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Finding time-robust fuel-efficient paths for a call-taxi in a stochastic city road network

T. Godwin[1]*, Karthik Sajeev[2] and Albert C. George[2]

[1] *Indian Institute of Management Tiruchirappalli, Tiruchirappalli, Tamil Nadu 620015, India*
[2] *Department of Production Engineering, National Institute of Technology Tiruchirappalli, Tiruchirappalli, Tamil Nadu 620015, India*

SUMMARY

Intra-city commuting is being revolutionized by call-taxi services in many developing countries such as India. A customer requests a taxi via phone, and it arrives at the right time and at the right location for the pick-up. This mode of intra-city travel has become one of the most reliable and convenient modes of transportation for customers traveling for business and non-business purposes. The increased number of vehicles on city roads and raising fuel costs has prompted a new type of transportation logistics problem of finding a fuel-efficient and quickest path for a call-taxi through a city road network, where the travel times are stochastic. The stochastic travel time of the road network is induced by obstacles such as the traffic signals and intersections. The delay and additional fuel consumption at each of these obstacles are calculated that are later imputed to the total travel time and fuel consumption of a path. A Monte-Carlo simulation-based approach is proposed to identify unique fuel-efficient paths between two locations in a city road network where each obstacle has a delay distribution. A multi-criteria score is then assigned to each unique path based on the probability that the path is fuel efficient, the average travel time of the path and the coefficient of variation of the travel times of the path. Copyright © 2016 John Wiley & Sons, Ltd.

KEY WORDS: call-taxi; fuel-efficient path; time-robust path; Monte-Carlo simulation; multi-criteria score

1. INTRODUCTION

In urban cities of developing nations such as India, personal transportation within the city, both for business and leisure, has increased greatly during the last decade. For example, in India, where the intra-city personal transportation was mainly restricted to own vehicles and hired auto-rickshaws until a decade back, a new form of hired transportation called the call-taxi has emerged. This coincided with the tremendous growth of the mobile phone users, which makes it even easier for customers to book a taxi. A customer requiring a call-taxi makes a call or requests through a mobile app to a service provider to book it, and one of the call-taxis is assigned to the customer. This call-taxi need not always be present at the customer's pick-up location and often has to travel through the city road network to reach the customer pick-up location. The call-taxi picks up the customer, travels through the city road network again to reach customer drop-off location, and then becomes available for next customer assignment. Reasonable fare, timely service, and 24-hour service are some of the reasons quoted for the popularity and success of this type of service [1].

The ability of a taxi to reach a pick-up location before the stipulated time and then reach drop-off location at the earliest time is crucial especially if the customer has to catch a train, bus, or flight. It should be noted that the emphasis here is not driving fast as it is not at all safe and feasible in a crowded city network but to intelligently identify a route that helps in reaching a location in a relatively faster time while driving within the speed limits in a city. The travel time in a less congested city is

*Correspondence to: T. Godwin, Indian Institute of Management Tiruchirappalli, Tiruchirappalli, Tamil Nadu 620015, India. E-mail: godwin@iimtrichy.ac.in

highly predictable and is mostly directly proportional to distance. This is not true in more congested larger cities of developing countries such as India where the travel time for the movement of vehicle through it is unpredictable. The absence of strict speed restriction enforcements within cities has forced the use of barricades and road bumps to reduce speed of vehicles on road. In addition, the roads have numerous intersections - both with and without traffic signals that induce delays to the vehicles passing through them. These delays are also a function of the day of the week as well as the time of the day as there is a variation in traffic level as well as the duration of “stop” and “go” traffic signals.

Finding the shortest time route in a congested city road network that also minimizes fuel consumption is what all call-taxi drivers would like to achieve. The stochastic nature of transit time through this network makes it challenging to find the best path for a call-taxi that seeks to optimize travel time as well as fuel consumption. The aim of this paper is to come up with a methodology that helps in finding time-robust fuel-efficient path for a call-taxi in a city road network.

2. RELATED LITERATURE

Taxis have been around for many decades in many countries but the type of service provided to customers did vary. The differences in regulations of taxis by market segments as well as different taxi service types were studied by Aarhaug and Skollerud [2]. Dial [3] developed a fully automated dial-a-ride system where the tasks of assigning trips and routing the vehicle are fully automated. Jung *et al*. [4] studied dynamic shared-taxi system and developed three algorithms for taxi dispatching. Simulation of two taxi operation strategies revealed the benefit of shared rides at various demand levels. Maciejewski and Bischoff [5] used floating car data to simulate taxi services on a large scale and developed two heuristics that dynamically dispatches taxis within the simulation. Lammoglia et al. [6] compared two types of taxi services through an agent-based simulation that uses optimization models to capture their cooperation and flexibility in responding to dynamic service requests. Salanova *et al*. [7] reviewed models developed for taxicab problems from a market organization, operational organization and regulation issues perspectives. Salanova *et al*. [8] reviewed formulations to estimate various metrics such as fleet size, generalized cost and optimum fleet of taxi services in urban regions. The variation of taxi fleet size and their passenger demand can influence the customer-search behavior of vacant taxis. A logit model to predict such behavior was proposed by Wong *et al*. [9]. A similar modeling approach was used by Wang *et al*. [10] to predict the passenger service choice behavior between shuttle bus and taxi service for the last mile travel. In a city network, the points of demand for taxi service need not always be the points of supply of vacant taxis, and hence, there is always a movement of both taxis and customers to the equilibrium points where they meet. The movement of vacant and occupied taxis in search of customers was modeled by Yang and Wong [11] and found that taxi utilization level is correlated with the customer waiting time. Kim *et al*. [12] analyzed the effect of passenger travel demand on the quality of service rendered by taxis using an agent based simulation model. The study showed the importance of having symmetric travel demand pattern for having a higher service quality for passengers and taxi drivers. Grau and Romeu [13] conducted a simulation study of taxi services in urban area using an agent-based model that considers temporal and spatial dimensions of supply and demand. In a congested road network, the shortest distance path need not be shortest time path. The variation in travel time of a taxi in a city road network greatly influences the scheduling decisions. Xiang *et al*. [14] developed an approach for scheduling dial-a-ride system in a city road network with time dependent and stochastic delays. Schilde *et al*. [15] presented metaheuristic based solution approaches for dynamic dial-a-ride problem in a road network with stochastic and time dependent travel speeds.

