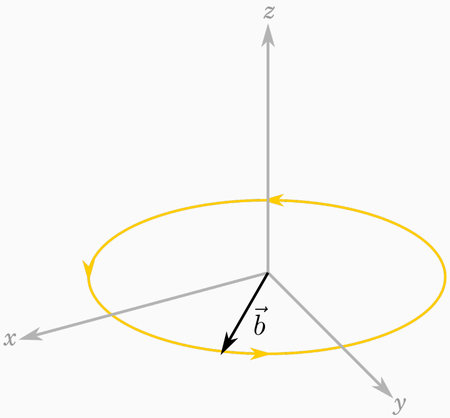


Consider a wire loop of radius b, around which an alternating current is flowing:

$$I(t) = I_0 \cos(\omega t)$$

The magnetic dipole moment is given by

$$\begin{aligned} \vec{m}(t) &= I(t)\,\vec{a} \\ &= I(t)\pi b^2\,\hat{z} \end{aligned} \tag{16}$$



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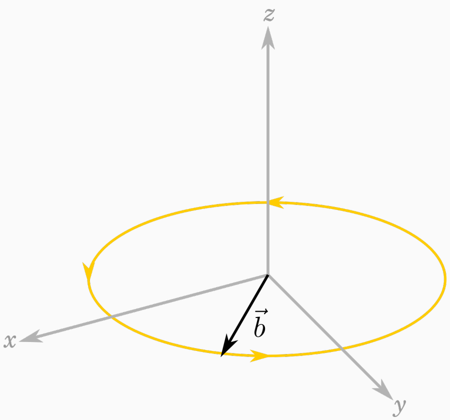

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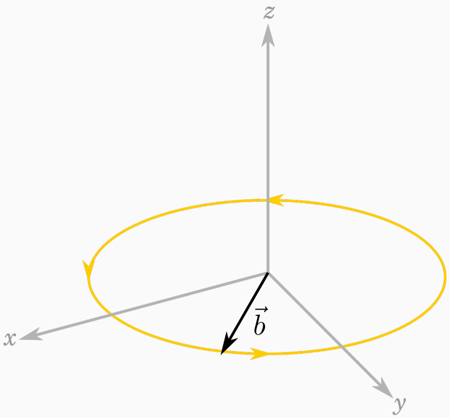

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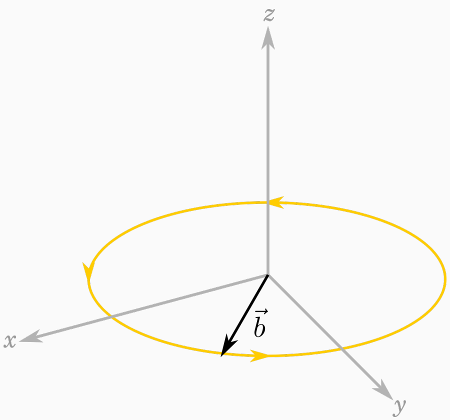
where $m_0 \equiv \pi b^2 I_0$ is the maximum value of the magnetic dipole moment.



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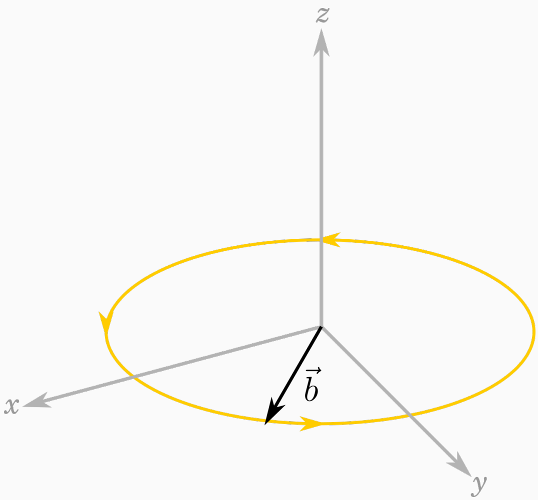
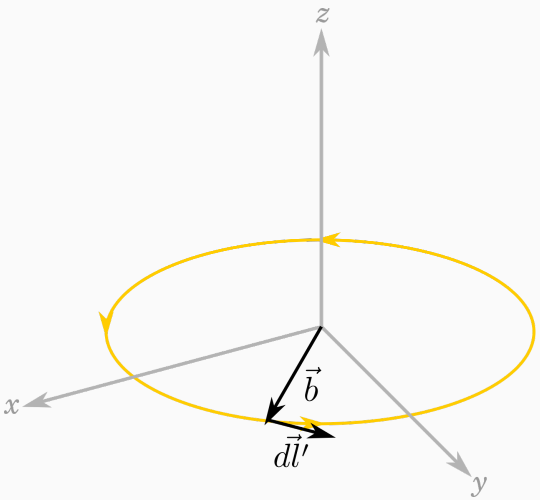
The loop is electrically neutral, so the electric potential

