# Beamforming Transmission in Cognitive AF Relay Networks with Feedback Delay

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***Abstract*—In this paper, we investigate the effect of feedback delay on outage probability (OP) and symbol error rate (SER) of cognitive amplify-and-forward (AF) relay networks with beamforming transmission in Rayleigh fading environment. It is assumed that the secondary transmitter and the secondary receiver are equipped with multiple antennas, whereas the relay and the primary user comprise of a single antenna. Furthermore, in this system, the secondary users use the underlay scheme in which the secondary transmitter and the relay can coexist with the primary user as long as their interference caused to the primary receiver is below a predefined threshold. Through our analysis, numerical results are provided to show the level of degradation of cognitive AF relay systems in proportion to the degree of feedback delay. In addition, we also present the effect of the number of antennas at the secondary transceiver on the performance of the considered system. Along with analytical results, we adopt Monte Carlo simulations to confirm the correctness of the theoretical analysis.**

***Index Terms*—Amplify-and-forward, beamforming, feedback delay, outage probability, symbol error rate.**

## I. Introduction

Along with emerging research on conventional cooperative technology, recently, cognitive radio (CR) technology has attracted great attention due to its opportunistic spectrum access. The combination of CR and cooperative diversity techniques, analyzed in [1], [2], and [3], is a promising approach not only to improve utilization of scarce radio frequency spectrum but also to achieve connectivity and guarantee a certain quality of service for radio applications. In particular, the works of [1] and [3] give a brief overview of techniques for a cognitive wireless relay network. In [4], the outage probability (OP) of cognitive relay networks with operation of secondary users based on the underlay scheme was investigated. Moreover, [5] presented several suboptimal distributed transmit power allocation schemes for relay-assisted cognitive radio networks. Nevertheless, all works mentioned above have not accounted for feedback delay. Therefore, in [6], the impact of feedback delay on the performance of dual-hop amplify-and-forward (AF) relay networks with beamforming over Rayleigh fading channels has been investigated. However, to the best of our knowledge, there is no previous work considering a cognitive AF relay network with beamforming transmission in the presence of feedback delay.

In this paper, we investigate the effect of feedback delay for underlay cognitive AF relay networks with beamforming transmission. In particular, we derive a closed-form expression for the OP and a tight approximation for the symbol error rate (SER) of the considered network in the Rayleigh fading environment. It is noted that considering the feedback delay from the secondary transmitter to the relay makes our analysis more complicated as compared to [4]. However, the impact of feedback delay should be accounted for in the considered systems since it is unavoidable in many practical cases. Finally, to verify the theoretical analysis, numerical results are given showing close agreement with Monte-Carlo simulations.

*Notation:* In this paper, we use the following notations. The bold lower case letter and the bold upper case letter denote vector and matrix, respectively. The Frobenius norm of a vector or matrix is represented as $\|\cdot\|_F$. Next, the transpose and the transpose conjugate of a vector or matrix are indicated by the superscript T and H, respectively. Furthermore, the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable $X$ are denoted as $f_X(\cdot)$ and $F_X(\cdot)$, respectively. Then, $\mathcal{CN}(\mu,\sigma^2)$ represents a complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. For brevity, we also use $\Omega = \sigma^2$. In addition, $\Gamma(n)$ is the gamma function [7, eq. (8.310.1)] and $\Gamma(n,x)$ is the incomplete gamma function defined in [7, eq. (8.350.2)]. Then, $\mathcal{K}_n(\cdot)$ denotes the $n$-th order modified Bessel function of the second kind [7, eq. (8.432.1)], and $\mathcal{W}_{\lambda,\mu}(x)$ represents the Whittaker function [7, eq. (9.222)]. Finally, $\Psi(a,b;x)$ is the confluent hypergeometric function [7, eq. (9.211.4)], and ${}_2\mathcal{F}_1(a,b;c;x)$ is the Gauss hypergeometric function defined in [7, eq. (9.111)].

## II. System and Channel Model

Consider a cognitive beamforming AF relay network under feedback delay in the Rayleigh fading environment as shown in Fig. 1. In this system, there exists a secondary transmitter, $\mathrm{SU_{TX}}$, which communicates with a secondary receiver, $\mathrm{SU_{RX}}$, through the help of a relay, $\mathrm{SU_{AF}}$, without any direct link from

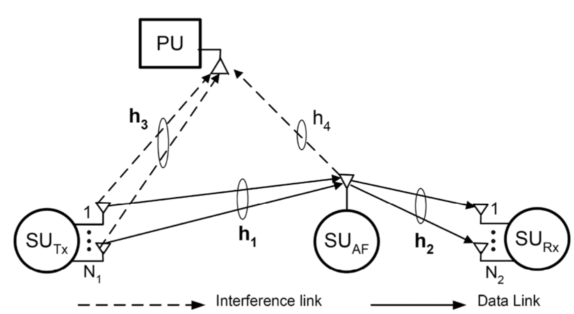


Fig. 1. System model for cognitive beamforming amplify-and-forward relay networks under feedback delay.