A prerequisite to implement a conventional shortest path algorithm in a road network is that the arc attributes are known and does not change. Finding the path through a stochastic road network where the arc attribute is not deterministic is challenging. *Patire et al*. [16] investigated on the amount of GPS data required to estimate accurate travel times and speeds of a vehicle. Ehmke *et al*. [17] used a database of historical floating car data to determine time-dependent travel time at various levels of aggregation in a city road network. Zhan *et al*. [18] used origin–destination (O-D) trip data of taxis to estimate link travel times. Unlike link travel times, the route travel time distribution was estimated by Rahmani *et al*. [19] based on floating car data. The arc attribute in such networks could be considered as a random variable, which could follow a statistical distribution. In such networks, predicting the arrival time at destination could be a challenging task. Azaron and Kianfar [20] and Peer and Sharma [21] developed solutions for the shortest path problem in a

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stochastic network where the arc attributes are exponentially distributed. Cheng and Lisser [22] and Kosuch and Lisser [23] modeled a stochastic network where the arc attributes followed normal distribution. A stochastic network where arc costs are deterministic while the delay on each arc is a random variable was modeled by Cheng *et al.* [24], and a semi-definite programming relaxation approach was developed to find the shortest path. Lin [25] addressed the problem of finding the quickest path in a stochastic network where the travel time on each arc is probabilistic. Evolution and heuristic algorithms to find the shortest path in a stochastic network were developed by Siddiqi *et al.* [26], Farhanchi *et al.* [27], and Fu and Rilett [28]. Ji *et al.* [29] developed a hybrid intelligent algorithm that integrates genetic algorithm and simulation to solve the shortest path problem with fuzzy arc lengths. Nielsen *et al.* [30] and Pan *et al.* [31] studied and developed solutions for finding the shortest path in a stochastic network where travel times on the arcs are time dependent. A review of literature on time dependent routing problems is presented in Gendreau *et al.* [32]

The raising fuel cost has made researchers focus on finding the path through a network that minimizes fuel consumption. Finding the fuel-efficient path helps sustain a business and makes it environmentally friendly. A review of operations research literature that contributed towards green logistics including reduced fuel consumption was presented by Dekker *et al.* [33] Prahara *et al.* [34] developed a model to measure instantaneous fuel consumption. Yao and Song [35] developed fuel consumption and vehicle emission models and proposed an eco-route planning algorithm. Mensing *et al.* [36] studied the trade-off between fuel saving and reduced emission. Kluge *et al.* [37] found that energy-optimal paths differ from fast paths and on an average have 10% lower energy consumption. Catay and Yildirim [38] observed that the quickest path need not be the fuel-efficient path and proposed an algorithm to find the fuel-efficient path on a time-dependent road network. Ben-Chaim *et al.* [39] described an analytical approach for finding vehicle fuel consumption under standard operating conditions. Kang *et al.* [40] developed a fuel consumption model for high-speed highways based on highway geometric characteristics, speed, and road surface condition. A detailed study on the vehicle fuel consumption calculation was made by Ferreira [41], reporting idle speed fuel consumption, constant speed fuel consumption, fuel consumption during acceleration, and fuel consumption during deceleration. The study found that most passenger vehicles achieve highest fuel efficiency between 40 and 60 km/hour based on engine size. Similar recent studies indicate that vehicles attain maximum fuel efficiency if driven in speeds between 50 and 80 km/hour [42] and it emits minimal emission between 40 and 80 km/hour [35]. The driving characteristics of a vehicle, which include the rate of acceleration and speed of driving, greatly influence the fuel consumption. Rouwendal [43] analyzed the impact of technical characteristics of cars and socio-economic characteristics of the drivers on fuel efficiency of private cars. A data-based Bayesian approach to estimate fuel consumption of vehicle was presented by Suzdaleva and Nagy [44] and demonstrated the importance of driving at recommended speed to reduce fuel consumption. Researchers have explored on developing driver assistance system to achieve fuel efficiency. A visual-auditory system for drivers to use accelerator pedal fuel-efficiently was developed by Jamson *et al.* [45]. Jamson *et al.* [46] used a driving simulator to investigate the rate of learning of eco-driving, where steady driving is important for fuel-efficient driving. A context-aware driving assist system was prototyped by Gilman *et al.* [47] Zhao *et al.* [48] developed eco-driving feedback system and reported idling for a long time as one of the contributing factors for non-eco-driving. Car navigation system is also evolving as a driver assist system for reduction in fuel consumption. Shokri *et al.* [49] presented a navigation system to find path in a road network that minimized fuel consumption and observed that a shortest distance route need not be the most fuel efficient route. Motavalli [50] reported the launch of a new navigation system by Ford that has three route options: quickest route, shortest distance route, and the fuel efficient “Eco-Route”. The system obtains real-time traffic information to compute the fuel efficient route.

The increased competition between call-taxi service providers and raising fuel costs necessitates finding paths for call-taxis that are fuel-efficient as well as quick. Hamacher *et al.* [51] developed an algorithm for bi-criteria shortest path problem with time-dependent data. Opasanon and Miller-Hooks [52] developed an exact algorithm to find Pareto-optimal paths in a stochastic network where the arc attributes are time varying. Mohamed *et al.* [53] proposed a solution procedure to find pareto optimal set using genetic algorithm for shortest path problem with dual criteria of minimizing transportation cost and travel time. An oriented spanning tree based genetic algorithm to find pareto optimal solutions for multi-criteria shortest path problems was developed by Liu et al.. [54] Siddiqi et al. [55] developed

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an evolutionary algorithm that identified single most dominant solution as well as achieved a pre-defined number of non-dominant solutions for multi-objective shortest path problem. An exact label-setting algorithm that identifies a set of pareto optimal paths based on lexicographic goals for multi-objective shortest path problem was developed by Pulido *et al*. [56] Duque *et al*. [57] developed an exact recursive approach for simultaneously minimizing two conflicting objectives while finding the shortest path in a network. A shortest time path need not be the fuel efficient path assuming that the travel time through the network is a function of vehicle speed and there are no speed restrictions. Gendreau *et al*. [32] emphasized the need for algorithms and models to be developed that exploits trade-off between travel time and fuel consumption. There is a strong correlation between minimum fuel consumption and fuel-efficient travel time that are achieved while driving the vehicle at fuel-efficient speed range. A highly congested path in a city road network with many obstacles will lead to higher fuel consumption and longer travel time. At the same time, traveling in a road network with minimal obstacles at high speed will lead to shorter travel time but higher fuel consumption as the travel speed would most likely be higher than the fuel-efficient speed range. We assume that the speed limit of vehicles in a city does not exceed the fuel-efficient speed range of call-taxis. If the vehicles in a city road network drive at their optimal speeds that maximize fuel efficiency, the shortest time path is likely to be the fuel efficient path. However, the obstacles in the road network delays the movement of vehicles, which in turn leads to increased fuel consumption.

