Karthik Rajagopalan
904918243

Homework 7

1. Base cases-

Let $n=1$

$Pr(a_1|\beta) = Pr(a_1|\beta)$ - true

Let $n=2$

$Pr(a_1, a_2|\beta) = Pr(a_1|a_2,\beta)\, Pr(a_2|\beta)$

$\frac{Pr(a_1, a_2, \beta)}{Pr(\beta)} = \frac{Pr(a_1, a_2, \beta)}{\cancel{Pr(a_2, \beta)}} \times \frac{\cancel{Pr(a_2, \beta)}}{Pr(\beta)}$ - true

Now let us assume that the identity is true for $n=k$.

i.e. $Pr(a_1, \ldots a_k|\beta) = Pr(a_1|a_2, \ldots, a_k, \beta) \ldots\ldots Pr(a_k|\beta)$ - (1)

If $n=k+1$ -

$Pr(a_1, \ldots\ldots, a_k, a_{k+1}|\beta) = \frac{Pr(a_1, \ldots, a_k, a_{k+1}, \beta)}{Pr(\beta)}$

$= \frac{Pr(a_1, \ldots a_k|a_{k+1}, \beta)\, Pr(a_{k+1}, \beta)}{Pr(\beta)}$

$= Pr(a_1, \ldots\ldots a_k|a_{k+1}, \beta)\, Pr(a_{k+1}|\beta)$

Now if (1) is true, we can expand out the first term with (a_{k+1}, β) substituted for β.

$= Pr(a_1|a_2, \ldots a_k, a_{k+1}, \beta)\, Pr(a_2|a_3 \ldots a_k, a_{k+1}, \beta) \ldots Pr(a_k|a_{k+1}, \beta)\, Pr(a_{k+1}|\beta)$

Thus proved.

So if the identity is true for $n=k$, it is also true for $n=k+1$. Since $n=1$ is true and $n=2$ is true, $n=3$ is true, $n=4$ is true and so on.

2. Let oil being present be event O, natural gas being present be event N and test coming back positive be event T.

$Pr(O) = 0.5$

$Pr(N) = 0.2$

$Pr(\neg O, \neg N) = 0.3$

$Pr(O, N) = 0$

$Pr(T \mid O) = 0.9$

$Pr(T \mid N) = 0.3$

$Pr(T \mid \neg O, \neg N) = 0.1$

We want to find $Pr(O \mid T)$.

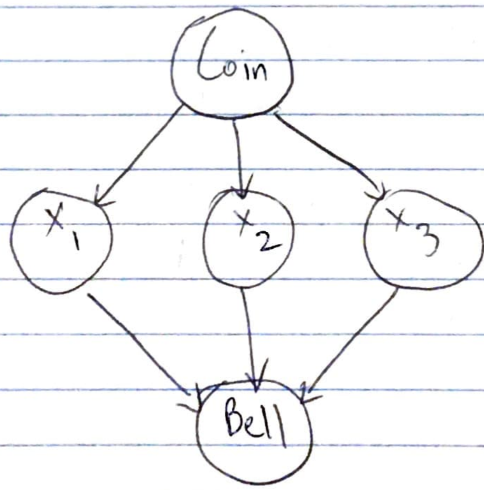
$$Pr(T) = Pr(T \mid O)P(O) + P(T \mid N)P(N) + Pr(T \mid O, N)Pr(O, N) + Pr(T \mid \neg O, \neg N)Pr(\neg O, \neg N)$$

$$= 0.9 \times 0.5 + 0.3 \times 0.2 + 0 + 0.1 \times 0.3$$

$$= 0.54$$

$$Pr(O \mid T) = \frac{Pr(O, T)}{Pr(T)} = \frac{Pr(T \mid O)Pr(O)}{Pr(T)} = \frac{0.9 \times 0.5}{0.54} = \boxed{0.833}$$

3.



Let C denote coin, B denote bell.

C	θ_c
a	1/3
b	1/3
c	1/3

C	X_1	$\theta_{X_1\mid C}$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

C	X_2	$\theta_{X_2\mid C}$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

C	X_3	$\theta_{X_3\mid C}$
a	heads	0.2
a	tails	0.8
b	heads	0.4
b	tails	0.6
c	heads	0.8
c	tails	0.2

X_1	X_2	X_3	B	$\theta_{B\mid X_1,X_2,X_3}$
heads	heads	heads	on	1
heads	heads	heads	off	0
heads	heads	tails	on	0
heads	heads	tails	off	1
heads	tails	heads	on	0
heads	tails	heads	off	1
heads	tails	tails	on	0
heads	tails	tails	off	1
tails	heads	heads	on	0
tails	heads	heads	off	1
tails	heads	tails	on	0
tails	heads	tails	off	1
tails	tails	heads	on	0
tails	tails	heads	off	1
tails	tails	tails	on	1
tails	tails	tails	off	0

4. a) $I(A, \phi, BE)$
$I(B, \phi, AC)$
$I(C, A, BDE)$
$I(D, AB, CE)$
$I(E, B, ACDFG)$
$I(F, CD, ABE)$
$I(G, F, ABCDEH)$
$I(H, EF, ABCDG)$

b) d-separated(A, F, E) = false, path ADBE is not blocked

d-separated(G, B, E) = true, valves B and H are closed given B, and block all paths from G to E.

d-separated(AB, CDE, GH) = true - all paths from AB to GH involve passing through C, D or E as a sequential valve which is closed, therefore all paths are blocked by CDE.

c) $Pr(a,b,c,d,e,f,g,h) = Pr(a) \times Pr(c|a) \times Pr(b) \times Pr(d|a,b) \times Pr(e|b) \times Pr(f|c,d) \times Pr(g|f) \times Pr(h|f,e)$

d) d-separated(A, ϕ, B) is true, therefore $Pr(A, B) = Pr(A)\,Pr(B)$

$Pr(A=1, B=1) = Pr(A=1) \times Pr(B=1) = 0.2 \times 0.7 = \boxed{0.14}$

d-separated(A, A, E) is true,

Therefore $Pr(E=0 | A=0) = Pr(E=0)$

$= Pr(E=0|B=0) \times Pr(B=0) + Pr(E=0|B=1) \times Pr(B=1)$

$= 0.1 \times 0.3 + 0.9 \times 0.7$

$= \boxed{0.66}$

5.a)

	A	B	$\alpha : A \Rightarrow B$
w_0	T	T	T
w_1	T	F	F
w_2	F	T	T
w_3	F	F	T

Models of α : w_0, w_2 and w_3

b) $Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3) = 0.3 + 0.1 + 0.4 = 0.8$

c)

A	B	$Pr(A, B \mid \alpha)$
T	T	$0.3/0.8 = 0.375$
T	F	0
F	T	$0.1/0.8 = 0.125$
F	F	$0.4/0.8 = 0.5$

d)

A	B	α	$A \Rightarrow \neg B$
T	T	T	F
T	F	F	T
F	T	T	T
F	F	T	T

Models of $\alpha \wedge A \Rightarrow \neg B$: w_2 and w_3

$$P(A \Rightarrow \neg B \mid \alpha) = \frac{Pr(A \Rightarrow \neg B, \alpha)}{Pr(\alpha)} = \frac{0.1 + 0.4}{0.8} = \boxed{0.625}$$