the $\mathrm{SU_{TX}}$ to the $\mathrm{SU_{RX}}$. Assume that the $\mathrm{SU_{TX}}$ and the $\mathrm{SU_{RX}}$ are equipped with $N_1$ and $N_2$ antennas, respectively, whereas the $\mathrm{SU_{AF}}$ has a single antenna. Moreover, there is only a single primary received user, PU, with a single antenna. Supposing that the $\mathrm{SU_{TX}}$ and the $\mathrm{SU_{AF}}$ use the underlay scheme which means that the $\mathrm{SU_{TX}}$ and the $\mathrm{SU_{AF}}$ can take advantage of the PU's frequency band as long as their interference incurred to the PU is below a predefined threshold, $Q$. At the $\mathrm{SU_{TX}}$, we deploy beamforming transmission with a beamforming vector, $\mathbf{w}_1(t)$. Therefore, the received signal at the $\mathrm{SU_{AF}}$ is written as

$$y_R(t) = \mathbf{h}_1(t)\mathbf{w}_1^{\mathrm{H}}(t)s(t) + n_1(t) \tag{1}$$

where $s(t)$ is the transmitted signal with average power per symbol, $P_1$, and $\mathbf{h}_1(t)$ is the $1 \times N_1$ Rayleigh channel coefficient vector of the link from the $\mathrm{SU_{TX}}$ to the $\mathrm{SU_{AF}}$ with independently and identically distributed (i.i.d.) complex Gaussian random variable (RV) entries defined as $\mathcal{CN}(0, \Omega_1)$. Further, $n_1(t)$ is the complex additive white Gaussian noise (AWGN) at the $\mathrm{SU_{AF}}$, denoted as $\mathcal{CN}(0, N_0)$. According to the principles of maximal ratio transmission, $\mathbf{w}_1(t)$ is chosen to be $\mathbf{w}_1(t) = \frac{\mathbf{h}_1(t-\tau)}{\|\mathbf{h}_1(t-\tau)\|_F}$ where $\tau$ is the feedback delay. As in [8] and [9], the relationship between $\mathbf{h}_1(t)$ and $\mathbf{h}_1(t-\tau)$ is expressed following the time-varying channel model

$$\mathbf{h}_1(t) = \rho\mathbf{h}_1(t-\tau) + \sqrt{1-(|\rho|)^2}\mathbf{e}(t) \tag{2}$$

where $\rho$ stands for the normalized correlation coefficient between $h_{1j}(t)$ and $h_{1j}(t-\tau)$ with $j = 1, \ldots, N_1$. For Clarke's fading spectrum, $\rho = J_0(2\pi f_d \tau)$, where $J_0(\cdot)$ is the 0th-order Bessel function of the first kind and $f_d$ is the Doppler frequency. Here, $\mathbf{e}(t)$ is the $1 \times N_1$ error vector. In case having no error in estimation $\mathbf{h}_1(t)$, $\mathbf{e}(t)$ is exactly the same as $\mathbf{h}_1(t)$ whose elements follow the complex Gaussian distribution, $\mathcal{CN}(0, \Omega_1)$.

At the second hop, the received signal at the $\mathrm{SU_{AF}}$ is amplified with a gain $G$ and then transmitted to the $\mathrm{SU_{RX}}$. Denoting the average symbol energy available at the $\mathrm{SU_{AF}}$ as $P_2$, the gain $G$ must satisfy

$$G^2 = \frac{P_2}{P_1|\mathbf{h}_1(t)\mathbf{w}_1^{\mathrm{H}}(t)|^2} \tag{3}$$

Consequently, the received signal at the $\mathrm{SU_{RX}}$ is given by

$$\mathbf{y}_D^{\mathrm{T}}(t) = G\mathbf{h}_2^{\mathrm{T}}(t)[\mathbf{h}_1(t)\mathbf{w}_1^{\mathrm{H}}(t)s(t) + n_1(t)] + \mathbf{n}_2^{\mathrm{T}}(t) \tag{4}$$

where $\mathbf{n}_2(t)$ is the $1 \times N_2$ AWGN vector at the $\mathrm{SU_{RX}}$, and $\mathbf{h}_2(t)$ is the $1 \times N_2$ Rayleigh channel coefficient vector of the link from the $\mathrm{SU_{AF}}$ to the $\mathrm{SU_{RX}}$ whose elements are i.i.d. complex Gaussian RVs $\mathcal{CN}(0, \Omega_2)$. Let us denote $\mathbf{h}_3(t)$ as the $1 \times N_1$ Rayleigh channel coefficient vector of the link from the $\mathrm{SU_{TX}}$ to the PU with all i.i.d. complex Gaussian RV entries denoted as $\mathcal{CN}(0, \Omega_3)$, and $h_4(t)$ as the channel coefficient of the link from the $\mathrm{SU_{AF}}$ to the PU whose distribution is $\mathcal{CN}(0, \Omega_4)$. To prevent PU from being interfered beyond an acceptable level $Q$, the average transmit power $P_1$ and $P_2$ must satisfy

$$P_1 = \frac{Q}{\|\mathbf{h}_3(t)\|_F^2}, \; P_2 = \frac{Q}{|\mathrm{h}_4(t)|^2} \tag{5}$$

As a result, the gain $G$ is formulated as

$$G^2 = \frac{\|\mathbf{h}_3(t)\|_F^2}{|\mathrm{h}_4(t)|^2|\mathbf{h}_1(t)\mathbf{w}_1^H(t)|^2} \tag{6}$$

At the $\mathrm{SU_{RX}}$, beamforming processing is applied by multiplying the received signal $\mathbf{y}_D^{\mathrm{T}}(t)$ with vector $\mathbf{w}_2$, i.e. the beamforming vector defined as $\mathbf{w}_2(t) = \frac{\mathbf{h}_2(t)}{\|\mathbf{h}_2(t)\|_F}$. Consequently, the output signal of the considered system is obtained as

$$\begin{aligned}\hat{y}_D(t) &= G\mathbf{w}_2(t)\mathbf{h}_2^{\mathrm{T}}(t)\mathbf{h}_1(t)\mathbf{w}_1^{\mathrm{H}}(t)s(t) \\ &\quad + G\mathbf{w}_2(t)\mathbf{h}_2^{\mathrm{T}}(t)n_1(t) + \mathbf{w}_2(t)\mathbf{n}_2^{\mathrm{T}}(t)\end{aligned} \tag{7}$$