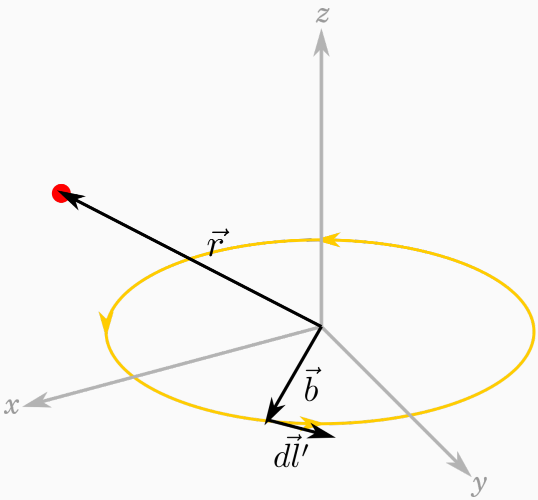
$$\boxed{V = 0} \tag{17}$$



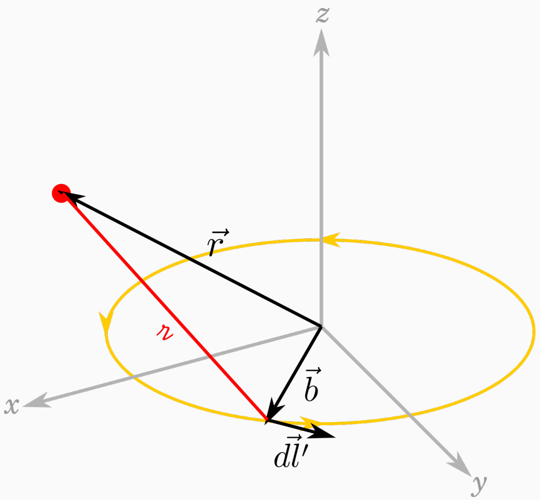
z
x
y
$\vec{b}$



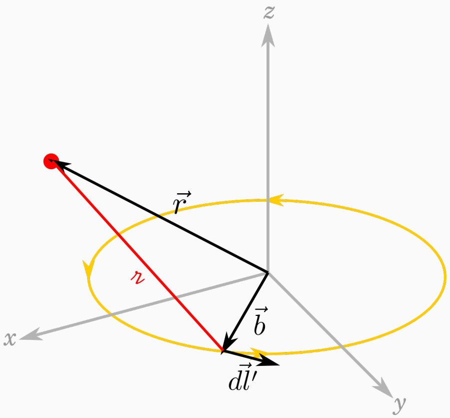
z
x
y
$\vec{b}$
$d\vec{l}'$



z
$\vec{r}$
x
$\vec{b}$
$d\vec{l}'$
y



z
$\vec{r}$
$\mathscr{r}$
$\vec{b}$
x
$d\vec{l}'$
y



z
$\vec{r}$
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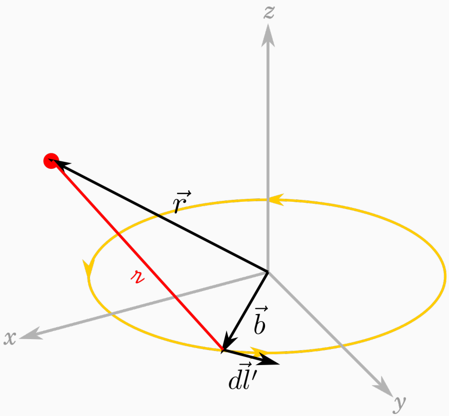
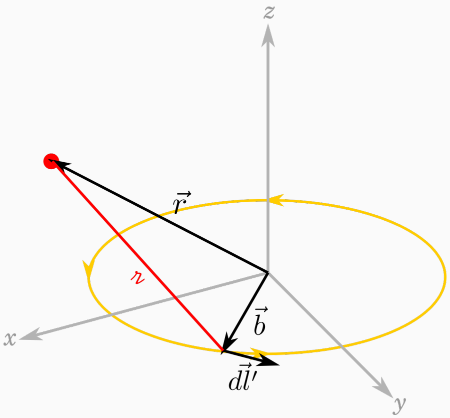


The general equation for retarded magnetic potential (equation (15)) is,



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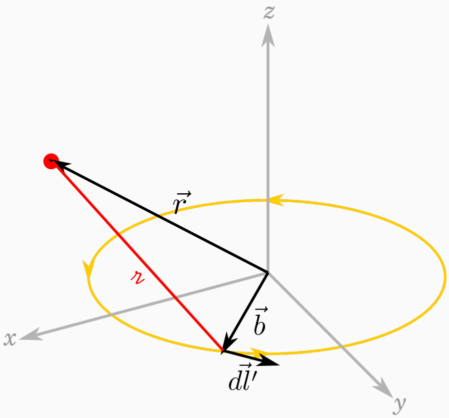

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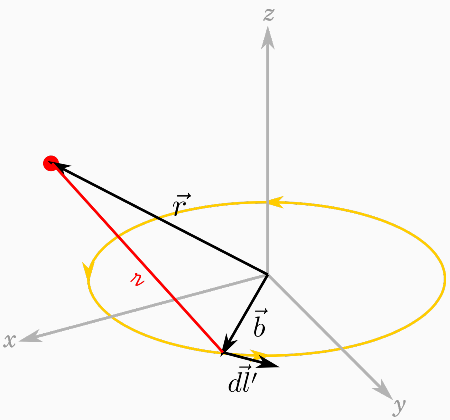

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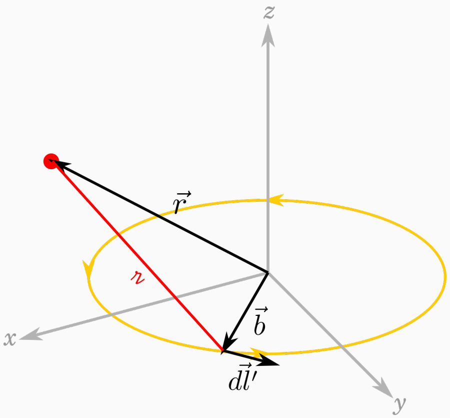
$$\begin{aligned}\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{I(t_r)}{\mathscr{r}}\,d\vec{l}' \\ &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos(\omega t_r)}{\mathscr{r}}\,d\vec{l}'\end{aligned}$$



