Forward & Inverse Kinematics

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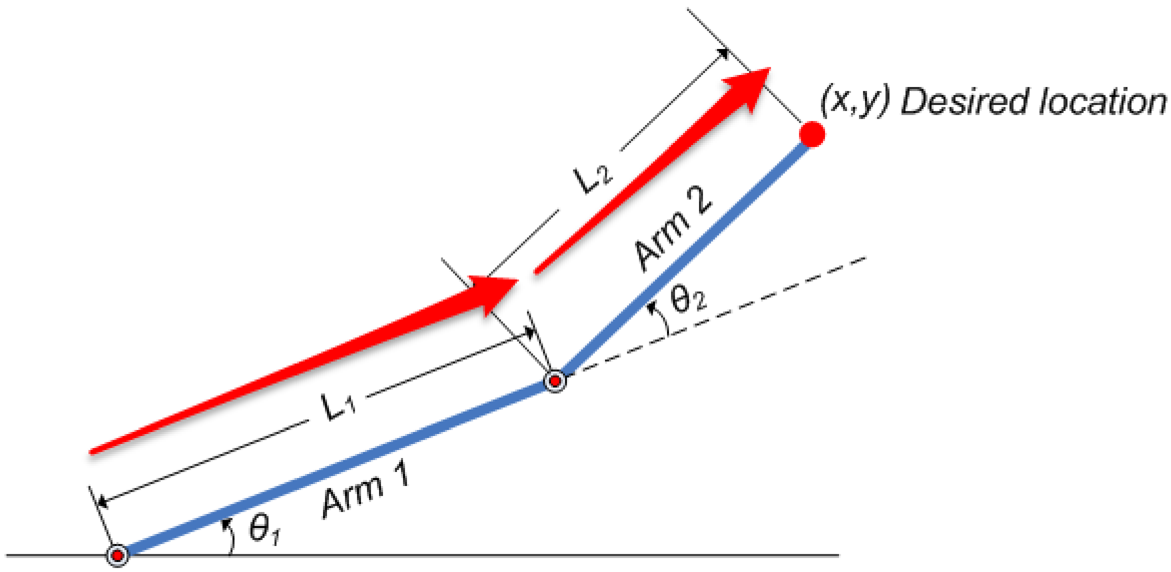
Forward & Inverse Kinematics

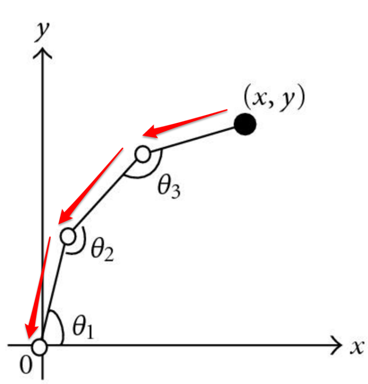
Keywords **:** forward kinematics,inverse kinematics,workspace,vectors,vector components,motion planning
Prerequisites : Linear Algebra,Scalars and Vectors,Matrices,Geometry,Trigonometry,MATLAB(optional)

Forward kinematics refers to the use of the **kinematic** equations of a robot to compute the position of the **end-effector** from specified values for the joint parameters.
The **kinematics** equations of the robot are used in robotics, computer games, and animation. (source : Wikipedia)



Forward Kinematics - origin to end-effector

Inverse kinematics makes use of the **kinematics** equations to determine the joint parameters that provide a desired position for each of the robot's **end-effectors**. Specification of the movement of a robot so that its end-effectors achieve the desired tasks is known as **motion planning**. (source : Wikipedia).



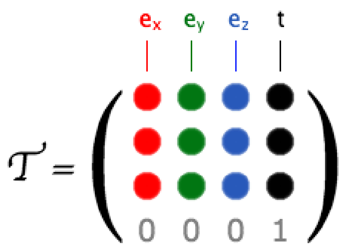
Inverse Kinematics - end-effector to origin

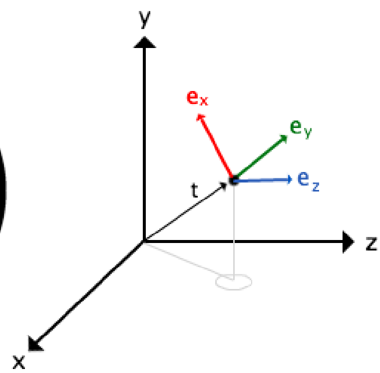
Homogeneous Transformation Matrix (HTM): If you ever wondered what matrices are used for, one of the applications is , to represent the forward kinematic chain of robots. Unlike the way it sounds , HTM is just a 4X4 matrix representing the position and orientation of a point in space wrt to some global frame of reference(mostly origin).