From (7), after some manipulations, we can express the instantaneous SNR at the $\mathrm{SU_{RX}}$, $\gamma_D$, as

$$\gamma_D = \eta\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4 + \gamma_2\gamma_3} = \eta\gamma \tag{8}$$

where $\gamma_1 = |\mathbf{h}_1(t)\mathbf{w}_1^{\mathrm{H}}(t)|^2$, $\gamma_k = \|\mathbf{h}_k(t)\|_F^2, \; k = 2, \; 3$, $\gamma_4 = |\mathrm{h}_4(t)|^2$, $\eta = \frac{Q}{N_0}$, and $\gamma = \frac{\gamma_1\gamma_2}{(\gamma_1\gamma_4+\gamma_2\gamma_3)}$. Now, we derive the statistical properties of the RVs $\gamma_l$ with $l = 1, \ldots, 4$. As in [6, eq. (9)] and [6], the PDF of $\gamma_1$ is given by

$$\begin{aligned}f_{\gamma_1}(\gamma_1) &= \sum_{i=0}^{N_1-1}\frac{C_i^{N_1-1}}{\Omega_1^{N_1-i}}\frac{|\rho|^{2(N_1-i-1)}}{(N_1-i-1)!} \\ &\quad \times (1-|\rho|^2)^i\gamma_1^{N_1-i-1}e^{-\frac{\gamma_1}{\Omega_1}}\end{aligned} \tag{9}$$

where $C_i^{N_1-1} = \frac{(N_1-1)!}{i!(N_1-1-i)!}$. To obtain the CDF of $\gamma_1$, we integrate $f_{\gamma_1}(x)$ with respect to variable $x$ over the interval $(0, \; \gamma_1)$. By using [7, eq. (8.350 .2)] to solve the related integral and [7, eq. (8.352.2)] to expand the incomplete gamma function to finite summations, we obtain the CDF of $\gamma_1$ as

$$\begin{aligned}F_{\gamma_1}(\gamma_1) &= 1 - \sum_{i=0}^{N_1-1}C_i^{N_1-1}|\rho|^{2(N_1-i-1)} \\ &\quad \times (1-|\rho|^2)^i\sum_{p=0}^{N_1-i-1}\frac{\gamma_1^p}{p!\Omega_1^p}e^{-\frac{\gamma_1}{\Omega_1}}\end{aligned} \tag{10}$$

Because $\gamma_k, \; k = 2, \; 3$, is the summation of $N_i$, ($i = 2$ if $k = 2$, and $i = 1$ if $k = 3$) i.i.d. complex Gaussian RVs $\mathcal{CN}(0, \Omega_k)$, its statistical distribution is expressed as

$$f_{\gamma_k}(\gamma_k) = \frac{\gamma_k^{N_j-1}}{\Omega_k^{N_j}\Gamma(N_j)}e^{-\frac{\gamma_k}{\Omega_k}} \tag{11}$$

$$F_{\gamma_k}(\gamma_k) = 1 - \sum_{t=0}^{N_j-1}\frac{\gamma_k^t}{t!\Omega_k^t}e^{-\frac{\gamma_k}{\Omega_k}} \tag{12}$$

Finally, since $\gamma_4$ is the Rayleigh channel coefficient with the distribution $\mathcal{CN}(0, \Omega_4)$, its PDF is given by

$$f_{\gamma_4}(\gamma_4) = \frac{1}{\Omega_4}e^{-\frac{\gamma_4}{\Omega_4}} \tag{13}$$

## III. End-to-End Performance Analysis

In this section, we first derive a closed-form expression for the CDF of the instantaneous SNR, $\gamma_D$, enabling us to obtain an exact closed-form expression for the OP of the cognitve AF relay system. In addition, we derive an upper bound on the CDF of $\gamma_D$ to obtain a tight approximation for the average SER of the considered system.

### A. Exact Outage Performance Analysis

OP is defined as the probability that instantaneous SNR $\gamma_D$ drops below a specified threshold $\gamma_{th}$, i.e.

$$P_{out} = Pr\{\gamma_D < \gamma_{th}\} = F_\gamma\left(\frac{\gamma_{th}}{\eta}\right) \tag{14}$$

where $F_\gamma(\gamma)$ is given by

$$F_\gamma(\gamma) = Pr\left\{\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4 + \gamma_2\gamma_3} < \gamma\right\} \tag{15}$$

Due to the statistical independence of the RVs $\gamma_l$, $l \in \{1,\ldots,4\}$, the CDF of $\gamma$ can be determined as

$$F_\gamma(\gamma) = \int_0^\infty\int_0^\infty\int_0^\infty Pr\{\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4+\gamma_2\gamma_3} < \gamma|\gamma_2,\gamma_3,\gamma_4\} \times f_{\gamma_2}(\gamma_2)f_{\gamma_3}(\gamma_3)f_{\gamma_4}(\gamma_4)d\gamma_2 d\gamma_3 d\gamma_4 \tag{16}$$

