NOTE: From now on, we'll use a 2D notation to simplify the statements. Let frame F_A be such that $\hat{\imath}_A$ points towards the East and $\hat{\jmath}_A$ point towards the North, and let w be a point. Then, the statement "P is at (x_p, y_p) m" implies that $\vec{r}_{P/w}|_A = \begin{bmatrix} x_p & y_p & 0 \end{bmatrix}^T$ m.

Problem 1 (20 points). Consider the case where the pursuer P starts at $(0, 0)$ m and the evader E starts at $(100, 0)$ m. In particular, E is sailing north without maneuvering at constant speed $V_E = 5$ m/s, and P pursues E by sailing with constant speed V_P.

Use ODE45 with events to solve the R&β equations for direct pursuit (DP). Your states should include the positions of P and E, bearing angle β and range R. Solve for $V_P = 6$ m/s and time $t \in \{0, 0.01, \ldots, 99.99, 100\}$ s, or until the stop condition $R = 0$ is met (value = R in the tutorial notation). In a figure, plot the trajectory of E, the trajectory of P calculated by ode45, ~~and the theoretical trajectory of~~ P for $x \in \{0, 0.001, \ldots, 99.999, 100\}$ m. ~~Do the theoretical and calculated trajectories of P match?~~ Also, does the theoretical time of capture T_c coincide with the calculated time of capture (time at which the stop conditions was triggered)? In another figure, plot R versus time.

Please skip the red-crossed parts (no need to provide an

Next, consider the cases $V_P = 8$ m/s and $V_P = 11$ m/s such that $\gamma > 3/2$ and $\gamma > 2$, respectively. Do as before and use ODE45 with events to solve the R&β equations for DP for $V_P \in \{6, 8, 11\}$ m/s, for time $t \in \{0, 0.01, \ldots, 99.99, 100\}$ s, or until the stop condition $R = 0$ is met. In a figure, plot the trajectory of E (choose the one with the northernmost terminal position), and the trajectories of P for every value of V_P. Then, plot $|\dot{\beta}|$ versus time ($t \in [0, T_c]$) and $|\ddot{\beta}|$ versus time in two different figures for all $V_P \in \{6, 8, 11\}$ m/s. Check the theoretical predictions concerning these variables as time t approaches T_c.

NOTE 1: To calculate $\ddot{\beta}$, you will have derive an algebraic equation that depends on $\beta, \dot{\beta}, R, \dot{R}, V_P, \theta, \dot{\theta}, V_E, \theta_E$, and $\dot{\theta}_E$, where θ_E is the flight-path angle of E ($\pi/2$ in this case).

NOTE 2: When you plot $|\dot{\beta}|$ and $|\ddot{\beta}|$, ignore the last value. Since your simulation will end when $R \approx 0$, the values of $|\dot{\beta}|$ and $|\ddot{\beta}|$ obtained in this last index will be too large.

HINT: The file "ode45_Stop_Conditions" should be helpful in setting up the program for this problem.

Problem 2 (20 points). Reconsider Problem 1, but now consider the case of constant bearing pursuit (CBP), that is, compute the heading θ of P such that $\dot{\beta} = 0$. Remember that, since E is non-maneuvering, you can obtain the initial heading of θ by solving an equation.

Use ODE45 with events to solve the R&β equations for DP and CBP for $V_E = 5$ m/s, and $V_P \in \{6, 8, 11\}$ m/s, for time $t \in \{0, 0.01, \ldots, 99.99, 100\}$ s, or until the stop condition $R = 0$ is met. In a figure, using a 3-by-1 subplot, plot the trajectory of E (choose the one with the northernmost terminal position), and the trajectories of P obtained via DP and CBP, one plot for each value of V_P. In another figure, using a 3-by-1 subplot, plot R versus time for P obtained via DP and CBP, one plot for each value of V_P. What happens to the difference between the times of capture obtained for DP and CBP as V_P increases?

Problem 3 (20 points). Reconsider Problem 1, but now consider the case of proportional pursuit (PP) using different values of λ.