The problem discussed in this paper supplements and complements the existing research by explicitly translating the delays in a city road network into fuel consumption while finding a fuel-efficient path. A methodology is developed that tries to minimize the deviation from the ideal fuel consumption of call-taxis for a given origin-destination pair by finding the time-robust fuel-efficient path in a city road network where the travel times are stochastic in nature. A Monte-Carlo simulation-based approach is proposed to find fuel-efficient paths for a call-taxi in a city road network where each obstacle has a delay distribution. A multi-criteria score is then assigned for each unique path based on the probability that the path is fuel-efficient, the average travel time of the path and the coefficient of variation of the travel times of the path. This is an extension of the work by Godwin [58], where the emphasis was only on finding the robust shortest time path. The solution approach proposed in this paper could be used for driver assist navigation system in call-taxis for finding time-robust fuel-efficient path.

The rest of the paper is organized as follows. Section 3 describes the problem characteristics, explaining the various obstacles in a city road network and how they translate into additional fuel consumption. Section 4 describes the proposed approaches for finding unique fuel-efficient paths between an origin and destination in a stochastic road network and the calculation of the multi-criteria score for these unique paths. Section 5 illustrates the proposed approach for a city road network. The main contributions of this study and scope for future work are summarized in section 6.

3. PROBLEM CHARACTERISTICS AND FUEL CONSUMPTION MODELING

A city road network can be viewed as an undirected graph $G=(V,A)$, where V represents the set of nodes and A represents the set of arcs. Most of the roads in a city road network are bidirectional, and hence, it is approximated to an undirected graph. The node equivalents in a city road network are the intersections where major roads meet and the arc equivalents are the major roads connecting these intersections. The travel time and fuel consumption along each arc $(i,j)\in A$ are of interest in this study. These attributes exhibit stochastic behavior because of the presence of various obstacles that slows down a vehicle.

NOTATIONS

i, j = index of nodes in the network
a_j^i = arc connecting nodes i and j
i_s = origin node for the call-taxi in the network
i_d = destination node for the call-taxi in the network

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d^{ij} = length of the arc a_j^i in meters (m)
s = call-taxi travel speed in meters per second (m/s)
f_e = average fuel consumption rate of the call-taxi measured in milliliter per second
f_e^{idle} = idle fuel consumption rate of the call-taxi measured in milliliter per second
$S_{ij}^{f_e}$ = ideal call-taxi travel speed in meters per second (m/s) that maximizes fuel economy
k = index of arc-intersections with signals
l = index of arc-intersections without signals
S_l^{ij} = speed in meters per second (m/s) at which a call-taxi passes through arc intersection without signal l on arc a_j^i
m = index of speed breakers
S_m^{ij} = speed in meters per second (m/s) at which a call-taxi passes through speed breaker m on arc a_j^i
α_i = actual delay in seconds (s) for the call-taxi at node intersection i
β_k^{ij} = actual delay in seconds (s) for the call-taxi at intersection k on arc a_j^i
γ_l^{ij} = actual delay in seconds (s) for the call-taxi at arc-intersection l on arc a_j^i
δ_m^{ij} = actual delay in seconds (s) for the call-taxi at speed breaker m on arc a_j^i
ρ = acceleration rate of a call-taxi in m/s^2 to accelerate from rest to the speed $S_{ij}^{f_e}$ m/second
χ = deceleration rate of a call-taxi in m/s^2 to decelerate from the speed $S_{ij}^{f_e}$ to 0 m/second
f_ρ = fuel consumed in milliliters (mL) while a call-taxi accelerates from rest to $S_{ij}^{f_e}$
f_χ = fuel consumed in milliliters (mL) while a call-taxi decelerates from $S_{ij}^{f_e}$ to rest
f_i = fuel consumed in milliliters (mL) at node-intersection i
f_k^{ij} = fuel consumed in milliliters (mL) at arc-intersection k on the arc a_j^i
f_l^{ij} = fuel consumed in milliliters (mL) at arc-intersection l on the arc a_j^i
f_m^{ij} = fuel consumed in milliliters (mL) at speed breaker m on the arc a_j^i
f^{ij} = fuel consumption in milliliters (mL) for a call-taxi along the arc a_j^i
t_ρ = time required by a call-taxi in seconds (s) to accelerate from rest to $S_{ij}^{f_e}$
t_χ = time required by a call-taxi in seconds (s) to decelerate from $S_{ij}^{f_e}$ to rest
t_i = time required by a call-taxi in seconds (s) to cross node-intersection i
t_k^{ij} = time required by a call-taxi in seconds (s) to cross arc-intersection k on the arc a_j^i
t_l^{ij} = time required by a call-taxi in seconds (s) to cross arc-intersection l on the arc a_j^i
t_m^{ij} = time required by a call-taxi in seconds (s) to cross speed breaker m on the arc a_j^i
t^{ij} = time required by a call-taxi in seconds (s) to cross the arc a_j^i
d_ρ = distance in meters (m) covered by a call-taxi while accelerating from rest to $S_{ij}^{f_e}$
d_χ = distance in meters (m) covered by a call-taxi while decelerating from $S_{ij}^{f_e}$ to rest
d_i = distance in meters (m) covered by a call-taxi corresponding to the time delay associated with crossing of node-intersection i
d_k^{ij} = distance in meters (m) covered by a call-taxi corresponding to the time delay associated with crossing of arc-intersection k on the arc a_j^i
d_l^{ij} = distance in meters (m) covered by a call-taxi corresponding to the time delay associated with crossing of arc-intersection l on the arc a_j^i
d_m^{ij} = distance in meters (m) covered by a call-taxi corresponding to the time delay associated with crossing of speed breaker m on the arc a_j^i

Consider an arc of length d_{ij} shown in Figure 1. A vehicle traveling at speed s_{ij} takes d_{ij}/s_{ij} amount of time to traverse through it. Free-flow fuel is the amount of fuel consumed when a vehicle is traveling at a speed $s = S_{ij}^{f_e}$ in the given route without any time delay en route. This is also known as fuel consumption at constant speed that excludes acceleration, deceleration, and slowdown. The value of $S_{ij}^{f_e}$ been estimated to be between 40 and 70 km/hour depending on vehicle engine size [34, 35, 41–43]. Any delay along the path would lead to additional fuel consumption and thus more emission.