To calculate $F_\gamma(\gamma)$, we adopt a common method known for other combinations of RVs such as $AB/(A+B)$ as in [10] to the more complicated relationship of RVs given in (8). Accordingly, we separate the integration domain of $\gamma_2$ into two subsets as $\gamma_2 < \gamma\gamma_4$ and $\gamma_2 > \gamma\gamma_4$. After some manipulations, we obtain $F_\gamma(\gamma)$ as

$$F_\gamma(\gamma) = \underbrace{\int_0^\infty\left(\int_0^{\gamma\gamma_4} f_{\gamma_2}(\gamma_2)d\gamma_2\right)f_{\gamma_4}(\gamma_4)d\gamma_4\int_0^\infty f_{\gamma_3}(\gamma_3)d\gamma_3}_{P} + \underbrace{\int_0^\infty\left(\int_0^\infty\left(\int_{\gamma\gamma_4}^\infty F_{\gamma_1}(\varphi)f_{\gamma_2}(\gamma_2)d\gamma_2\right)f_{\gamma_3}(\gamma_3)d\gamma_3\right)f_{\gamma_4}(\gamma_4)d\gamma_4}_{I} \tag{17}$$

where $\varphi = \frac{\gamma\gamma_2\gamma_3}{\gamma_2-\gamma\gamma_4}$. It is realized that $P$ can be simplified as

$$P = \int_0^\infty F_{\gamma_2}(\gamma\gamma_4)f_{\gamma_4}(\gamma_4)d\gamma_4 \tag{18}$$

For brevity, let us denote $I_1$ and $I_2$, respectively, as the deepest and the middle integral of $I$ given as

$$I_1 = \int_0^\infty F_{\gamma_1}\left(\gamma\gamma_3 + \frac{\gamma^2\gamma_3\gamma_4}{\gamma_2}\right)f_{\gamma_2}(\gamma_2+\gamma\gamma_4)d\gamma_2 \tag{19}$$

$$I_2 = \int_0^\infty I_1 f_{\gamma_3}(\gamma_3)d\gamma_3 \tag{20}$$

Then, we can rewrite $I$ as

$$I = \int_0^\infty I_2 f_{\gamma_4}(\gamma_4)d\gamma_4 \tag{21}$$

In order to calculate $I$, we need to compute $I_1$ and $I_2$. Firstly, we calculate the deepest integral $I_1$. By substituting (10) and (11) into (19), then applying the binomial theorem in [7, eq. (1.111)], i.e., $(a+b)^n = \sum_0^n C_k^n x^k y^{n-k}$, and performing some manipulations, we transform $I_1$ into the following form

$$\begin{aligned} I_1 &= 1 - F_{\gamma_2}(\gamma\gamma_4) - \sum_{i=0}^{N_1-1} C_i^{N_1-1}|\rho|^{2(N_1-i-1)}(1-|\rho|^2)^i \\ &\times \sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\sum_{r=0}^{N_2-1}\frac{C_r^{N_2-1}}{\Gamma(N_2)}\frac{\gamma_3^p\gamma_4^{N_2+q-r-1}}{\Omega_1^p\Omega_2^{N_2}}e^{-\gamma(\frac{\gamma_4}{\Omega_2}+\frac{\gamma_3}{\Omega_1})} \\ &\times \gamma^{N_2+p+q-r-1}\int_0^\infty \gamma_2^{r-q}e^{-\frac{\gamma^2\gamma_3\gamma_4}{\Omega_1\gamma_2}-\frac{\gamma_2}{\Omega_2}}d\gamma_2 \end{aligned} \tag{22}$$

The remaining integral in (22) is solved by applying [7, eq. (3.471 .9)] together with rearranging terms, which yields the closed-form expression for $I_1$ as

$$\begin{aligned} I_1 &= 1 - F_{\gamma_2}(\gamma\gamma_4) - 2\sum_{i=0}^{N_1-1} C_i^{N_1-1}|\rho|^{2(N_1-i-1)}(1-|\rho|^2)^i \\ &\times \sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\sum_{r=0}^{N_2-1}\frac{C_r^{N_2-1}}{\Gamma(N_2)}\frac{\gamma_3^{\frac{2p+r-q+1}{2}}\gamma_4^{\frac{2N_2+q-r-1}{2}}}{\Omega_1^{\frac{2p+r-q+1}{2}}\Omega_2^{\frac{2N_2-r+q-1}{2}}} \\ &\times \gamma^{N_2+p}e^{-(\frac{\gamma\gamma_4}{\Omega_2}+\frac{\gamma\gamma_3}{\Omega_1})}K_{r-q+1}\left(2\sqrt{\frac{\gamma^2\gamma_4\gamma_3}{\Omega_1\Omega_2}}\right) \end{aligned} \tag{23}$$

Secondly, we calculate the middle integral $I_2$. By substituting (23) and (11) into (20), after some manipulations, we rewrite the expression for $I_2$ as

$$\begin{aligned} I_2 &= 1 - F_{\gamma_2}(\gamma\gamma_4) - 2\sum_{i=0}^{N_1-1}\frac{C_i^{N_1-1}}{\Gamma(N_1)}|\rho|^{2(N_1-i-1)}(1-|\rho|^2)^i \\ &\times \sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\sum_{r=0}^{N_2-1}\frac{C_r^{N_2-1}}{\Gamma(N_2)}\frac{\gamma_4^{\frac{2N_2+q-r-1}{2}}\gamma^{N_2+p}}{\Omega_1^{\frac{2p+r-q+1}{2}}\Omega_2^{\frac{2N_2-r+q-1}{2}}\Omega_3^{N_1}} \\ &\times \int_0^\infty \gamma_3^{\frac{2N_1+2p+r-q-1}{2}}e^{-\frac{(\gamma\Omega_3+\Omega_1)\gamma_3}{\Omega_1\Omega_3}}K_{r-q+1}\left(2\sqrt{\frac{\gamma^2\gamma_4\gamma_3}{\Omega_1\Omega_2}}\right)d\gamma_3 \end{aligned} \tag{24}$$