Use ODE45 with events to solve the R& β equations for CBP and PP for $V_{\mathrm{E}} = 5$ m/s, and $V_{\mathrm{P}} = 6$ m/s, $\lambda \in \{0.25, 0.5, 0.75, 0.9, 1, 2, 5, 50\}$, for time $t \in \{0, 0.01, \ldots, 99.99, 100\}$ s, or until the stop condition $R = 0$ is met. Suppose that, initially, $\theta = \beta$.

In a figure, plot the trajectory of E (choose the one with the northernmost terminal position), and the trajectories of P obtained via PP for $\lambda \in \{0.25, 0.5, 0.75, 0.9\}$, and, in another figure, plot R versus time for P obtained via PP for $\lambda \in \{0.25, 0.5, 0.75, 0.9\}$. For $\lambda < 1$, how does the trajectory of P obtained via PP behave? Does it overshoot or undershoot? What happens as you increase λ?

Furthermore, in a figure, plot the trajectory of E (choose the one with the northernmost terminal position), and the trajectories of P obtained via CBP and PP for $\lambda \in \{1, 2, 5, 50\}$, and, in another figure, plot R versus time for P obtained via CBP and PP for $\lambda \in \{1, 2, 5, 50\}$. For $\lambda > 1$, what happens as you increase λ? How does it compare to the CBP trajectory?

Problem 4 (20 points). Reconsider Problem 1, but now consider the case of DP and PP in the presence of noisy bearing measurement. Let T_{s} be the sampling period and let u be the bearing measurement used for calculating θ in DP and PP, such that, for all $k \geq 0$ and $t \in [kT_{\mathrm{s}}, (k+1)T_{\mathrm{s}})$, $u(t) = \beta_k$, where

$$\beta_k \triangleq \beta(kT_{\mathrm{s}}) + 0.25 w_k, \tag{1}$$

and $w_k \sim \mathcal{N}(0, 1)$.

Use ODE45 with events and solve timestep by timestep ($T_{\mathrm{s}} = 0.01$ s) the R&B equations for DP and PP for $V_{\mathrm{E}} = 5$ m/s, and $V_{\mathrm{P}} = 6$ m/s, $\lambda \in \{1.5, 2, 2.5\}$, for time $t \in \{0, 0.01, \ldots, 99.99, 100\}$ s, or until the stop condition $R = 0.5$ is met (value = R - 0.5 in the tutorial notation). In a figure, plot the trajectory of E (choose the one with the northernmost terminal position), and the trajectories of P obtained via DP and PP for all values of λ. In another figure, plot R versus time for P obtained via DP and PP for all values of λ. In contrast to what you observed in Problem 3, what happens as you increase λ? Can you intuitively explain why?

HINT: The file "ode45_Stop_Conditions_Ts_by_Ts" should be helpful in setting up the program for this problem.

Problem 5 (20 points). Reconsider Problem 3, but suppose that $\theta_{\mathrm{E}} = \pi/2 + \cos t$ rad. (in Problem 3, $\theta_{\mathrm{E}} = \pi/2$ rad). Do these results differ from those observed in Problem 3 ?

NOTE: For CBP, remember that, for all $t \geq 0$, you have to calculate $\theta(t)$ such that $\dot{\beta}(t) = 0$.

Problem 6 (20 points). Reconsider Problem 4, but suppose that $\theta_{\mathrm{E}} = \pi/2 + \cos t$ rad. (in Problem 4, $\theta_{\mathrm{E}} = \pi/2$ rad) Do these results differ from those observed in Problem 4 ?

Problem 7 (20 points). Consider the case of linearized PG with a missile autopilot with first-order dynamics (the model used in the analysis in Lecture 22). Let the autopilot dynamics be given by 1/(Ts+1), where s is the Laplace operator and $T = 1$ s.

a) Use the adjoint method to obtain $G(\bar{t}_{\mathrm{f}}, t)$ for $t \in \{0, 0.001, \ldots, 9.999\}$ s, $\Lambda \in \{3, 4, 5\}$, and $\bar{t}_{\mathrm{f}} = 10$ s. In a single figure, plot $-TG(\bar{t}_{\mathrm{f}}, t)$ versus $(\bar{t}_{\mathrm{f}} - t)/T$ for all $\Lambda \in \{3, 4, 5\}$. **NOTE:** You should obtain a plot similar to the one in Figure 5.8 in the Navigation and Guidance book by Kabamba and Girard, in page 135. Furthermore, unlike the previous problems, you can use ode45 regularly in this problem.