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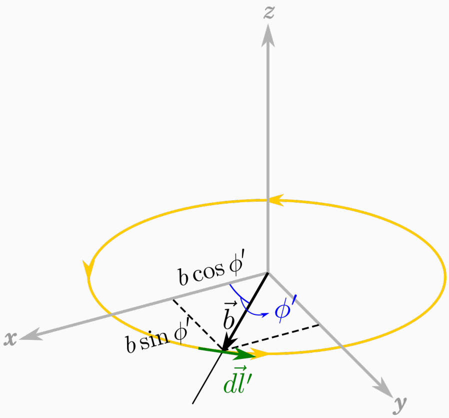
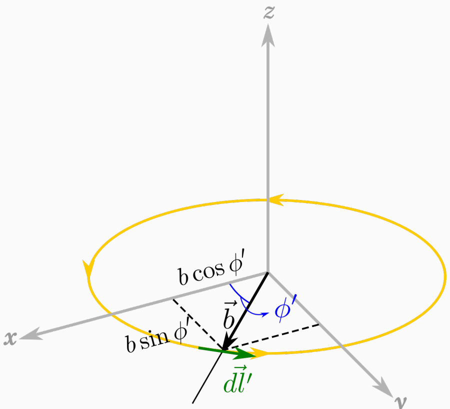
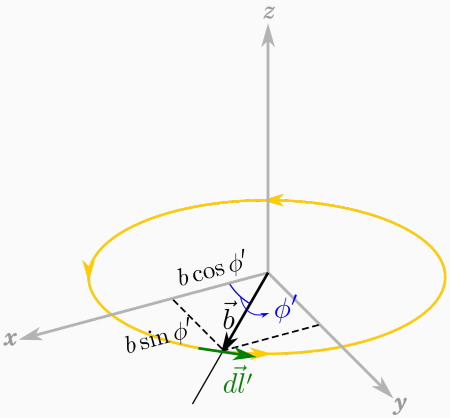
\vec{A}(\vec{r}, t) &= \frac{\mu_0}{4\pi} \int \frac{I(t_r)}{\mathscr{r}} \, d\vec{l}' \\
&= \frac{\mu_0}{4\pi} \int \frac{I_0 \cos(\omega t_r)}{\mathscr{r}} \, d\vec{l}' \\
&= \frac{\mu_0}{4\pi} \int \frac{I_0 \cos\left[\omega \left(t - \mathscr{r}/c\right)\right]}{\mathscr{r}} \, d\vec{l}'
\end{aligned}$$


z
$b\cos\phi'$
$\vec{b}$
ϕ'
x
$b\sin\phi'$
$\vec{dl'}$
y

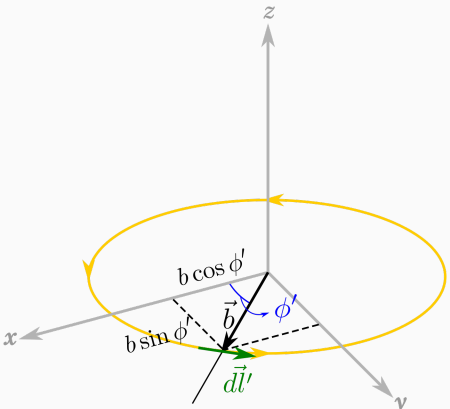


$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$



$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

For the source co-ordinates,



$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

For the source co-ordinates,

$$
\begin{aligned}
x' &= b\cos\phi' && \Longrightarrow && dx' = -\,b\sin\phi'\,d\phi' \\
y' &= b\sin\phi' && \Longrightarrow && dy' = b\cos\phi'\,d\phi' \\
z' &= 0 && \Longrightarrow && dz' = 0
\end{aligned}
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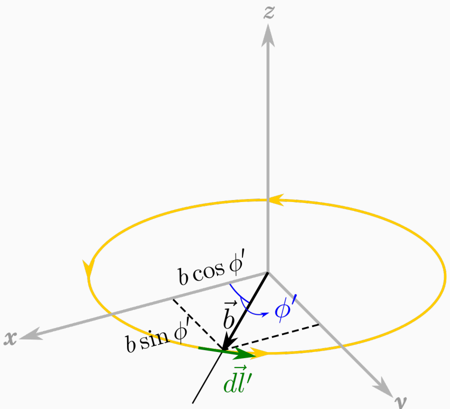

$$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

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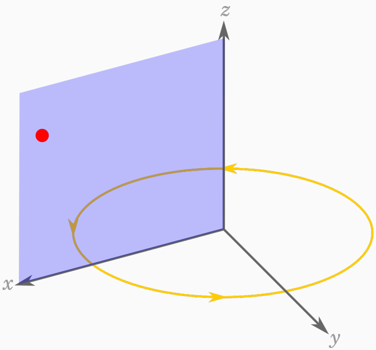
$$\begin{aligned}
d\vec{l}' &= dx'\,\hat{x} + dy'\,\hat{y} + dz'\,\hat{z} \\
&= -b\sin\phi'\,d\phi'\,\hat{x} + b\cos\phi'\,d\phi'\,\hat{y}
\end{aligned}$$

Consider a point on the xz plane.