By utilizing [7, eq. (6.643.3)] to calculate the integral in (24), $I_2$ is formulated as

$$\begin{aligned} I_2 &= 1 - F_{\gamma_2}(\gamma\gamma_4) - \sum_{i=0}^{N_1-1} C_i^{N_1-1}|\rho|^{2(N_1-i-1)}(1-|\rho|^2)^i \\ &\times \sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\sum_{r=0}^{N_2-1}C_r^{N_2-1}\frac{\Gamma(N_1+p+r-q+1)}{\Gamma(N_1)\Gamma(N_2)} \\ &\times \Gamma(N_1+p)\frac{\Omega_1^{N_1}\Omega_3^{\frac{(2p+r-q)}{2}}}{\Omega_2^{\frac{2N_2-r+q-2}{2}}}\frac{\gamma_4^{\frac{2N_2-r+q-2}{2}}\gamma^{N_2+p-1}}{(\gamma\Omega_3+\Omega_1)^{\frac{(2N_1+2p+r-q)}{2}}} \\ &\times e^{-\frac{(\Omega_3\gamma^2+2\Omega_1\gamma)\gamma_4}{2\Omega_2\Omega_3\gamma+2\Omega_1\Omega_2}}W_{-\frac{2N_1+2p+r-q}{2},\frac{r-q+1}{2}}\left(\frac{\Omega_3\gamma^2\gamma_4}{\Omega_2\Omega_3\gamma+\Omega_1\Omega_2}\right) \end{aligned} \tag{25}$$

Now, we are ready to calculate the outer integral $I$. By substituting (25) and (13) into (21), and then performing some algebraic manipulations, we can rewrite $I$ as

$$
\begin{aligned}
I &= 1 - P - \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \\
&\times \sum_{p=0}^{N_1-i-1} \sum_{q=0}^{p} \frac{C_q^p}{p!} \sum_{r=0}^{N_2-1} C_r^{N_2-1} \frac{\Omega_1^{N_1} \Omega_3^{\frac{2p+r-q}{2}}}{\Omega_2^{\frac{2N_2-r+q-2}{2}} \Omega_4} \\
&\times \frac{\Gamma(N_1+p)\Gamma(N_1+p+r-q+1)\gamma^{N_2+p-1}}{\Gamma(N_2)\Gamma(N_1)(\gamma\Omega_3+\Omega_1)^{\frac{2N_1+2p+r-q}{2}}} \\
&\times \int_0^\infty \gamma_4^{\frac{2N_2-r+q-2}{2}} e^{-\frac{(\gamma^2\Omega_3\Omega_4+2\gamma\Omega_1\Omega_4+2\gamma\Omega_2\Omega_3+2\Omega_1\Omega_2)\gamma_4}{2\gamma\Omega_2\Omega_3\Omega_4+2\Omega_1\Omega_2\Omega_4}} \\
&\times W_{-\frac{2N_1+2p+r-q}{2}, \frac{r-q+1}{2}} \left( \frac{\Omega_3 \gamma^2 \gamma_4}{\gamma\Omega_2\Omega_3+\Omega_1\Omega_2} \right) d\gamma_4 \qquad (26)
\end{aligned}
$$

Applying [7, eq. (7.621 .3)] to solve the integral in (26), then performing some simplifications, and reorganizing the order of terms, we attain the equation for $I$. Substituting this outcome into (17), we obtain the closed-form expression for $F_\gamma(\gamma)$ as in (27). Finally, substituting (27) into (14), we obtain a closed-form expression for the OP.

*B. Symbol Error Rate Performance Analysis*

For many modulation formats, the SER is given directly in terms of the CDF of the instantaneous SNR $\gamma_D$ as in [11]

$$
P_E = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-b\gamma_D}}{\sqrt{\gamma_D}} F_{\gamma_D}(\gamma_D) d\gamma_D \qquad (28)
$$

where $a$, $b$ are modulation parameters determined by modulation schemes. For $M$-ary phase shift keying, $M$-PSK, we have $a = 2,\ b = \sin(\pi/M)^2$; and for $M$-ary pulse amplitude modulation, $M$-PAM, we have $a = 2(M-1)/M,\ b = 3/(M^2-1)$. It is clear that using the exact expression of $F_\gamma(\gamma)$ in (27) to calculate the SER is intractable, so we use another approach. In particular, first, we derive a tightly bounded expression for $\gamma_D$, and then we apply this outcome to obtain an approximation for the SER. In the high SNR regime, we can approximate $\gamma$ as $\gamma \approx \gamma_U = \min(\gamma_{U1}, \gamma_{U2})$ where $\gamma_{U1} = \gamma_2/\gamma_4$ and $\gamma_{U2} = \gamma_1/\gamma_3$. Hence, the CDF of $\gamma_U$ is given by

$$
F_{\gamma_U}(\gamma_U) = 1 - [1 - F_{\gamma_{U1}}(\gamma_U)][(1 - F_{\gamma_{U2}}(\gamma_U)] \qquad (29)
$$

where $F_{\gamma_{U1}}(\gamma_U)$ and $F_{\gamma_{U2}}(\gamma_U)$ are given by

$$
F_{\gamma_{U1}}(\gamma_U) = \int_0^\infty F_{\gamma_2}(\gamma_U\gamma_4) f_{\gamma_4}(\gamma_4) d\gamma_4 \qquad (30)
$$

$$
F_{\gamma_{U2}}(\gamma_U) = \int_0^\infty F_{\gamma_1}(\gamma_U\gamma_3) f_{\gamma_3}(\gamma_3) d\gamma_3 \qquad (31)
$$