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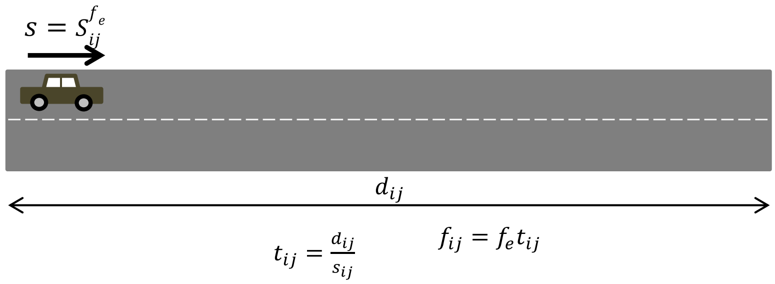

Figure 1. Free-flow path.

3.1. Obstacles in a city road network

Fuel-efficient path corresponds to the path that consumes the least amount of fuel and hence emits minimum pollutants. In reality, the fuel consumed through a city road network is more than the free-flow fuel because of the presence of various obstacles en route as shown in Figure 2. The obstacles include node-intersections with traffic signals, arc intersections with traffic signals, arc intersections without traffic signals, and speed breakers. The delays caused at arc intersections without traffic signals and speed breakers depends on road traffic intensity while the delays at node-intersections with traffic signals and arc intersections with traffic signals are a function of road traffic intensity and the duration of traffic signals. The traffic intensity and duration of traffic signal could vary by the time of the day as well as by the day of the week.

3.1.1. Node-intersection with traffic signals

A city road network consists of both major and minor roads where vehicles travel. Node-intersections are the meeting points of the major roads in a road network, and the nodes in V in the graph G represent them. This study considers the movement of call-taxis only on major roads where nodes in V serve as the pick-up and drop-off locations. This simplification could be justified by the fact that most of their travel happens through major roads and only the last mile pick-up and drop-off happen through minor roads. Node-intersections also provide an opportunity for a call-taxi to change route, and there could be more than one road diverging from it. The traffic signals present at these intersections make a vehicle idle for considerable amount of time. A vehicle waiting at this intersection for a green signal might not always clear the intersection at the first instance of green signal, that is, the signal could turn red before the vehicle actually crosses the intersection.

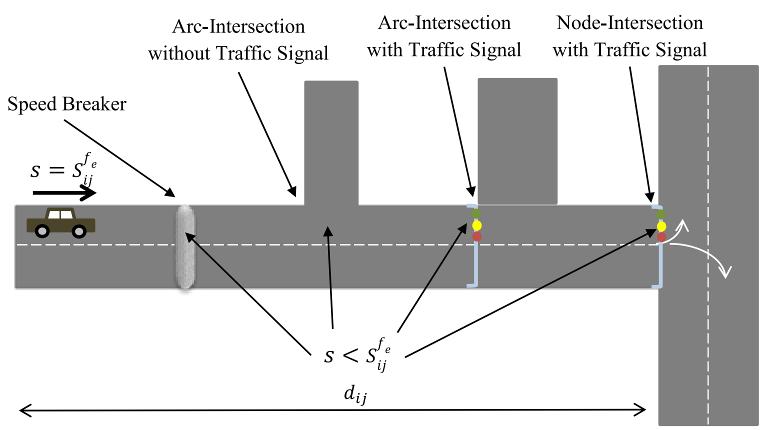

Figure 2. Obstacle in a city road network.

3.1.2. Arc-intersection with traffic signals

These are intersections on the arcs of the road network where minor roads meet or cross a major road. In addition, there could be pedestrian crossings as well. These intersections are present on the arcs in A of the road network graph G, and the traffic signals present at these intersections have a shorter duration compared with node-intersection with traffic signals. However, the same notion of a vehicle not clearing the intersection at the first instance of a green signal applies here as well. Hence, a vehicle spends considerable amount of time idling at this intersection.

3.1.3. Arc-intersection without traffic signals

These intersections are present on the arcs in A in the road network graph G and are similar to arc-intersections with traffic signals with the only difference of not having traffic signals. However, the crossing of vehicles and pedestrians at these intersections would cause delay and hence additional fuel consumption. A vehicle passing through this intersection might stop for a vehicle or pedestrian to cross or simply slow down. Hence, the deceleration and acceleration of a vehicle passing through this intersection considerably contribute to the additional fuel consumption.

3.1.4. Speed breaker

Speed breakers are the bumps or barricades that are present on the arcs in A in the road network graph G. They are located near schools, colleges, and hospitals to slow down a vehicle passing through them. A vehicle passing through a speed breaker seldom stops and just slows down to safely cross it. Hence, in most instances, the additional fuel consumption at this obstacle is caused by the deceleration and acceleration of a vehicle while passing through it.

3.2. Travel at the obstacles

A vehicle achieves its specified fuel economy f_e only when driven at speed S_{ij}^{fe}. The findings by Ferreira [41], Rouwendal [43], Prahara *et al.*, [34] Yao and Song [35], and Avoid High Speeds [42] indicate that a vehicle achieves minimal emission and maximum fuel economy at speeds between 40 and 70 km/hour based on the vehicle type. We consider an instance where all vehicles strive to drive at their ideal speed of S_{ij}^{fe} in a city road network. However, owing to the delay on the arcs and at nodes, there would be additional fuel consumption through the path, and the goal is to find the path that minimizes fuel consumption and travel time. There are four types of obstacles as described in Section 3.1 of the paper, of which, two of them are dependent on traffic signals. The delay time at these obstacles follows statistical distributions, and the actual delay could be sampled using one of the random variate generation techniques [59]. In addition, the deceleration of the call-taxi, the acceleration of the call-taxi, and the less than optimal travel speed of the call-taxi are microscopically modeled while arriving at the actual delay at an obstacle and the corresponding fuel consumption.

