Global Form of Flavor Symmetry Groups in Twisted A_{2N} Class S Theories

Karthik Prasad
Advisors: Dr. Anderson Trimm[1]

[1]Department of Mathematics,
Illinois Mathematics and Science Academy

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History of Particle Physics

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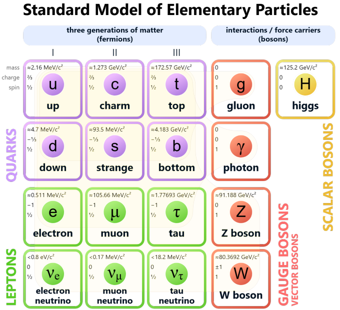

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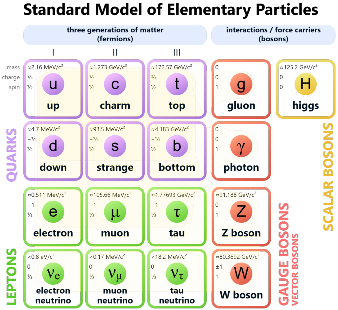

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- At the heart of this progress was an increased understanding of the importance of symmetry.

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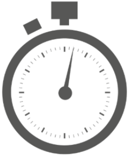 

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  - Spatial-translation and rotation are symmetries.

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- "Continuous" (smooth) symmetries become Lie Groups.
- The symmetries in a neighborhood of the identity are encoded in the infinitesmial generators which form a structure called a Lie algebra.

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- If an observer measures time interval t and position $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ of an event, then another in relative (differing only by a constant velocity) motion with respect to the first measures the time interval to be t' and position to be $(\mathrm{x}', \mathrm{y}', \mathrm{z}')$, then the quantity:

$$(\Delta \mathrm{s})^2 := \mathrm{c}^2\mathrm{t}^2 - \mathrm{x}^2 - \mathrm{y}^2 - \mathrm{z}^2.$$

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- When objects at speed $\mathrm{v} = \left(\frac{\mathrm{dx}}{\mathrm{dt}}, \frac{\mathrm{dy}}{\mathrm{dt}}, \frac{\mathrm{dz}}{\mathrm{dt}}\right) << \mathrm{c}$, the Lorentz group is the same as the Galilean Group, the symmetry group of spacetime in Newtonian mechanics.

Maxwell and Special Relativity

- Maxwell's equation, which describe the electric and magnetic fields, in the absence of charges are given by:

$$\nabla \cdot \mathrm{E} = 0, \qquad \nabla \cdot \mathrm{B} = 0$$

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}}, \qquad \nabla \times \mathrm{B} = \frac{\partial \mathrm{E}}{\partial \mathrm{t}}.$$

  The electric and magnetic fields themselves can be derived from a scalar potential ϕ and vector potential A as:

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- To see Maxwell's equations are compatible with special relativity, we combine the potentials into a 4-potential $\mathrm{A}_\mu = (\phi, \mathrm{A})$ and define the electromagnetic field strength tensor in each entry by $\mathrm{F}_{\mu\nu} := \partial_\mu \mathrm{A}_\mu - \partial_\nu \mathrm{A}_\mu$, which transforms as a 2-tensor under Lorentz transformations. Maxwell's equations become:

$$\partial^\mu \mathrm{F}_{\mu\nu} = 0, \quad \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\mu \mathrm{F}^{\rho\sigma} = 0.$$

Lagrangian Perspective

- We can find equations of motion for classical field theories by using the action functional

$$\mathrm{S}[\phi] = \int \mathrm{d}^4\mathrm{x}\ \mathcal{L}(\phi, \partial_\mu \phi),$$

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- One can verify that Maxwell's equations are the Euler-Lagrange equations arising from the Lagrangian:

$$\mathcal{L}_{\mathrm{Maxwell}} = -\frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}.$$

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Quantum Field Theories

- The full QED Lagrangian of the electromagnetic interaction (coupled to matter) is given by:

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \bar{\psi}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - \mathrm{m})\psi - \mathrm{e}\bar{\psi}\gamma^{\mu}\psi\mathrm{A}_{\mu}.$$

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- In Feynman's path-integral formulation of QFT, we quantize Maxwell's theory by integrating over all field configurations (weighted by the exponential of the classical action):

$$\int \mathcal{D}[\mathrm{A}, \psi]\mathrm{e}^{\mathrm{i}\mathrm{S}[\mathrm{A},\psi]/\hbar}.$$

  This is called the path integral.