Substituting (12), (13) into (30), and (10), (11) into (31), together with the help of [7, eq. (3.381.4)], we obtain the CDF of $\gamma_{U1}$ and $\gamma_{U2}$ as

$$
F_{\gamma_{U1}}(\gamma_U) = 1 - \sum_{t=0}^{N_2-1} \frac{\Omega_2 \Omega_4^t \gamma_U^t}{(\gamma_U\Omega_4+\Omega_2)^{(t+1)}} \qquad (32)
$$

$$
\begin{aligned}
F_{\gamma_{U2}}(\gamma_U) &= 1 - \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \\
&\times \sum_{p=0}^{N_1-i-1} \frac{1}{p!} \frac{\Gamma(N_1+p)}{\Gamma(N_1)} \frac{\Omega_1^{N_1} \Omega_3^p \gamma_U^p}{(\gamma_U\Omega_3+\Omega_1)^{N_1+p}} \qquad (33)
\end{aligned}
$$

By substituting (32) and (33) into (29), we finally obtain the CDF of $\gamma_U$ as

$$
\begin{aligned}
F_{\gamma_U}(\gamma_U) &= 1 - \sum_{t=0}^{N_2-1} \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \\
&\times \sum_{p=0}^{N_1-i-1} \frac{1}{p!} \frac{\Gamma(N_1+p)}{\Gamma(N_1)} \frac{\Omega_1^{N_1} \Omega_3^p \Omega_2 \Omega_4^t \gamma_U^{t+p}}{(\gamma_U\Omega_4+\Omega_2)^{(t+1)} (\gamma_U\Omega_3+\Omega_1)^{N_1+p}} \qquad (34)
\end{aligned}
$$

Due to $\gamma_D \approx \eta\gamma_U$, the expression for SER in (28) can be rewritten in term of $\gamma_U$ as

$$
P_E = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty F_{\gamma_U}\left(\frac{\gamma_U}{\eta}\right) \gamma_U^{-\frac{1}{2}} e^{-b\gamma_U} d\gamma_U \qquad (35)
$$

By changing variable $\gamma_5 = \gamma_U/\eta$ for the integral of (35), and then substituting (34) into (35), we derive the expression for SER as

$$
\begin{aligned}
P_E &= \frac{a\sqrt{b\eta}}{2\sqrt{\pi}} \int_0^\infty \gamma_5^{-\frac{1}{2}} e^{-b\eta\gamma_5} d\gamma_5 - \frac{a\sqrt{b\eta}}{2\sqrt{\pi}} \sum_{t=0}^{N_2-1} \sum_{i=0}^{N_1-1} \\
&\times C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \sum_{p=0}^{N_1-i-1} \frac{\Gamma(N_1+p)}{p!\Gamma(N_1)} \Omega_1^{N_1} \\
&\times \Omega_2 \Omega_3^p \Omega_4^t \int_0^\infty \frac{\gamma_5^{\frac{2t+2p-1}{2}} e^{-b\eta\gamma_5}}{(\gamma_5\Omega_4+\Omega_2)^{(t+1)} (\gamma_5\Omega_3+\Omega_1)^{N_1+p}} d\gamma_5 \quad (36)
\end{aligned}
$$

Applying [7, eq. (3.361.2)] to calculate the first integral, and then using partial fraction in [7, eq. (2.102)] to expand the polynomial in the second integral of (36) into finite sums, we rewrite the expression for SER as

$$
\begin{aligned}
P_E &= \frac{a}{2} - \frac{a\sqrt{b\eta}}{2\sqrt{\pi}} \sum_{t=0}^{N_2-1} \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \\
&\times \sum_{p=0}^{N_1-i-1} \frac{1}{p!} \frac{\Gamma(N_1+p)}{\Gamma(N_1)} \frac{\Omega_1^{N_1} \Omega_2}{\Omega_3^{N_1} \Omega_4} \Bigg[ \sum_{l=1}^{t+1} \kappa_{tl} \int_0^\infty \frac{\gamma_5^{\frac{2t+2p-1}{2}}}{(\gamma_5+\Omega_2\Omega_4^{-1})^l} \\
&\times e^{-b\eta\gamma_5} d\gamma_5 + \sum_{j=1}^{N_1+p} \theta_{ij} \int_0^\infty \frac{\gamma_5^{\frac{2t+2p-1}{2}} e^{-b\eta\gamma_5}}{(\gamma_5+\Omega_1\Omega_3^{-1})^j} d\gamma_5 \Bigg] \qquad (37)
\end{aligned}
$$

$$F_{\gamma}(\gamma) = 1 - \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \sum_{p=0}^{N_1-i-1} \frac{1}{p!} \sum_{q=0}^{p} C_q^p \sum_{r=0}^{N_2-1} C_r^{N_2-1} \Omega_1^{N_1} \Omega_2 \Omega_3^{r-q+p+1} \Omega_4^{N_2}$$
$$\times \frac{\Gamma(N_1+p)\Gamma(N_1+p+r-q+1)\Gamma(N_2+1)\Gamma(N_2-r+q)}{\Gamma(N_1)\Gamma(N_2)\Gamma(N_2+N_1+p+1)} \frac{\gamma^{N_2+p+r-q+1}}{(\gamma\Omega_3+\Omega_1)^{N_1+p+r-q+1}(\gamma\Omega_4+\Omega_2)^{(N_2+1)}}$$
$$\times {}_2\mathcal{F}_1\left(N_2+1, N_1+p+r-q+1, N_2+N_1+p+1; \frac{\gamma\Omega_1\Omega_4+\gamma\Omega_2\Omega_3+\Omega_1\Omega_2}{\gamma^2\Omega_3\Omega_4+\gamma\Omega_1\Omega_4+\gamma\Omega_2\Omega_3+\Omega_1\Omega_2}\right) \tag{27}$$