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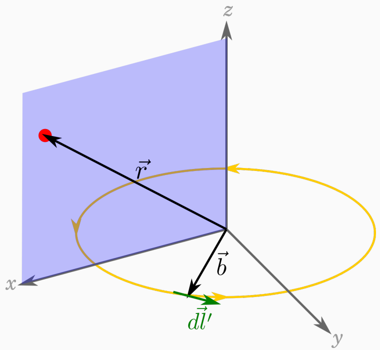

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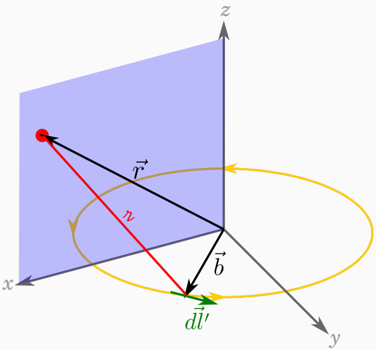

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos\left[\omega\left(t - \mathscr{r}/c\right)\right]}{\mathscr{r}}\, d\vec{l}'$$

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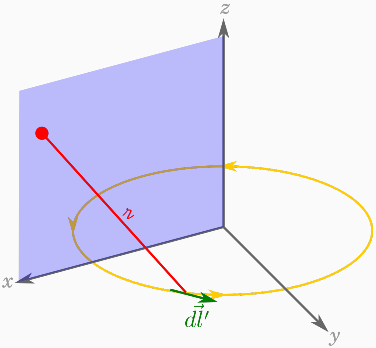

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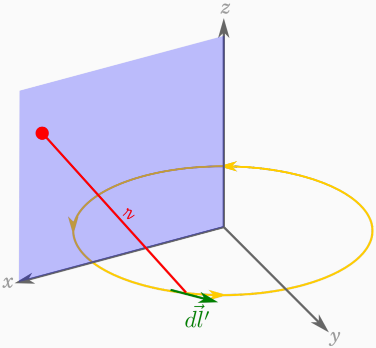

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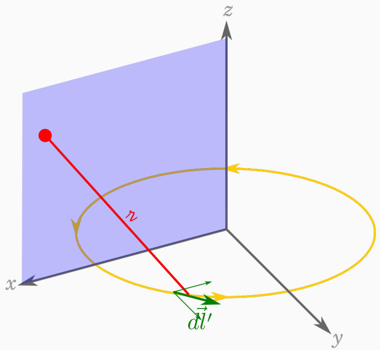

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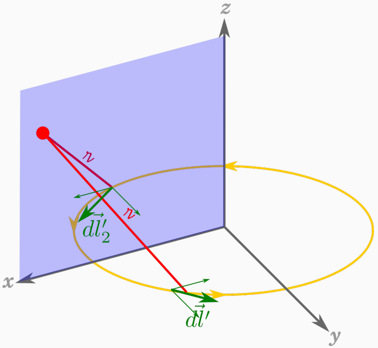

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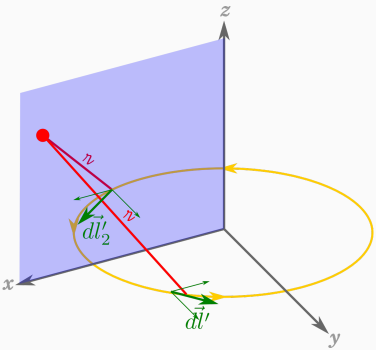

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Hence, $\vec{A}$ has $\hat{y}$-component only.

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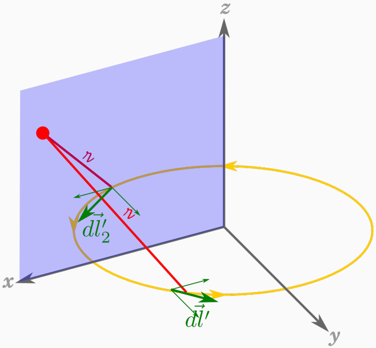

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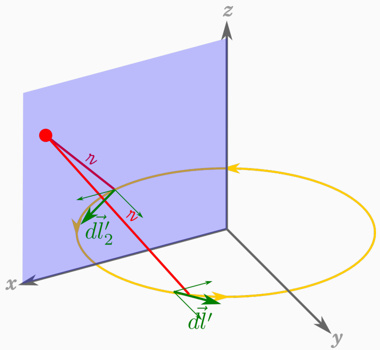

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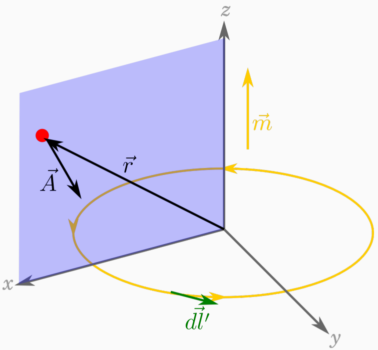
$$\vec{dl}' = -b\sin\phi'\, d\phi'\, \hat{x} + b\cos\phi'\, d\phi'\, \hat{y}$$

$$\therefore\ \vec{dl}' = b\cos\phi'\, d\phi'\, \hat{y}$$

Consider a point on the xz plane.



$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{I_0 \cos\left[\omega\left(t - \mathscr{r}/c\right)\right]}{\mathscr{r}}\, d\vec{l}'$$

Hence, $\vec{A}$ has $\hat{y}$-component only.