The delay at obstacles with traffic signal could be zero (if the signal is green) or the cycle time of all the signals in the intersection if it had turned red just at the arrival of call-taxi or somewhere in between. In addition, even if the signal is green, a call-taxi may not be able to clear it in that window because of congested traffic, and many have to additionally wait for another cycle before clearing that intersection. This particular aspect is not modeled at microscopic level as it would become very complex to capture it at network level. However, using the distribution of waiting time of a call-taxi at intersections with traffic signal is a good approximation of what would happen in real world, especially when finding the path between two locations in a city road network where there are multiple obstacles of various types along the path. In addition, the consideration of deceleration time if the call-taxi had to stop at a signal, the acceleration time when the call-taxi clears a signal, and the actual waiting time of the call-taxi at the signal in the approach developed in this section helps in simulating the real-world scenario at the network level.

We use the findings by Ferreira [41] to impute fuel consumptions during acceleration, deceleration, steady speed, and idling.

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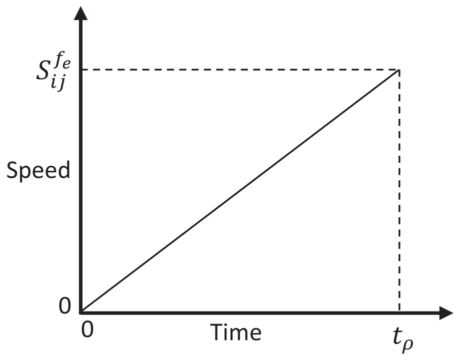

Figure 3. Call-taxi accelerating from rest to S_{tf}^{fe}.

3.2.1. *Travel characteristics of a call-taxi that starts from its origin node-intersection*

A call-taxi starting from its origin node-intersection i_s has to accelerate from rest to S_{ij}^{fe} (Figure 3) at an acceleration rate ρ.

The fuel-consumption of a call-taxi during acceleration [41] from its origin i_s is given by the following:

$$f_\rho = 0.13 S_{ij}^{fe^2} + 0.42 \frac{S_{ij}^{fe}}{\rho} \tag{1}$$

The time taken by the call-taxi to accelerate from rest to S_{ij}^{fe} is given by the following:

$$t_\rho = \frac{S_{ij}^{f_e}}{\rho} \tag{2}$$

The distance covered by the call-taxi while accelerating from rest to $S_{ij}^{f_e}$ is given by the following:

$$d_\rho = \frac{\rho t_\rho^{\,2}}{2} \tag{3}$$

3.2.2. *Travel characteristics of a call-taxi that reaches its destination node-intersection*

A call-taxi reaching its destination node-intersection i_d has to decelerate from $S_{ij}^{f_e}$ to rest (Figure 4) at a deceleration rate χ.

The fuel-consumption of a call-taxi during deceleration [41] to its destination i_d is given by the following:

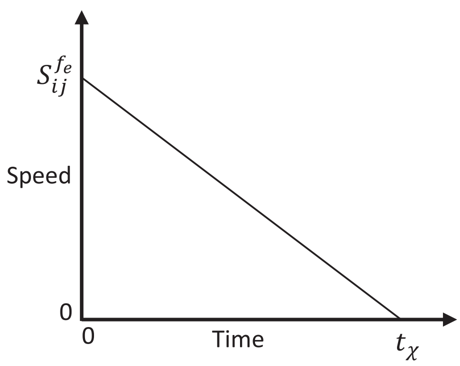

Figure 4. Call-taxi decelerating from S_{tf}^{fe} to rest.

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$$f_{\chi} = 0.537 \frac{S_{ij}^{fe}}{\chi} \tag{4}$$

The time taken by the call-taxi to decelerate from S_{ij}^{fe} to rest is given by the following:

$$t_{\chi} = \frac{S_{ij}^{fe}}{\chi} \tag{5}$$

The distance covered by the call-taxi while decelerating from S_{ij}^{fe} to rest is given by the following:

$$d_{\chi} = S_{ij}^{fe} t_{\chi} - \frac{\chi t_{\chi}^{2}}{2} \tag{6}$$

3.2.3. Travel characteristics of a call-taxi at node-intersections with traffic signals

The actual delay at this obstacle $\alpha_i > 0$ is a combination of idling and brief travel between successive red lights at a signal (assuming that a call-taxi does not always clear this obstacle at the first instance of green signal). Parida and Gangopadya [60] reported that about 98% of drivers do not turn off their vehicles while waiting at node-intersections for green (go) signal. A call-taxi has to decelerate from S_{ij}^{fe} to rest, idle for α_i, and then accelerate back to $S_{ij}^{f_e}$ (Figure 5).

Ferreira [41] reported the idle fuel consumption of the vehicles to be between 20.67 and 21.83 mL/minute. Parida and Gangopadya [60] listed the idling fuel consumption across 21 vehicle types (mostly cars), and it averages to 12.7 mL/minute. Neha *et al.* [61] reported average fuel consumption by vehicles during idling as 14 mL/minute. We use the parameter f_e^{idle} while calculating idle fuel consumption, which could always be substituted with the actual value based on the vehicle specification.

The results obtained in 1 and 4 in conjunction with f_e^{idle}, and α_i are used to impute the total fuel consumption for a call-taxi at a node intersection with traffic signals.

$$f_{i} = 0.537 \frac{S_{ij}^{fe}}{\chi} + \alpha_i f_e^{idle} + 0.13 S_{ij}^{fe^2} + 0.42 \frac{S_{ij}^{fe}}{\rho} \tag{7}$$

The total time taken by call-taxi to clear this node intersection with traffic signal is given by the following:

$$t_i = \frac{S_{ij}^{fe}}{\chi} + \alpha_i + \frac{S_{ij}^{fe}}{\rho} \tag{8}$$

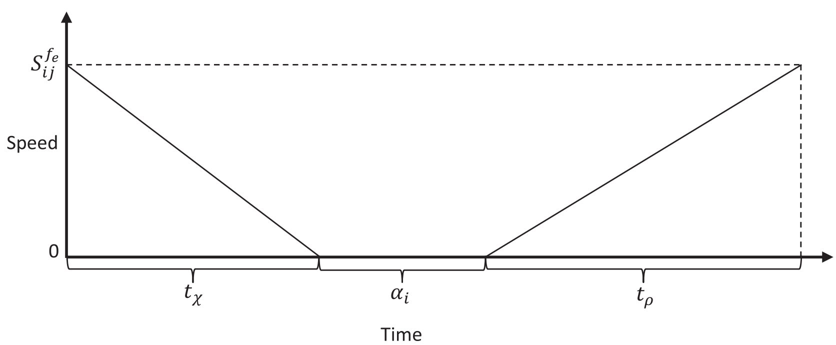

Figure 5. Call-taxi passing through node-intersection with traffic signal.