Why is QFT successful

- We can calculate path integrals by expanding the interaction in a series:

$$e^{-ie\overline{\psi}\gamma^{\mu}\psi A_{\mu}} = 1 - \frac{ie}{\hbar}\overline{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{e^2}{2!\hbar^2}(\overline{\psi}\gamma^{\mu}\psi A_{\mu})^2 + \dots$$

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- These calculations are hugely successful. The theoretical prediction and experimental measurement of the magnetic moment of the electron agree to one part in a trillion (which is astonishingly good).

Generalized Theory

- Yang and Mills generalized QED in the 1950s. The QED Lagrangian is invariant under the gauge transformation:

$$\psi(\mathrm{x}) \leftarrow \mathrm{e}^{\mathrm{i}\alpha(\mathrm{x})\psi(\mathrm{x})}, \qquad \mathrm{A}_\mu \to \mathrm{A}_\mu - \frac{1}{\mathrm{e}}\partial_\mu\alpha(\mathrm{x}),$$

which acts on the spinor field ψ as a (local) phase rotation, as the $\mathrm{e}^{\mathrm{i}\alpha(\mathrm{x})}$ is an element of the group U(1) of phase rotations.

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  which acts on the spinor field ψ as a (local) phase rotation, as the $\mathrm{e}^{\mathrm{i}\alpha(\mathrm{x})}$ is an element of the group U(1) of phase rotations.
- Yang and Mills replaced the abelian Lie group U(1) with a non-abelian compact Lie group G. Then, the gauge field $\mathrm{A}_\mu = \mathrm{A}_\mu^\mathrm{a} \mathrm{T}^\mathrm{a} \in \mathfrak{g}$, the Lie algebra of G, with the T^α the generators of G in the adjoint representation of $\mathfrak{g}$. The field strength is then:

$$\mathrm{F}_{\mu\nu} = \partial_\mu \mathrm{A}_\nu - \partial_\nu \mathrm{A}_\mu + [\mathrm{A}_\mu, \mathrm{A}_\nu],$$

  where $[\cdot, \cdot]$ is the commutator. Notice since U(1) is commutative, its Lie algebra has trivial commutator, and indeed we recover the field strength of the electromagnetic field.

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- Let us look at the sector charged under $SU(3)$. This describes the strong interaction, which binds quarks into hadrons, and is called Quantum Chromodynamics.The QCD Lagrangian is:

$$\mathcal{L}_{QCD} = -\frac{1}{4} \sum_{a=1}^{\dim SU(3)} F^{a}_{\mu\nu} F^{\mu\nu a} + \sum_{f=1}^{N_f} \overline{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f .$$

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- Here, $\mathrm{F}^{\mathrm{a}}_{\mu\nu} = \partial_\mu \mathrm{A}^{\mathrm{a}}_\nu - \partial_\nu \mathrm{A}^{\mathrm{a}}_\mu + \mathrm{g_s} \mathrm{f}^{\mathrm{abc}} \mathrm{A}^{\mathrm{b}}_\mu \mathrm{A}^{\mathrm{c}}_\nu$, where $\mathrm{A}^{\mathrm{a}}_\mu$ is one of the $\dim \mathrm{SU}(3) = 8$ gluon fields and $\mathrm{g_s}$ is the strong coupling constant. The first term in the Lagrangian is the kinetic term for gluons.

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- Here, $\overline{\psi}_{\mathrm{f}} (\mathrm{i}\gamma^{\mu} \mathrm{D}_{\mu} - \mathrm{m_f}) \psi_{\mathrm{f}}$, where there are $\mathrm{N_f} = 6$ quarks for $\mathrm{f} \in \{1, ..., 6\}$. $\mathrm{m_f}$ is the mass of the quark flavor f, and ψ_{f} is a Dirac spinor describing the quark flavor f and its antiquark. The three components of ψ_{f} are called colors.

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- Theories with $\mathrm{N_f} = 4$ flavors are examples of superconformal field theories (SCFTs), which look the same at all energy scales. In the Wilsonian picture of QFT, renormalizable are relevant or marginal deformations of UV fixed points, which are believed to be conformal.

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    - As such, a starting point to understand QFTs is to classify conformal field theories (SCFTs in supersymmetric cases).

S-duality

- For the SU(2), $\mathcal{N} = 2$ with $\mathrm{N_f} = 4$ we get a family of SCFTs parametrized by the complexified gauge coupling $\tau = \frac{\theta}{\pi} + \frac{8\pi \mathrm{i}}{\mathrm{g}^2}$.