b) Let $-TG(\bar{t}_{\mathrm{f}}, t)$ be the normalized miss distance of the case where the evader performs an impulsive maneuver (remember that G is the impulse response function) and $(\bar{t}_{\mathrm{f}} - t)/T$ be the normalized time-to-go. Intuitively, how would you interpret the plot you obtained in a)? Furthermore, can you use this insight to explain the pursuit results displayed in Figure 1 ? These were obtained by computing

$$y_{\mathrm{P}}(t) = \int_0^t G(\bar{t}_{\mathrm{f}}, \tau) y_{\mathrm{E}}(\tau) d\tau, \tag{2}$$

for all $t \in \{0, 0.001, \ldots, \bar{t}_{\mathrm{f}} - 0.001\}$ s and various evader maneuvers shown in the figures.

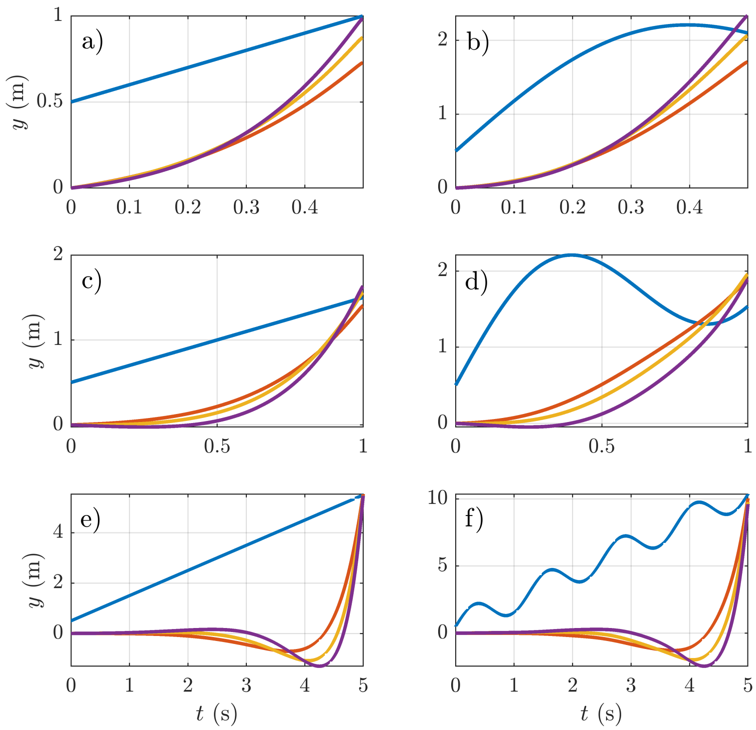

Figure 1: Problem 7 b): y versus time for y_{E} (blue line), and y_{P} for $\Lambda = 3$ (orange line), $\Lambda = 4$ (yellow line), and $\Lambda = 5$ (purple line), for various values of $\bar{t}_{\mathrm{f}}$ and y_{E} ($y_{\mathrm{E},1}(t) = 0.5 + t$ m, $y_{\mathrm{E},2}(t) = 0.5 + 2t + \sin 5t$ m). a) $y_{\mathrm{E}} = y_{\mathrm{E},1}$, $\bar{t}_{\mathrm{f}} = 0.5$ s. b) $y_{\mathrm{E}} = y_{\mathrm{E},2}$, $\bar{t}_{\mathrm{f}} = 0.5$ s. c) $y_{\mathrm{E}} = y_{\mathrm{E},1}$, $\bar{t}_{\mathrm{f}} = 1$ s. d) $y_{\mathrm{E}} = y_{\mathrm{E},2}$, $\bar{t}_{\mathrm{f}} = 1$ s. e) $y_{\mathrm{E}} = y_{\mathrm{E},1}$, $\bar{t}_{\mathrm{f}} = 5$ s. f) $y_{\mathrm{E}} = y_{\mathrm{E},2}$, $\bar{t}_{\mathrm{f}} = 5$ s.