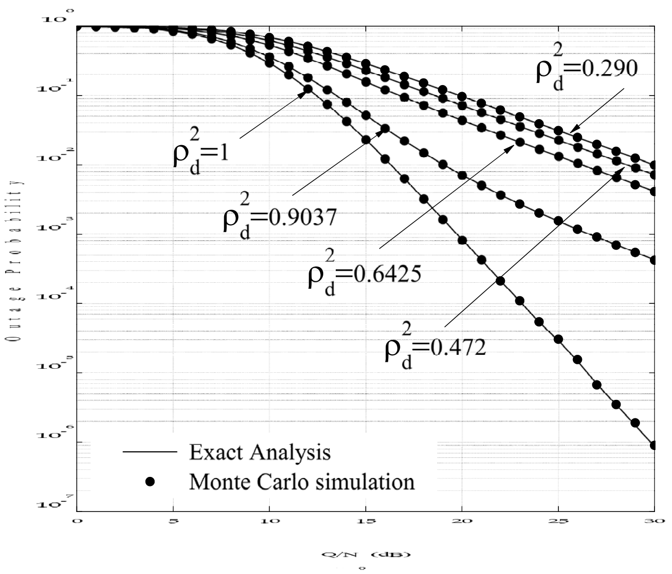


Fig. 2. Impact of different normalized correlation coefficient channels on the OP of cognitive amplify-and-forward relay networks in Rayleigh fading.

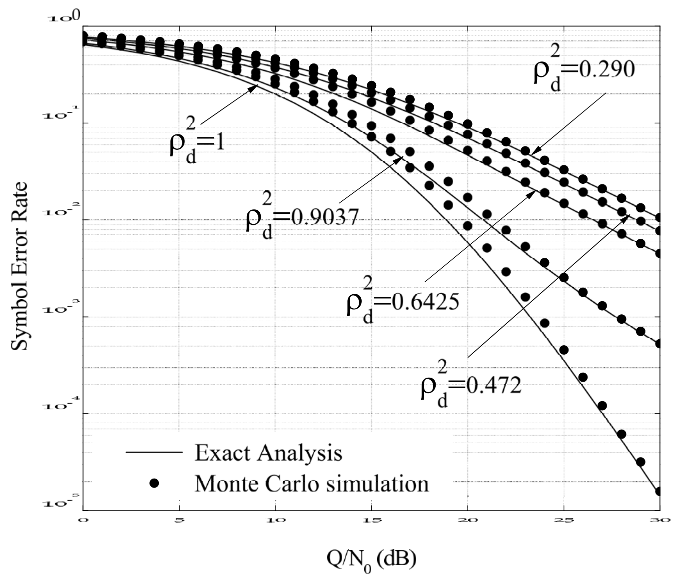


Fig. 3. Impact of different normalized correlation coefficient channels on the symbol error rate of 8-PSK of cognitive amplify-and-forward relay networks in Rayleigh fading.

$$\kappa_{tl} = \frac{1}{(t+1-l)!} \frac{d^{t+1-l}}{d\gamma_5^{t+1-l}} \left[\frac{1}{(\gamma_5+\Omega_2\Omega_4^{-1})^{N_1+p}}\right]\Bigg|_{\gamma_5=-\frac{\Omega_2}{\Omega_4}} \tag{38}$$

$$\theta_{ij} = \frac{1}{(N_1+p-j)!} \frac{d^{N_1+p-j}}{d\gamma_5^{N_1+p-j}} \left[\frac{1}{(\gamma_5+\Omega_1\Omega_3^{-1})^{t+1}}\right]\Bigg|_{\gamma_5=-\frac{\Omega_1}{\Omega_3}} \tag{39}$$

By using [12, eq. (2.3.6.9)] to calculate the two integrals in (37), we finally obtain a tight approximation for the SER as in (40).

## IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we verify the correctness of the analytical results through comparison with Monte Carlo simulations. After that we discuss the impact of feedback delay and the number of antennas at the transceiver on the system performance. For all illustrations, we examine the considered system with the channel mean powers given as $\Omega_1 = 0.8,\ \Omega_2 = 0.6,\ \Omega_3 = 1,\ \Omega_4 = 1.2$. The SNR threshold is selected as $\gamma_{th} = 5\,\text{dB}$.

Fig. 2 and Fig. 3 depict the impact of feedback delay on OP and average SER performance for 8-PSK modulation of a cognitive AF relay network in Rayleigh fading environment. In particular, we analyze four cases of Jakes fading spectrum, $\rho = 0.9037, 0.6425, 0.472, 0.290$, corresponding to, $f_dT_d = 0.1, 0.2, 0.25, 0.3$, respectively, with $N_1 = N_2 = 3$. For reference, we also illustrate the OP and SER when there is no feedback delay $\rho = 1$. As expected, a high feedback delay, i.e. small value of $\rho$, considerably degrades system performance while degradation reduces with increasing $\rho$. Hence, the best performance is achieved when the normalized correlation coefficient $\rho$ approaches to one. Although feedback delay is inevitable in some practical cases, it should be kept at a very low level in order to not compromise the system performance.