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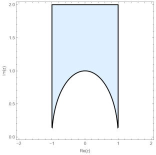
- Seiberg and Witten found an $\mathrm{SL}(2, \mathbb{Z})$ duality group acting on the space of gauge couplings, where τ produces distinct theories in the fundamental domain of $\mathrm{SL}(2, \mathbb{Z})$:



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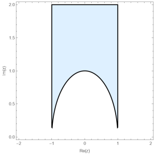

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- This is weakly coupled as $\tau \to i\infty$. Seiberg and Witten found that as τ approaches the cusps at $-\frac{1}{2}, \frac{1}{2}$, we obtain the original SU(2) and $N_f = 4$ theory but now with gauge coupling $\tau_D = -\frac{1}{\tau}$, becoming weakly coupled.

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S-duality

- For the SU(2), $\mathcal{N}=2$ with $\mathrm{N_f}=4$ we get a family of SCFTs parametrized by the complexified gauge coupling $\tau = \frac{\theta}{\pi} + \frac{8\pi \mathrm{i}}{\mathrm{g}^2}$.
- Seiberg and Witten found an $\mathrm{SL}(2,\mathbb{Z})$ duality group acting on the space of gauge couplings, where τ produces distinct theories in the fundamental domain of $\mathrm{SL}(2,\mathbb{Z})$:

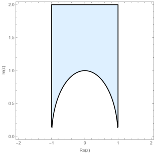

Figure: Fundamental Domain of $\mathrm{SL}(2,\mathbb{Z})$.

- This is weakly coupled as $\tau \to \mathrm{i}\infty$. Seiberg and Witten found that as τ approaches the cusps at $-\frac{1}{2}, \frac{1}{2}$, we obtain the original SU(2) and $\mathrm{N_f}=4$ theory but now with gauge coupling $\tau_{\mathrm{D}} = -\frac{1}{\tau}$, becoming weakly coupled.
- This strong/weak duality is called S-duality.

Electric-Magnetic Duality

- Maxwell's equations with electric and magnetic charges become:

$$\nabla \cdot \mathrm{E} = \rho_{\mathrm{e}}, \qquad \nabla \cdot \mathrm{B} = \rho_{\mathrm{m}}$$

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}} + \mathrm{J}_{\mathrm{m}}, \qquad \nabla \times \mathrm{B} = \frac{\partial \mathrm{E}}{\partial \mathrm{t}} + \mathrm{J}_{\mathrm{e}}.$$

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- These are now invariant under the combined transformation $\mathrm{E} \to \mathrm{B}$, $\mathrm{B} \to -\mathrm{E}$, $(\rho, \mathrm{J}_{\mathrm{e}}) \to (-\rho_{\mathrm{m}}, \mathrm{J}_{\mathrm{m}})$, $(\rho_{\mathrm{m}}, \mathrm{J}_{\mathrm{m}} \to \rho_{\mathrm{e}}, -\mathrm{J}_{\mathrm{e}})$.

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- S-duality can be considered a generalization of this.

Flavor Symmetry

- Flavor symmetry is the symmetries of the matter fields of a theory. Mathematically, a flavor symmetry is global symmetry commuting with the superconformal gauge symmetry of the theory. This is an invariant of the SCFT, independent of the gauge-coupling τ. It is therefore invariant under S-duality.

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  - For example, $\mathfrak{su}(2)$ has covering group SU(2). But, both the groups SU(2) and SO(3) have lie algebra $\mathfrak{su}(2)$.
- The manifest flavor symmetry of a theory is always one such simply-connected universal covering group of the algebra of the theory.

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- This is a subgroup of the center of the flavor symmetry group $\mathcal{F}$, and quotienting out gives another Lie group F'/N.
- This group F'/N is called the global form of flavor symmetry, and represents the "nontrivial" symmetries of the matter content of the theory.

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- These 4d N = 2 SCFTs are called "class S" theories.

SU(3) + 6(3) Argyres-Seiberg Duality

The SU(3) + 6(3) theory Argyes and Seiberg studied has the following dual Class S descriptions:

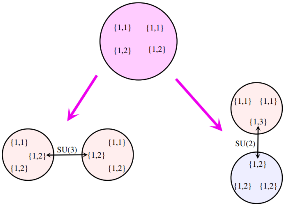

Figure: SU(3) + 6(3) Duality Frame

We have an SU(3) gauge theory with 6 hypermultiplets in the fundamental, and a dual SU(2) gauge theory coupled with one fundamental hypermultiplet, with the SU(2) a gauged subgroup of the original E_6 algebra. In both cases, the global symmetry group is $SU(6) \times U(1)$.