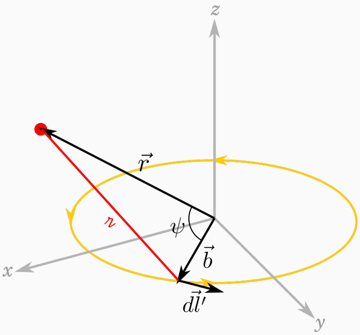
$$d\vec{l}' = -b\sin\phi'\, d\phi'\, \hat{x} + b\cos\phi'\, d\phi'\, \hat{y}$$

$$\therefore\ d\vec{l}' = b\cos\phi'\, d\phi'\, \hat{y}$$

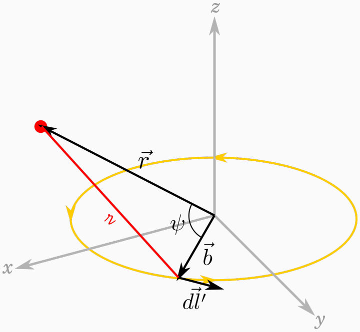
$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{I_0 \cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \, (b \cos\phi' \, d\phi') \, \hat{y} \tag{18}$$

$$
\begin{aligned}
\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\ (b\cos\phi'\,d\phi')\,\hat{y} \\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\frac{\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\cos\phi'\,d\phi'
\end{aligned}
\tag{18}
$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{I_0 \cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \, (b\cos\phi'\, d\phi')\,\hat{y} \\ = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi'\, d\phi' \tag{18}$$

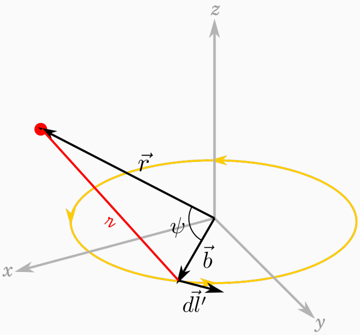


$$
\begin{aligned}
\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\,(b\cos\phi'\,d\phi')\,\hat{y} \\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\frac{\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\cos\phi'\,d\phi'
\end{aligned}
\tag{18}
$$

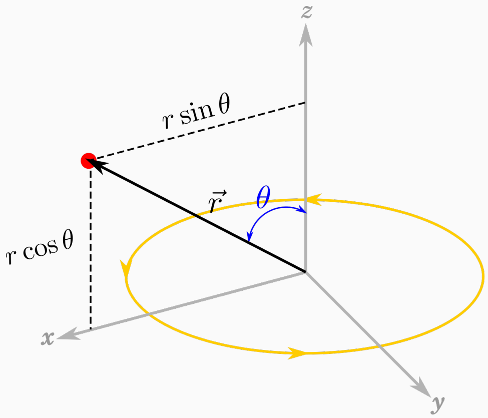


$$
\mathscr{r} = \sqrt{r^2 + b^2 - 2rb\cos\psi}
$$

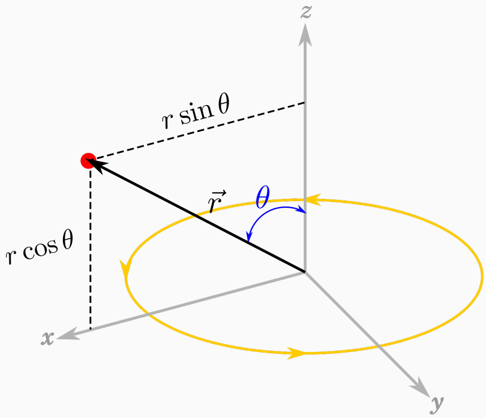
$$
\begin{aligned}
\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\ (b\cos\phi'\,d\phi')\,\hat{y} \\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\frac{\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}}\cos\phi'\,d\phi'
\end{aligned}
\tag{18}
$$



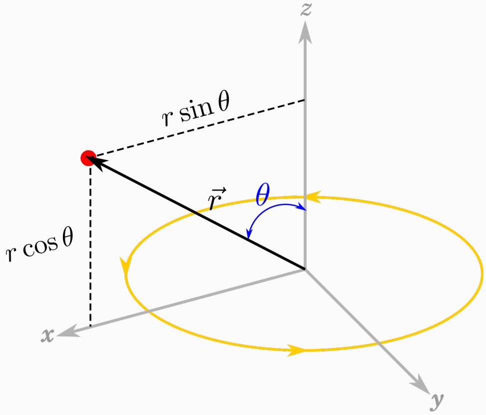
$$
\begin{aligned}
\mathscr{r} &= \sqrt{r^2+b^2-2rb\cos\psi} \\
&= r\sqrt{1+\left(\frac{b^2}{r^2}\right)-2\left(\frac{b}{r}\right)\cos\psi}
\end{aligned}
$$

