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CS 161 - Homework 5

1. a) $P \Rightarrow Q$
$= \neg P \vee Q$
$\neg Q \Rightarrow \neg P$
$= Q \vee \neg P$

	P	Q	$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$
w_1	0	0	1	1
w_2	0	1	1	1
w_3	1	0	0	0
w_4	1	1	1	1

$M(P \Rightarrow Q) = M(\neg Q \Rightarrow \neg P) = \{w_1, w_2, w_4\}$ - equivalent

b) $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

	P	Q	$P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
W_1	0	0	0	0
W_2	0	1	1	1
W_3	1	0	1	1
W_4	1	1	0	0

$M(P \Leftrightarrow \neg Q) = M((P \wedge \neg Q) \vee (\neg P \wedge Q)) = \{W_2, W_3\}$ - equivalent

2. Let smoke be S, fire be F and heat be H

a) $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
$= (\neg S \vee F) \Rightarrow (S \vee \neg F)$
$= (S \wedge \neg F) \vee (S \vee \neg F)$

	S	F	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
W_1	0	0	1
W_2	0	1	0
W_3	1	0	1
W_4	1	1	1

$M((S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)) = \{w_1, w_3, w_4\}$

This sentence is satisfied in 3 out of 4 worlds, and is therefore not valid (for it to be valid it must satisfiable by all worlds).

b) $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

$= (\neg S \vee F) \Rightarrow ((\neg S \wedge \neg H) \vee F)$

$= (S \wedge \neg F) \vee ((\neg S \wedge \neg H) \vee F)$

	S	F	H	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
w_1	0	0	0	1
w_2	0	0	1	0
w_3	0	1	0	1
w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

This sentence is not valid and satisfiable by w_1, $w_3 - w_8$.

c) $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow ((\neg S \vee F) \vee (\neg H \vee F))$

$= (\neg S \vee \neg H \vee F) \Leftrightarrow (\neg S \vee \neg H \vee F)$

	S	F	H	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
w_1	0	0	0	1
w_2	0	0	1	1
w_3	0	1	0	1
w_4	0	1	1	1
w_5	1	0	0	1
w_6	1	0	1	1
w_7	1	1	0	1
w_8	1	1	1	1

This sentence is satifiable by all words, and therefore also valid.

3. a) Mythical $\Rightarrow$ Immortal

$\neg$ Mythical $\Rightarrow$ ($\neg$ Immortal $\wedge$ Mammal)

(Immortal $\vee$ Mammal) $\Rightarrow$ Horned

Horned $\Rightarrow$ Magical

b) Let the following symbols represent -

Y - Mythical

I - Immortal

M - Mammal

H - Horned

G - Magical

We have -

$\neg Y \vee I$

$Y \vee (\neg I \wedge M)$

$(\neg I \wedge \neg M) \vee H$

$\neg H \vee G$

$= (\neg Y \vee I) \wedge (\neg I \vee Y) \wedge (M \vee Y) \wedge (\neg I \vee H) \wedge (\neg M \vee H) \wedge (\neg H \vee G)$

c) i) Assume the unicorn is not mythical.

Knowledge base -

1. $\neg Y$
2. $\neg Y \vee I$
3. $\neg I \vee Y$
4. $M \vee Y$
5. $\neg I \vee H$
6. $\neg M \vee H$
7. $\neg H \vee G$

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8. $\neg I$ (1 and 3)
9. $M \vee I$ (2 and 4)
10. $Y \vee H$ (4 and 6)
11. $\neg I \vee G$ (5 and 7)
12. M (8 and 9)
13. H (1 and 10)
14. G (7 and 13)

- satisfied by $\neg Y \wedge \neg I \wedge M \wedge H \wedge G$

Not mythical is satisfiable, therefore we can not prove the unicorn is mythical with the given knowledge base.

ii) Assume the unicorn is not magical.

Knowledge base -

1. $\neg G$
2. $\neg Y \vee I$
3. $\neg I \vee Y$
4. $M \vee Y$
5. $\neg I \vee H$
6. $\neg M \vee H$
7. $\neg H \vee G$

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8. $\neg H$ (1 and 7)
9. $Y \vee H$ (4 and 6)
10. $I \vee M$ (2 and 4)
11. $\neg M \vee G$ (6 and 7)
12. Y (8 and 9)
13. $I \vee G$ (10 and 11)
14. I (1 and 13)
15. H (5 and 14)

8 and 15 are contradictions, therefore $\neg G$ is unsatisfiable.