- **Homogeneous transformations** combine rotation and displacement into a single transformation matrix:

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{\begin{matrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{matrix}}_{\text{This does the rotation}} & \overbrace{\begin{matrix} d_x \\ d_y \\ d_z \end{matrix}}^{\text{This does the displacement}} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$





$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

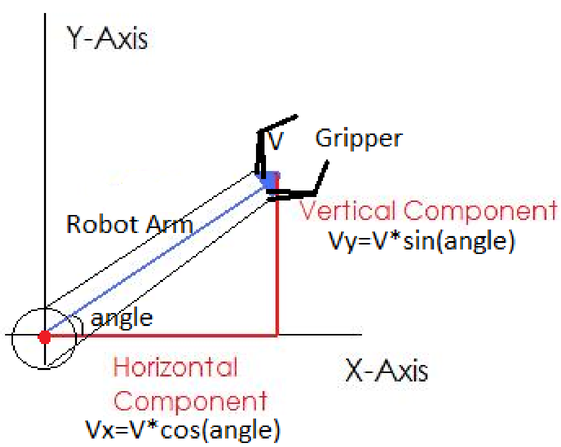
$$R_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We will focus more on **Analytical Inverse Kinematics** as it can quickly get you started with your robotics project.

1. One R Robotic arm Forward and Inverse Kinematics
2. Two R Robotic arm Forward and Inverse Kinematics
3. Three R Robotic arm Forward and Inverse Kinematics
4. Four R Robotic arm Forward and Inverse Kinematics

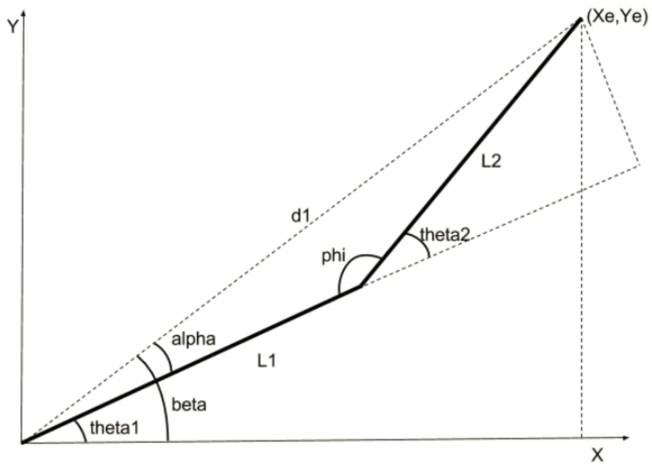
1. One R Robotic Arm



Forward Kinematics for 1R robotic arm is given by...
Vx = V*cos(angle)
Vy = V*sin(angle)

Inverse Kinematics for 1R robotic arm is given by...
angle = acos(Vx/V) **OR** angle = asin(Vy/V)
[acos - cos inverse , asin -sin inverse]

2. Two R Planar Robotic Arm



Forward Kinematics for 2R robotic arm is given by...

$$Xe = L1 \times cos(theta1) + L2 \times cos(theta1 + theta2)$$

$$Ye = L1 \times sin(theta1) + L2 \times sin(theta1 + theta2)$$

Inverse Kinematics for 2R robotic arm is given by...

$d1 = \sqrt{Xe^2 + Ye^2}$ [pythagoras theorem]

$$theta2 = 180 - phi$$

$d1^2 = L1^2 + L2^2 + 2 \times L1^2 \times L2^2 \times cos(phi)$ [cosine rule]

$$D = cos(phi) = (L1^2 + L2^2 - d1^2)/(2 \times L1 \times L2) = (L1^2 + L2^2 - (Xe^2 + Ye^2))/(2 \times L1 \times L2)$$

$$phi = atan2(D \pm \sqrt{1 + D^2})$$

$$theta2 = 180 - atan2(D \pm \sqrt{1 + D^2})$$

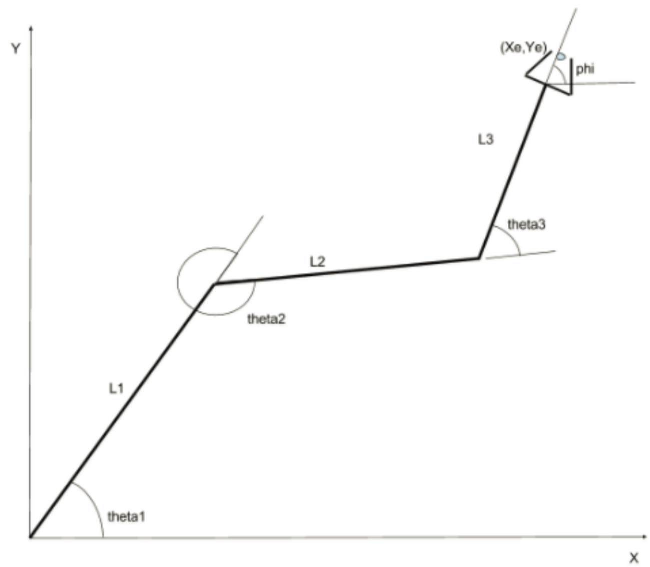
$$theta1 = beta - alpha$$

$$beta = atan2(Xe, Ye)$$

$$alpha = atan2(L1 + L2 \times cos(theta2), L2 \times sin(theta2))$$

$$theta1 = beta - alpha$$

3. Three R Planar Robotic Arm



Forward Kinematics for 3R robotic arm is given by...