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The total distance covered by the call-taxi to decelerate from S_{ij}^{fe} to rest, idle for α_i, and then accelerate back to S_{ij}^{fe} is given by the following:

$$d_i = S_{ij}^{fe} t_\chi - \frac{\chi t_\chi{}^2}{2} + \frac{\rho t_\rho{}^2}{2} \tag{9}$$

3.2.4. Travel characteristics of a call-taxi at Arc-Intersections with Traffic Signals

The delay and fuel consumption characteristics at this obstacle are similar to the node-intersection with signal, where the delay $\beta_k^{ij} > 0$ approximates idling and brief travel between successive red lights at a signal (if a call-taxi does not always clear this obstacle at the first instance of green signal). A call-taxi has to decelerate from S_{ij}^{fe} to rest, idle for β_k^{ij}, and then accelerate back to S_{ij}^{fe} (Figure 6).

The results obtained 7, 8, and 9 are adapted to find the fuel consumption, travel time, and distance covered by a call-taxi at this obstacle.

The fuel consumed by a call-taxi while crossing an arc-intersection with traffic signal is given by the following:

$$f_k^{ij} = 0.537 \frac{S_{ij}^{fe}}{\chi} + \beta_k^{ij} f_e^{idle} + 0.13 S_{ij}^{fe2} + 0.42 \frac{S_{ij}^{fe}}{\rho} \tag{10}$$

The total time taken by call-taxi to clear this arc intersection with traffic signal is given by the following:

$$t_k^{ij} = \frac{S_{ij}^{fe}}{\chi} + \beta_k^{ij} + \frac{S_{ij}^{fe}}{\rho} \tag{11}$$

The total distance covered by the call-taxi to decelerate from S_{ij}^{fe} to rest, idle for β_k^{ij}, and then accelerate back to S_{ij}^{fe} is given by the following:

$$d_k^{ij} = S_{ij}^{fe} t_\chi - \frac{\chi t_\chi{}^2}{2} + \frac{\rho t_\rho{}^2}{2} \tag{12}$$

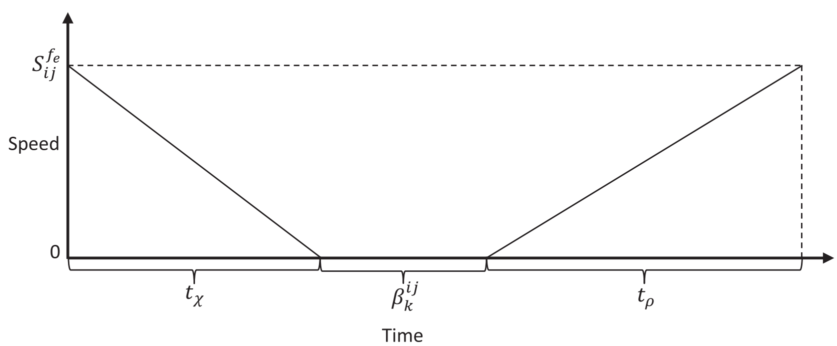

Figure 6. Call-taxi passing through arc-intersection with traffic signal.

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3.2.5. Travel characteristics of call-taxi at Arc-intersections without traffic signals

There are no traffic signals at this obstacle, and the delay $\gamma_l^{ij} \geq 0$ is associated with slowing down of a call-taxi because of the crossing of people and vehicles. A call-taxi decelerates to $0 \leq S_l^{ij} < S_{ij}^{f_e}$, travels at S_l^{ij}, idles for γ_l^{ij} duration, and then accelerates back to S_{ij}^{fe}. The four possible scenarios at arc-intersections without traffic signal are described in Figures 7–10.

Based on the results obtained in Sections 3.2.1–3.2.4 and basic travel kinematics equations, the fuel consumption for a call-taxi passing through an arc intersection without traffic signal is calculated under two scenarios: $S_l^{ij} = 0$ and $S_l^{ij} > 0$. These two scenarios symbolizes whether the call-taxi stopped or slowed down at this obstacle.

The fuel consumption when $S_l^{ij} = 0$ is given by the following:

$$f_l^{ij} = 0.537\frac{S_{ij}^{fe}}{\chi} + \gamma_l^{ij} f_e^{idle} + 0.13 S_{ij}^{fe^2} + 0.42\frac{S_{ij}^{fe}}{\rho} \tag{13}$$

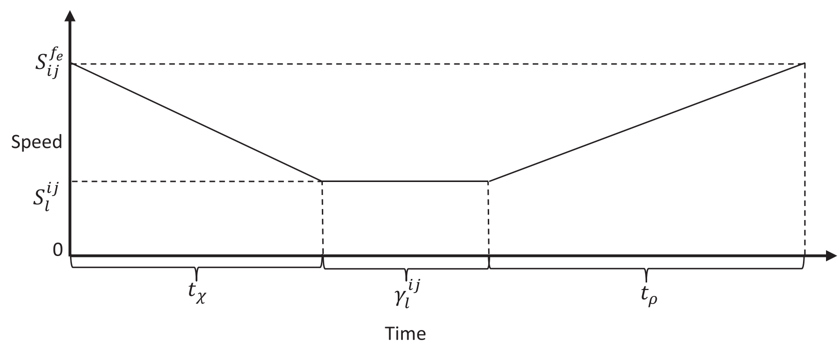

Figure 7. Call-taxi passing through arc-intersection without traffic signal where $S_t^{tf} > 0$ and $\gamma_t^{tf} > 0$.

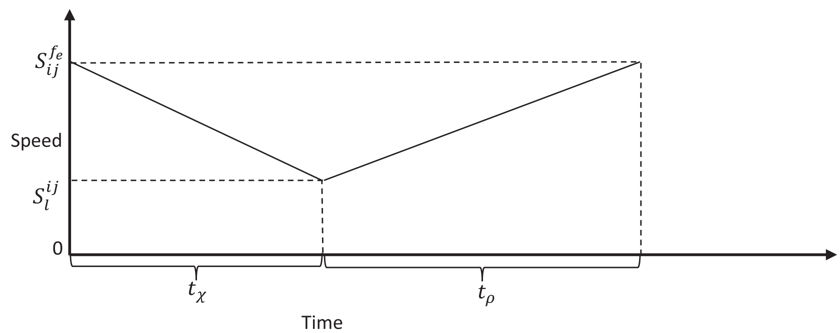

Figure 8. Call-taxi passing through arc-intersection without traffic signal where $S_t^{tf} > 0$ and $\gamma_t^{tf} = 0$.