Fig. 4 and Fig. 5 present the OP and SER for various antenna configurations versus average SNR. As we can see from these two figures, a large number of antennas at the transceiver significantly improves the OP and the SER. Thus, deployment of multiple antennas is a suitable way to improve the system performance. Furthermore, the analytical result for the OP closely agrees with the Monte Carlo simulation. Also, the approximation result in the high SNR regime of SER in (40) tightly converges to the Monte-Carlo simulation which validates our analysis results.

$$P_E = \frac{a}{2} - \frac{a\sqrt{b\eta}}{2\sqrt{\pi}} \sum_{t=0}^{N_2-1} \sum_{i=0}^{N_1-1} C_i^{N_1-1} |\rho|^{2(N_1-i-1)} (1-|\rho|^2)^i \sum_{p=0}^{N_1-i-1} \frac{\Gamma(N_1+p)}{p!\Gamma(N_1)} \frac{\Omega_1^{N_1}\Omega_2}{\Omega_3^{N_1}\Omega_4} \Gamma\left(t+p+\frac{1}{2}\right) \left[\sum_{l=1}^{t+1} \kappa_{tl} \left(\frac{\Omega_2}{\Omega_4}\right)^{t+p-l+\frac{1}{2}} \right.$$
$$\left. \times \Psi\left(t+p+\frac{1}{2}, t+p-l+\frac{3}{2}; \frac{b\eta\Omega_2}{\Omega_4}\right) + \sum_{j=1}^{N_1+p} \theta_{ij} \left(\frac{\Omega_1}{\Omega_3}\right)^{t+p-j+\frac{1}{2}} \Psi\left(t+p+\frac{1}{2}, t+p-j+\frac{3}{2}; \frac{b\eta\Omega_1}{\Omega_3}\right)\right] \quad (40)$$

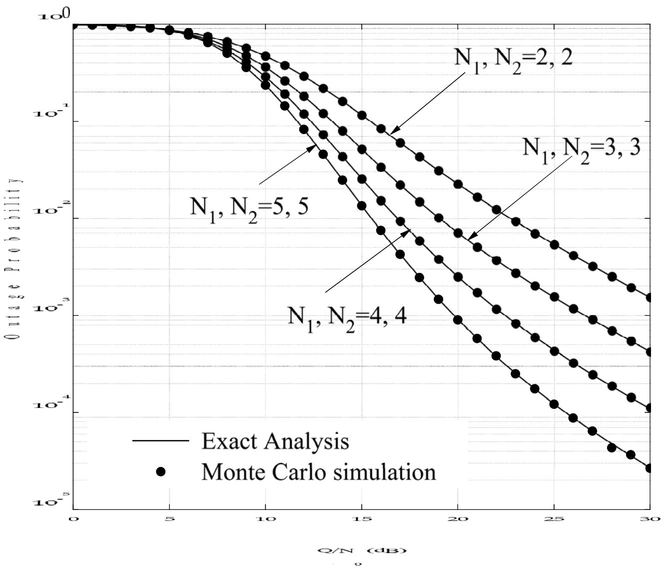


Fig. 4. Outage probability for various antenna configurations on cognitive amplify-and-forward relay networks in Rayleigh fading.

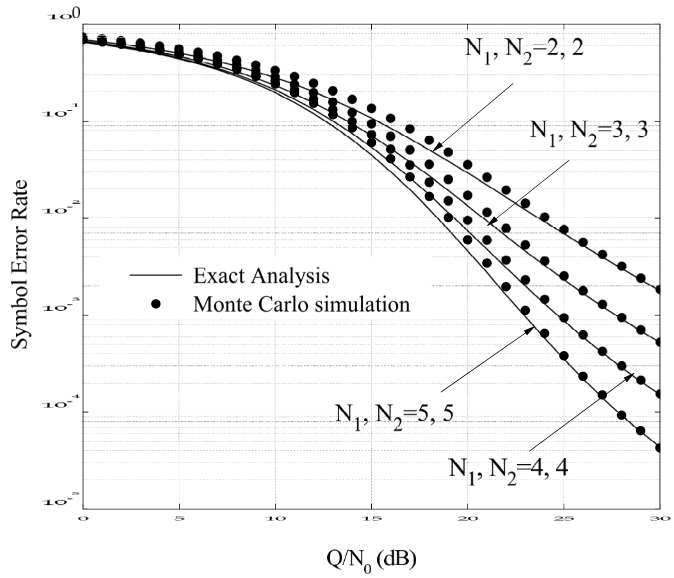


Fig. 5. Symbol error rate of 8-PSK for various antenna configurations on cognitive amplify-and-forward relay networks in Rayleigh fading.

## V. CONCLUSIONS

In this paper, we investigated the effect of feedback delay on the performance of cognitive AF relay networks in Rayleigh fading environment. In particular, we derived an exact closed-form expression for the OP and an approximation for the SER of the considered cognitive relay network with beamforming. System parameters were selected to illustrate clearly the impact of feedback delay as well as the number of antennas at the transceiver on the system performance. The numerical results obtained show that feedback delay severely decreases the system performance. However, we can reduce the effect of feedback delay by increasing the number of antennas at the $SU_{TX}$ and the $SU_{RX}$. Finally, we presented numerical results and compared them with Monte-Carlo simulations. The excellent agreement between the simulations and analytical results validates our analysis for the considered scenarios.

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