$$Xe = L1 \times cos(theta1) + L2 \times cos(theta1 + theta2) + L3 \times cos(theta1 + theta2 + theta3)$$

$$Ye = L1 \times sin(theta1) + L2 \times sin(theta1 + theta2) + L3 \times sin(theta1 + theta2 + theta3)$$

$$phi = theta1 + theta2 + theta3$$

Inverse Kinematics for 3R robotic arm is given by...

$$Xe' = Xe - L3 \times cos(phi) = L1 \times cos(theta1) + L2 \times cos(theta1 + theta2)$$

$$Ye' = Ye - L3 \times sin(phi) = L1 \times sin(theta) + L2 \times sin(theta1 + theta2)$$

$$Xe' - L1 \times cos(theta1) = L2 \times cos(theta1 + theta2)$$

$$Ye' - L1 \times sin(theta1) = L2 \times sin(theta1 + theta2))$$

Squaring and adding both the terms

$$(-2 \times Xe' \times L1) \times cos(theta1) + (-2 \times Ye' \times L1) \times sin(theta2) + Xe'^2 + Ye'^2 + L1^2 - L2^2 = 0$$

$$P \times cos(alpha) + Q \times sin(alpha) + R = 0$$

$$theta1 = gamma + sigma \times cos^{-1}\left(\frac{-(Xe'^2 + Ye'^2 + L1^2 - L2^2)}{(2 \times L1 \times \sqrt{Xe'^2 + Ye'^2})}\right)$$

$$gamma = atan2\left(\left(\frac{-Ye'^2}{\sqrt{Xe'^2 + Ye'^2}}\right), \left(\frac{-Xe'^2}{\sqrt{Xe'^2 + Ye'^2}}\right)\right)$$

$$sigma = \pm 1$$

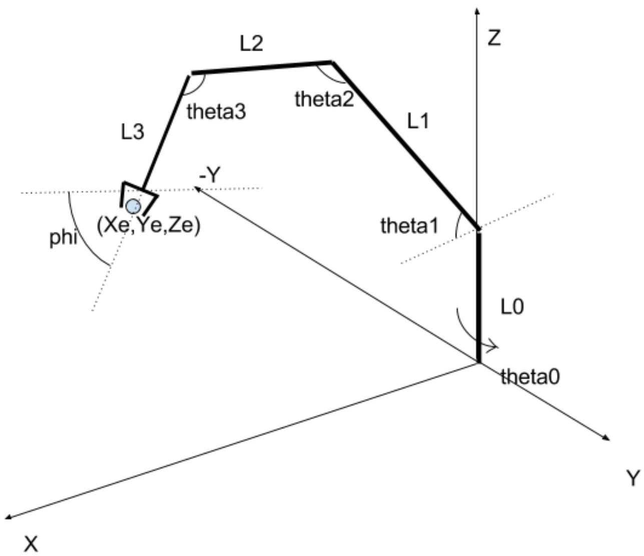
$$theta2 = atan2\left(\frac{Ye' - L1 \times sin(theta1)}{L2}, \frac{Xe' - L1 \times cos(theta1)}{L2}\right) - theta1$$

$$theta3 = phi - (theta1 + theta2)$$

You can plug in the hi lighted equations to get started with 3R IK.

4. Four R Robotic Arm

This configuration is not planar, hence its convenient to use HTM to represent this configuration



Forward Kinematics for 4R robotic arm is given by...

Let RX(theta), RY(theta), RZ(theta) respectively be

```
[ 1,          0,           0, 0]
[ 0, cos(theta), -sin(theta), 0]
[ 0, sin(theta),  cos(theta), 0]
[ 0,          0,           0, 1]
```

```
[  cos(theta), 0, sin(theta), 0]
[           0, 1,          0, 0]
[ -sin(theta), 0, cos(theta), 0]
[           0, 0,          0, 1]
```

```
[ cos(theta), -sin(theta), 0, 0]
[ sin(theta),  cos(theta), 0, 0]
[          0,           0, 1, 0]
[          0,           0, 0, 1]
```

Let T(X,Y,Z) be

```
[ 1, 0, 0, X]
[ 0, 1, 0, Y]
[ 0, 0, 1, Z]
[ 0, 0, 0, 1]
```

F1 = RZ(theta0)
F2 = F1 * T(0,0,L0)
F3 = F2 * RY(theta1) * T(0,0,L1)
F4 = F3 * RY(theta2) * T(0,0,L2)
F5 = F4 * RY(theta3) * T(0,0,L3)

F5 will contain the end-effector orientation and position

Inverse Kinematics for 4R robotic arm is given by...

The inverse kinematics can be solved separately in sagittal plane(X-Z plane) and transverse plane(X-Y plane)

In X-Z plane the solution is similar to 3R planar robotic arm. So the entire 3R solution explained in the previous section is applicable here,for input values of x position and z position.However, when the input y is given , the 3R solution changes. Intuitively as theta 0 is changed X position of the arm reduces, and needs to be compensated.

This compensation is given by

$$Xe' = Xe - L3 \times cos(phi) + \sqrt{Xe^2 + Ze^2} - Xe$$

to calculate angle theta0

$$theta0 = atan2(Ye, Xe)$$