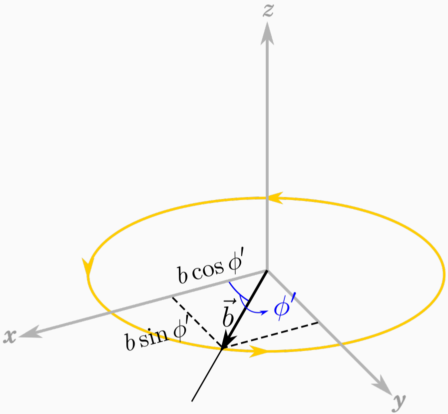


z
$r \sin\theta$
$\vec{r}$
θ
$r \cos\theta$
x
y

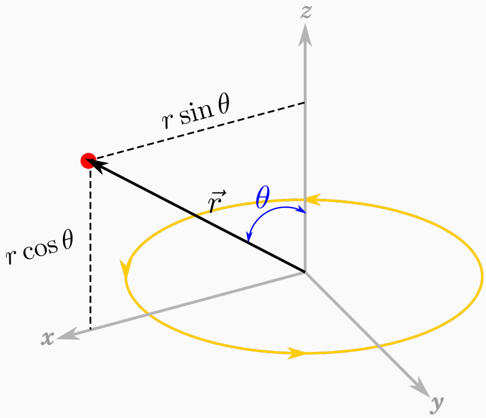


$$\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$$

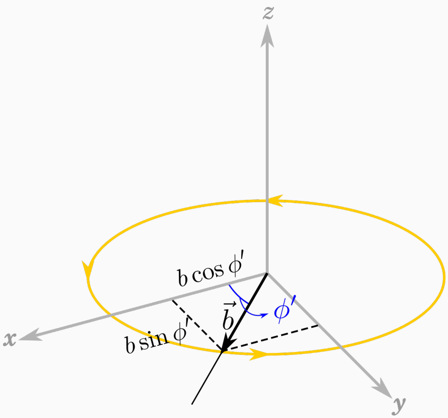




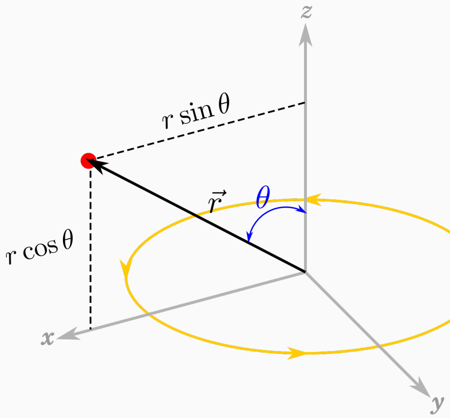
$$\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$$

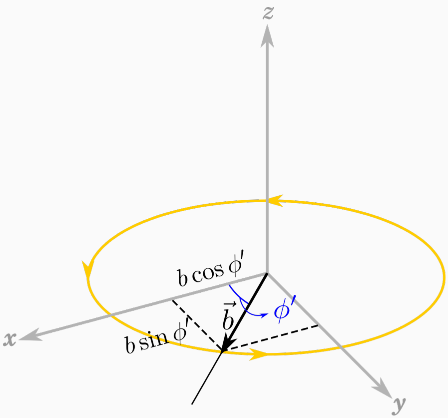


$$\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$$



$$\vec{b} = b\cos\phi'\hat{x} + b\sin\phi'\hat{y}$$





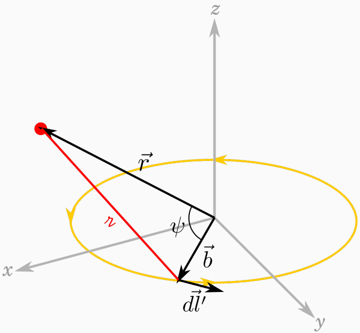
$$\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$$

$$\vec{b} = b\cos\phi'\hat{x} + b\sin\phi'\hat{y}$$

$$\vec{r}\cdot\vec{b} = rb\cos\psi = rb\sin\theta\cos\phi'$$

$$\cos\psi = \sin\theta\cos\phi'$$

$$
\begin{aligned}
\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}}\;(b\cos\phi'\,d\phi')\,\hat{y} \\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}}\cos\phi'\,d\phi'
\end{aligned}
$$



$$
\begin{aligned}
\mathscr{r} &= \sqrt{r^2 + b^2 - 2rb\cos\psi} \\
&= r\sqrt{1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\cos\psi}
\end{aligned}
$$

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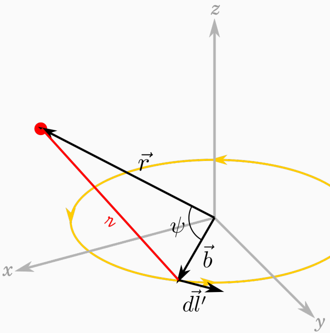

$$\mathscr{r} = \sqrt{r^2+b^2-2rb\cos\psi}$$
$$= r\sqrt{1+\left(\frac{b^2}{r^2}\right)-2\left(\frac{b}{r}\right)\cos\psi}$$
$$= r\sqrt{1+\left(\frac{b^2}{r^2}\right)-2\left(\frac{b}{r}\right)\sin\theta\cos\phi'}$$

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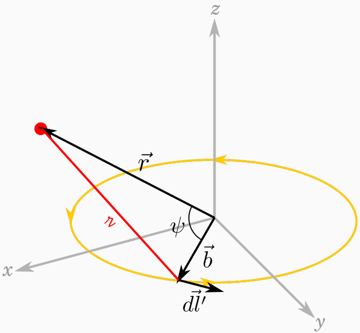

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&= r\sqrt{1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\cos\psi} \\
&= r\sqrt{1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'} \\
&= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}
\end{aligned}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$\boxed{\text{Approximation 1}: \; b \ll r}$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$$\boxed{\text{Approximation 1} : b \ll r}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

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$$\boxed{\text{Approximation 1}: b \ll r}$$

$$\begin{aligned}\mathscr{r} &= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\ &\approxeq r\left[1 - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}\end{aligned}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

Approximation 1 : $b \ll r$

$$\begin{aligned}
\mathscr{r} &= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\
&\approxeq r\left[1 - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\
&\approxeq r\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]
\end{aligned}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{-1}$$