Since $\neg G$ is unsatisfiable, G must be valid - the unicorn is magical.

iii) Assume the unicorn is not horned.

Knowledge base -

1. $\neg H$
2. $\neg Y \vee I$
3. $\neg I \vee Y$
4. $M \vee Y$
5. $\neg I \vee H$
6. $\neg M \vee H$
7. $\neg H \vee G$

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8. $\neg I$ (1 and 5)
9. $\neg M$ (1 and 6)
10. $\neg Y$ (2 and 8)
11. M (4 and 10) - 9 and 11 are contradictions

$\neg H$ is unsatisfiable, therefore H is valid - the unicorn is horned.

4. Figure 1 is:
- decomposable, as for each and gate, the subcircuits have no overlapping variables.
- not deterministic as the assignment $(A, \neg B, C, \neg D)$ results in two true inputs to the topmost or gate.
- not smooth because the two central or gates on the third level do not have same variables on each side.

Figure 2 is:
- decomposable, as for each and gate, the subcircuits have no overlapping variables.
- not deterministic, as the variable assignment $(\neg A, B)$ causes two OR gates on the third level to have two true inputs.
- smooth, as for each or gate, the subcircuits have the same variables.

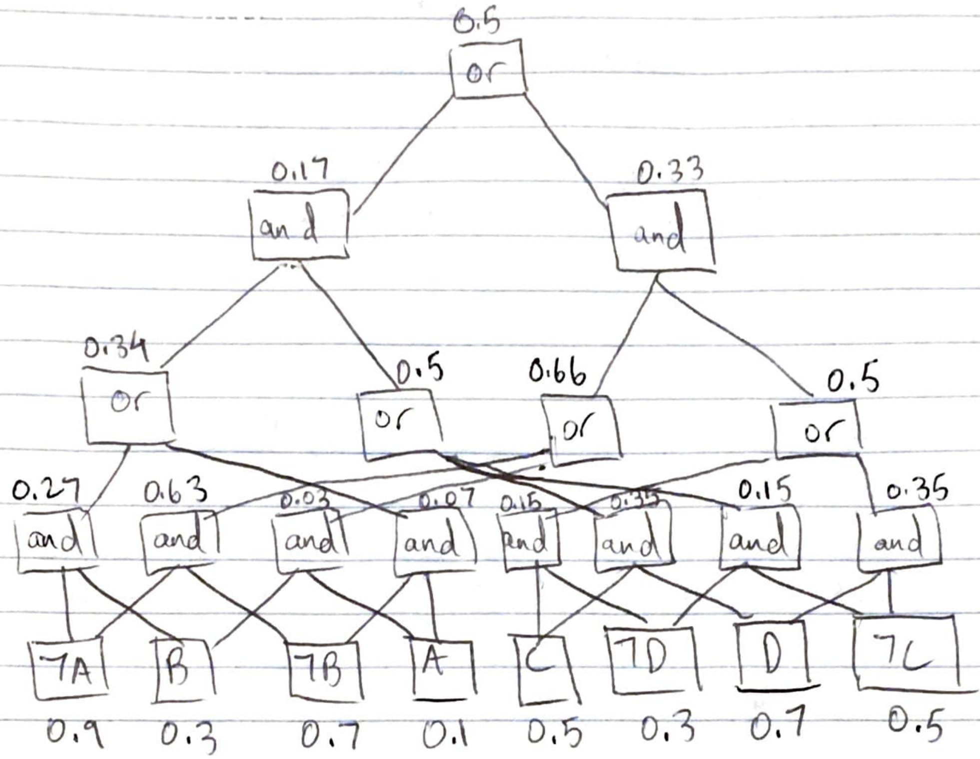
5. a)

A	B	$(\neg A \land B) \lor (\neg B \land A)$
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{aligned} WMC &= w(\neg A)w(B) + w(A)w(\neg B) \\ &= 0.9 \times 0.3 + 0.1 \times 0.7 \\ &= 0.34 \end{aligned}$$

b) The count on the root is the same as the Weighted Model Count for the formula from part (a). This shows us that if we have a decomposable, deterministic NNF form for a particular propositional formula, we can very easily linearly compute the number of assignments that satisfy the proposition, and in this problem, find the WMC in linear time.

c)



Weighted Model Count = 0.5