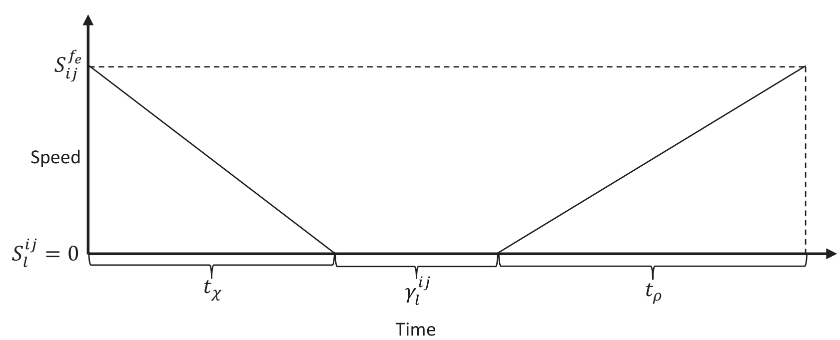

Figure 9. Call-taxi passing through arc-intersection without traffic signal where $S_t^{tf} = 0$ and $\gamma_t^{tf} > 0$.

DOI: 10.1002/atr

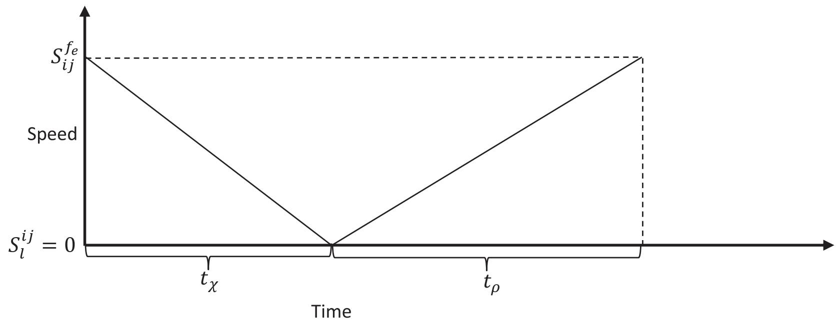

Figure 10. Call-taxi passing through arc-intersection without traffic signal where $S_t^{tf} = 0$ and $\gamma_t^{tf} = 0$.

A call-taxi that travels at a speed lower or higher than S_{ij}^{fe} has higher fuel consumption than f_e. The relationship between vehicle speed and fuel consumption has been studied by many researchers [34, 35, 41–43]. We use the data from the parabolic graph in Avoid High Speeds [42] where the fuel consumption is noted for increase in speed from 20 to 60 km/hour in increments of 10 km/hour. Table I lists the obtained data where the speed is translated to m/second and the fuel consumption is translated to mL/m.

A regression model with speed as independent variable and fuel consumption as dependent variable resulted in the following relationship with $R^2 = 0.938$.

$$Fuel\ Consumption = 0.118 - 0.00306\ speed\ mL/\mathrm{m} \tag{14}$$

Equation (14) calculates fuel consumption for a given speed in mL/m, but we need the fuel consumption in mL. The distance traveled by the call-taxi during γ_l^{ij} at speed S_l^{ij} is $\gamma_l^{ij} S_l^{ij}$ meters. Substituting S_l^{ij} for *Speed* in (14), the fuel consumed by a call-taxi during γ_l^{ij} at speed S_l^{ij} is given by $\gamma_l^{ij} S_l^{ij} \times \left(0.118 - 0.00306 S_l^{ij}\right)$.

The fuel consumption at arc intersection without traffic signal where $S_l^{ij} > 0$ is given by the following:

$$f_l^{ij} = 0.537 \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\chi} + \gamma_l^{ij} S_l^{ij} \times \left(0.118 - 0.00306 S_l^{ij}\right) + 0.13\left(S_{ij}^{fe^2} - S_l^{ij^2}\right) + 0.42 \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\rho} \tag{15}$$

The total time taken by call-taxi to clear this arc intersection without traffic signal is given by the following:

$$t_l^{ij} = \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\chi} + \gamma_l^{ij} + \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\rho} \tag{16}$$

The total distance covered by the call-taxi to decelerate from S_{ij}^{fe} to S_l^{ij}, travel or idle for γ_l^{ij}, and then accelerate back to S_{ij}^{fe} is given by the following:

Table I. Speed and fuel consumption (source: avoid high speeds [42]).

Speed (m/second)	Fuel consumption (mL/m)
5.55	0.105
8.33	0.09
11.11	0.08
13.88	0.075
16.66	0.07

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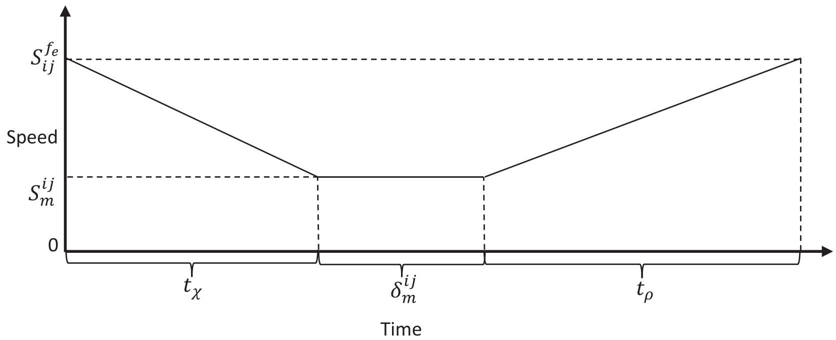

Figure 11. Call-taxi passing through speed breaker.

$$d_l^{ij} = S_{ij}^{fe}\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\chi}\right) - \frac{1}{2}\chi\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\chi}\right)^2 + S_l^{ij}\gamma_l^{ij} + S_l^{ij}\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\rho}\right) + \frac{1}{2}\rho\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\rho}\right)^2 \tag{17}$$

3.2.6. Travel characteristics of call-taxi at speed breaker

The purpose of a speed breaker is to slow down a vehicle and not stop it. Hence, we model it accordingly and assume that there is always a non-zero delay, which implies that the actual delay at the speed breaker $\delta_m^{ij} > 0$ and the speed at which a call-taxi clears a speed breaker is $0 < S_m^{ij} < S_{ij}^{fe}$. The call-taxi decelerates from S_{ij}^{fe} to S_m^{ij}, crosses the speed breaker at S_m^{ij} in δ_m^{ij} seconds, and then accelerates back to S_{ij}^{fe} (Figure 11).