Approximation 1 : $b \ll r$

$$\begin{aligned}\mathscr{r} &= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\ &\approxeq r\left[1 - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\ &\approxeq r\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\end{aligned}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$$\boxed{\text{Approximation 1 : } b \ll r}$$

$$\begin{aligned}\mathscr{r} &= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\ &\approxeq r\left[1 - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\ &\approxeq r\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\end{aligned}$$

$$\begin{aligned}\frac{1}{\mathscr{r}} &= \frac{1}{r}\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{-1} \\ &\approxeq \frac{1}{r}\left[1 + \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\end{aligned}$$

$$\mathscr{r} = r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2}$$

$$\boxed{\text{Approximation } 1 : b \ll r}$$

$$\begin{aligned}
\mathscr{r} &= r\left[1 + \left(\frac{b^2}{r^2}\right) - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\
&\approxeq r\left[1 - 2\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{1/2} \\
&\approxeq r\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\mathscr{r}} &= \frac{1}{r}\left[1 - \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]^{-1} \\
&\approxeq \frac{1}{r}\left[1 + \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]
\end{aligned}$$

$$\boxed{\therefore \frac{1}{\mathscr{r}} \approxeq \frac{1}{r}\left[1 + \left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]} \tag{19}$$

$$\cos\left[\omega(t - \mathscr{r}/c)\right] =$$

$$\cos\left[\omega(t - \mathscr{r}/c)\right] = \cos\left\{\omega\left[t - \frac{r}{c}\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\}$$

$$\cos\left[\omega(t-\mathscr{r}/c)\right] = \cos\left\{\omega\left[t-\frac{r}{c}\left(1-\frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\}$$

$$= \cos\left[\omega(t-r/c)+\frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

$$\cos\left[\omega(t - r/c)\right] = \cos\left\{\omega\left[t - \frac{r}{c}\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\}$$
$$= \cos\left[\omega(t - r/c) + \frac{\omega b}{c}\sin\theta\cos\phi'\right]$$
$$= \cos[\omega(t - r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right) - \sin[\omega(t - r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)$$

$$
\begin{aligned}
\cos\left[\omega(t-\mathscr{r}/c)\right] =\ & \cos\left\{\omega\left[t-\frac{r}{c}\left(1-\frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\} \\
&= \cos\left[\omega(t-r/c)+\frac{\omega b}{c}\sin\theta\cos\phi'\right] \\
&= \cos[\omega(t-r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)-\sin[\omega(t-r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)
\end{aligned}
$$

$$\boxed{\text{Approximation 2 : } b \ll \tfrac{c}{\omega}}$$

$$\cos\left[\omega(t - \mathscr{r}/c)\right] = \cos\left\{\omega\left[t - \frac{r}{c}\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\}$$

$$= \cos\left[\omega(t - r/c) + \frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

$$= \cos[\omega(t - r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right) - \sin[\omega(t - r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)$$

Approximation 2 : $b \ll \frac{c}{\omega}$

$$\cos[\omega(t - \mathscr{r}/c)] \approx$$

$$\cos\left[\omega(t-\mathscr{r}/c)\right] = \cos\left\{\omega\left[t-\frac{r}{c}\left(1-\frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\}$$

$$= \cos\left[\omega(t-r/c)+\frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

$$= \cos[\omega(t-r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)-\sin[\omega(t-r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)$$

Approximation 2 : $b \ll \frac{c}{\omega}$

$$\boxed{\cos[\omega(t-\mathscr{r}/c)] \approx \cos[\omega(t-r/c)]-\frac{\omega b}{c}\sin\theta\cos\phi'\sin[\omega(t-r/c)]} \tag{20}$$

$$
\begin{aligned}
\cos\left[\omega(t-\mathscr{r}/c)\right] &= \cos\left\{\omega\left[t-\frac{r}{c}\left(1-\frac{b}{r}\sin\theta\cos\phi'\right)\right]\right\} \\
&= \cos\left[\omega(t-r/c)+\frac{\omega b}{c}\sin\theta\cos\phi'\right] \\
&= \cos[\omega(t-r/c)]\cos\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)-\sin[\omega(t-r/c)]\sin\left(\frac{\omega b}{c}\sin\theta\cos\phi'\right)
\end{aligned}
$$

Approximation 2 : $b \ll \frac{c}{\omega}$

$$
\boxed{\cos[\omega(t-\mathscr{r}/c)] \approx \cos[\omega(t-r/c)]-\frac{\omega b}{c}\sin\theta\cos\phi'\sin[\omega(t-r/c)]} \tag{20}
$$

$$
\begin{pmatrix}
\cos x = & 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\cdots \\
\sin x = & x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\cdots
\end{pmatrix}
$$

Referring to equation (18)

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$$\vec{A}(\vec{r}, t) = \frac{\mu_0 I_0 b}{4\pi} \hat{y} \int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi' \, d\phi'$$

Referring to equation (18)

$$\vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi'\, d\phi'$$

Substituting equations (19) and (20),

$$\vec{A}(\vec{r},t)$$

$$= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \left\{\cos\left[\omega(t - r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi' \sin\left[\omega(t - r/c)\right]\right\}\left(\frac{1}{\mathscr{r}}\right)\cos\phi'\, d\phi'$$

Referring to equation (18)

$$\vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi'\, d\phi'$$

Substituting equations (19) and (20),

$$\vec{A}(\vec{r},t)$$

$$= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \left\{ \cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi' \sin\left[\omega(t-r/c)\right] \right\} \left(\frac{1}{\mathscr{r}}\right) \cos\phi' d\phi'$$

$$= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \left\{ \cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi' \sin\left[\omega(t-r/c)\right] \right\} \frac{1}{r}\left[1+\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right] \cos\phi' d\phi'$$

Referring to equation (18)

$$\vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi'\, d\phi'$$

Substituting equations (19) and (20),

$$\vec{A}(\vec{r},t)$$

$$= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\left(\frac{1}{\mathscr{r}}\right)\cos\phi' d\phi'$$

$$= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\frac{1}{r}\left[1+\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\cos\phi' d\phi'$$

$$= \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\left[1+\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\cos\phi' d\phi'$$

Referring to equation (18)

$$\vec{A}(\vec{r}, t) = \frac{\mu_0 I_0 b}{4\pi} \hat{y} \int_0^{2\pi} \frac{\cos\left[\omega(t - \mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi' \, d\phi'$$