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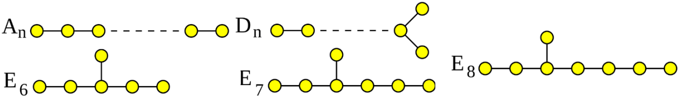

Figure: Simply-Laced Dynkin Diagrams (Wikipedia)

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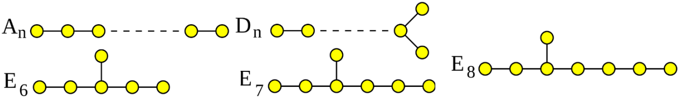

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- By listing the possible fixtures and cylinders connecting them, we can build theories by building the surface in a tinkertoy fashion.

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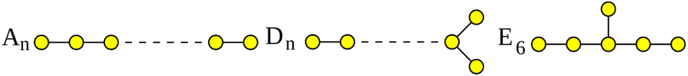

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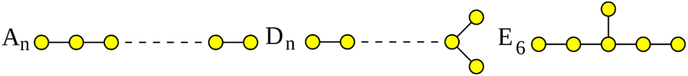

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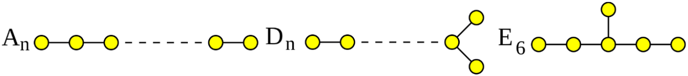

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  - $\mathrm{E_6}$ has a $\mathbb{Z}_2$.
- Twisted theories exist for $\mathrm{A_n}$, $\mathrm{D_n}$, $\mathrm{E_6}$ theories, and have been classified for all but $\mathrm{A_{2n}}$, which is more difficult as noted by Tachikawa. Distler et. al have made significant progress on classifying these recently.

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  - These can be broken, anomalous, or otherwise provide constraints on the low-energy effective theory at a point.
- The 0-form symmetry that goes into the class S 2-group symmetries is the global form of flavor symmetry.
  - For Lagrangian theories, this is straightforward to compute.
  - For non-Lagrangian theories, we can compute this using a proposal due to Bhardwaj for arbitrary class S theories in terms of the algebraic and geometric class S data.

Our Research

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- So far, we have calculated the global form of the $\mathrm{R}_{2,2\mathrm{n}}$ theory
- For $\mathrm{n} = 2$, we have explicitly calculated:

$$\frac{\mathrm{SU}(2) \times \mathrm{SU}(2)}{\mathbb{Z}_2} \times \mathrm{U}(1)$$

and for general n, we have (to be checked):

$$\frac{\mathrm{Sp}(\mathrm{n}) \times \mathrm{Sp}(\mathrm{n})}{\mathbb{Z}_2} \times \mathrm{U}(1).$$

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- Use these symmetries to contain the low-energy effective theory (i.e the "close-to-physical" theory) arising from A_{2n}.

The End.

Thanks for Listening! Questions!

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- The minimal untwisted picture carries a $\mathfrak{u}(1)$ symmetry algebra and has representation $\mathcal{R}^{\vee}_{\mathrm{o},\mathcal{P}} = 2\nVdash_1 \otimes \nVdash_{-2}$ and therefore we have:

$$\begin{aligned}
\mathcal{R}^{\vee}_{\mathrm{o},\mathcal{P}} \otimes \overline{\mathcal{R}}^{\vee}_{\mathrm{o},\mathcal{P}} &= (2\mathbf{1}_1 \oplus \mathbf{1}_{-2}) \otimes (2\mathbf{1}_{-1} \oplus \mathbf{1}_2) \\
&= 4(\mathbf{1}_1 \otimes \mathbf{1}_{-1}) \oplus 2(\mathbf{1}_1 \otimes \mathbf{1}_2) \oplus 2(\mathbf{1}_{-2} \otimes \mathbf{1}_{-1}) \oplus (\mathbf{1}_{-2} \otimes \mathbf{1}_2) \\
&= 5\mathbf{1}_0 + 2\mathbf{1}_3 + 2\mathbf{1}_{-3}.
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Calculating the Global Form for $\mathrm{R}_{2,4}$

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- This shows Y'_{F} is the sublattice of $\mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$ generated by $(0,0,3)$.

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- The representation $\mathcal{R}$ of $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ (the universal covering group) is:

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- So, this determines $\mathrm{Y}_{\mathrm{F}} \cup \mathrm{Y}_{\mathrm{F}'}$.

Calculating the Global Form for $\mathrm{R}_{2,4}$

- To finish calculating the quotient, we calculate:

$$\frac{\hat{\mathrm{G}}}{\widehat{\left(\frac{\hat{\mathrm{G}}}{\mathrm{Y_F} \cup \mathrm{Y_{F'}}}\right)}},$$

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- Taking the Pontrayagin dual and quotienting gives:

$$\frac{\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)}{\mathbb{Z}_2},$$

  as desired.