Substituting equations (19) and (20),

$$\vec{A}(\vec{r}, t)$$

$$= \frac{\mu_0 I_0 b}{4\pi} \hat{y} \int_0^{2\pi} \left\{ \cos\left[\omega(t - r/c)\right] - \frac{\omega b}{c} \sin\theta \cos\phi' \sin\left[\omega(t - r/c)\right] \right\} \left(\frac{1}{\mathscr{r}} \right) \cos\phi' d\phi'$$

$$= \frac{\mu_0 I_0 b}{4\pi} \hat{y} \int_0^{2\pi} \left\{ \cos\left[\omega(t - r/c)\right] - \frac{\omega b}{c} \sin\theta \cos\phi' \sin\left[\omega(t - r/c)\right] \right\} \frac{1}{r} \left[1 + \left(\frac{b}{r} \right) \sin\theta \cos\phi' \right] \cos\phi' d\phi'$$

$$= \frac{\mu_0 I_0 b}{4\pi r} \hat{y} \int_0^{2\pi} \left\{ \cos\left[\omega(t - r/c)\right] - \frac{\omega b}{c} \sin\theta \cos\phi' \sin\left[\omega(t - r/c)\right] \right\} \left[1 + \left(\frac{b}{r} \right) \sin\theta \cos\phi' \right] \cos\phi' d\phi'$$

$$\approx \frac{\mu_0 I_0 b}{4\pi r} \hat{y} \int_0^{2\pi} \left\{ \cos[\omega(t - r/c)] + b \sin\theta \cos\phi' \left(\frac{1}{r} \cos[\omega(t - r/c)] - \frac{\omega}{c} \sin[\omega(t - r/c)] \right) \right\} \cos\phi' d\phi'$$

Referring to equation (18)

$$\vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi} \frac{\cos\left[\omega(t-\mathscr{r}/c)\right]}{\mathscr{r}} \cos\phi'\, d\phi'$$

Substituting equations (19) and (20),

$$\begin{aligned}
&\vec{A}(\vec{r},t)\\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\left(\frac{1}{\mathscr{r}}\right)\cos\phi' d\phi'\\
&= \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\frac{1}{r}\left[1+\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\cos\phi' d\phi'\\
&= \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\int_0^{2\pi}\left\{\cos\left[\omega(t-r/c)\right] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin\left[\omega(t-r/c)\right]\right\}\left[1+\left(\frac{b}{r}\right)\sin\theta\cos\phi'\right]\cos\phi' d\phi'\\
&\approx \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\int_0^{2\pi}\left\{\cos[\omega(t-r/c)] + b\sin\theta\cos\phi'\left(\frac{1}{r}\cos[\omega(t-r/c)] - \frac{\omega}{c}\sin[\omega(t-r/c)]\right)\right\}\cos\phi' d\phi'\\
&= \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\left\{\int_0^{2\pi}\cos[\omega(t-r/c)]\cos\phi' d\phi' + \int_0^{2\pi} b\sin\theta\cos^2\phi'\left(\frac{1}{r}\cos[\omega(t-r/c)] - \frac{\omega}{c}\sin[\omega(t-r/c)]\right)d\phi'\right\}
\end{aligned} \tag{21}$$

$$\vec{A}(\vec{r},t)$$
$$= \frac{\mu_0 I_0 b}{4\pi r}\hat{y}\left\{\int_0^{2\pi} \cos[\omega(t-r/c)]\cos\phi' d\phi' + \int_0^{2\pi} b\sin\theta\cos^2\phi'\left(\frac{1}{r}\cos[\omega(t-r/c)] - \frac{\omega}{c}\sin[\omega(t-r/c)]\right)d\phi'\right\}$$

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$$\int_0^{2\pi}\cos\phi' d\phi' = 0 \qquad \int_0^{2\pi}\cos^2\phi' d\phi' = \pi$$

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$$\boxed{\vec{A}(r, \theta, t) = \frac{\mu_0 m_0}{4\pi}\left(\frac{\sin\theta}{r}\right)\left\{\frac{1}{r}\cos[\omega(t - r/c)] - \frac{\omega}{c}\sin[\omega(t - r/c)]\right\}\hat{\phi} \qquad (22)}$$

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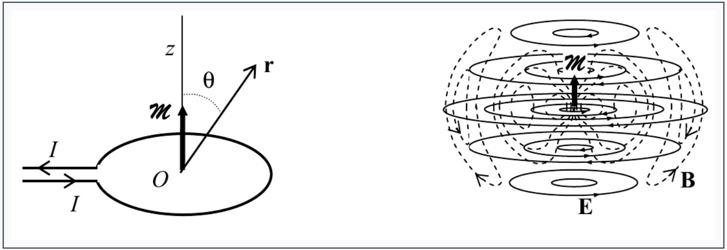

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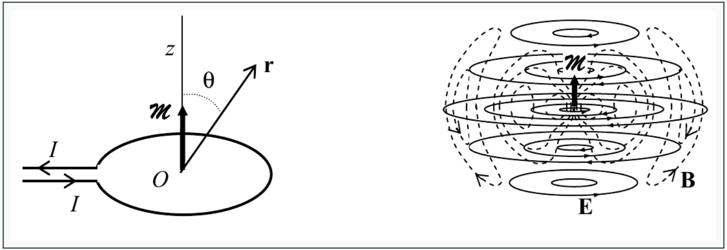

- These fields are in phase, mutually perpendicular, and transverse to the direction of propagation ($\hat{r}$), and the ratio of their amplitudes is $E_0/B_0 = c$, all of which is as expected for electromagnetic waves.

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