The travel characteristics obtained in Section 3.2.5 are used to calculate the necessary metrics in this section.

The fuel consumption at speed breaker is given by the following:

$$f_m^{ij} = 0.537\frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\chi} + \delta_m^{ij}S_m^{ij}\times\left(0.118 - 0.00306S_m^{ij}\right) + 0.13\left({S_{ij}^{fe}}^2 - {S_m^{ij}}^2\right) + 0.42\frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\rho} \tag{18}$$

The total time taken by call-taxi to clear this speed breaker is given by the following:

$$t_m^{ij} = \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\chi} + \delta_m^{ij} + \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\rho} \tag{19}$$

The total distance covered by the call-taxi to decelerate from S_{ij}^{fe} to S_m^{ij}, travel for δ_m^{ij}, and then accelerate back to S_{ij}^{fe} is given by the following:

$$d_m^{ij} = S_{ij}^{fe}\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\chi}\right) - \frac{1}{2}\chi\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\chi}\right)^2 + S_m^{ij}\delta_m^{ij} + S_m^{ij}\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\rho}\right) + \frac{1}{2}\rho\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\rho}\right)^2 \tag{20}$$

3.3. Reconciliation of obstacle travel characteristics of an arc

An arc in the graph G could have all the obstacles discussed in Section 3.2, and there could be multiple instances of them. The results obtained in Section 3.2 help in quantifying the fuel consumption for a given delay at the four types of obstacles. This implies that the free-flow travel time and fuel consumption are the lower bounds on the actual travel time and fuel consumption of a call-taxi.

Lemma 1.
The length of each obstacle on an arc is negligible when compared with the length of the arc in a city road network.

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Lemma 2.
The delay at each obstacle on an arc is significant when compared with the free-flow travel time across the obstacle.

It is important to reconcile the travel characteristics of call-taxi at the obstacles to help us determine the actual travel time and fuel consumption along the arc. This reconciliation is achieved by carefully replacing the free-flow travel time and fuel consumption at each obstacle with the actual values computed in Section 3.2. The following are some of the arc properties of the road network that we consider for reconciliation purpose:

a. The arcs linking the source node-intersection of a call-taxi will always have an acceleration travel characteristics as it has to accelerate from rest to $S_{ij}^{f_e}$ through one of them.
b. The arcs linking the destination node-intersection of a call-taxi will always have a deceleration travel characteristic as it has to decelerate from $S_{ij}^{f_e}$ to rest through one of them.
c. All node-intersections other than the source and destination of a call-taxi will have travel characteristics corresponding to node-intersection with traffic signal.
d. The delay and fuel consumption due to deceleration and idling at a node-intersection with traffic signal is reconciled with the inbound arcs connected to it.
e. The delay and fuel consumption due to acceleration after idling at a node-intersection with traffic signal is reconciled with all outbound arcs connected to it.
f. The arc intersections with traffic signals can be present in all arcs, and their numbers in each arc could vary including not having this obstacle.
g. The arc intersections without traffic signals can be present in all arcs, and their numbers in each arc could vary including not having this obstacle.
h. The speed breakers can be present in all arcs, and their numbers in each arc could vary including not having this obstacle.

The length of an arc is reduced by the distance covered by a call-taxi corresponding to the delays associated with various obstacles. In the case of node-intersection with traffic signal, a call-taxi decelerates, idles for some time, and then accelerates back to free-flow speed. The time delay, fuel consumption, and the corresponding distance covered for deceleration and idling is reconciled with the inbound arc of this node. The time delay, fuel consumption, and the corresponding distance covered for accelerating back to free-flow speed from a node-intersection with traffic signal are reconciled with the outbound arc of this node. Hence, for every arc in the road network, the distance corresponding to acceleration of a call-taxi from rest to free flow speed is added to reconcile with inbound node-intersection, and the distance corresponding to acceleration of a call-taxi from rest to free-flow speed is subtracted to reconcile with outbound node-intersection. For all obstacles on an arc, the distance covered by the call-taxi due to the resulting delay (deceleration, idling/slowing, and acceleration) is subtracted from the length of the arc. At the end of the subtraction process, the leftover length of the arc is the free-flow distance for which a call-taxi would travel at free-flow speed.

A similar procedure is adopted to calculate the travel time and fuel consumption of a call-taxi along an arc. Their values are first initialized to zero, and for each obstacle, the corresponding delay and fuel consumption due to deceleration, idling/slowing, and acceleration are added. Care is taken to ensure that for a node-intersection, the values corresponding to deceleration and idling reconcile with the inbound arc while the values corresponding to acceleration reconcile with the outbound arc. At the end of this step, the travel time and fuel consumption values corresponding to the free flow distance of this arc are added.

Travel Characteristics Reconciliation Pseudo Code

INPUT

$V, A, d_{ij}, i_s, i_d, t_\rho, t_\chi, t_j, t_k^{ij}, t_l^{ij}, t_m^{ij}, f_\rho, f_\chi, f_j, f_k^{ij}, f_l^{ij}, f_m^{ij}, d_\rho, d_\chi, d_j, d_k^{ij}, d_l^{ij}, d_m^{ij}$

INITIALIZE

For $\forall\, a_j^i$

$t_{ij} \leftarrow 0$

$f_{ij} \leftarrow 0$

$\hat{d}_{ij} \leftarrow d_{ij}$

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PROCEDURE Reconciliation

For $\forall\, a_j^i,\ i \in i_s$

$$t_{ij} \leftarrow t_{ij} + t_\rho + t_j + \sum t_k^{ij} + \sum t_l^{ij} + \sum t_m^{ij} - t_\rho$$
$$f_{ij} \leftarrow f_{ij} + f_\rho + f_j + \sum f_k^{ij} + \sum f_l^{ij} + \sum f_m^{ij} - f_\rho$$
$$\hat{d}_{ij} \leftarrow \hat{d}_{ij} - d_\rho - d_j - \sum d_k^{ij} - \sum d_l^{ij} - \sum d_m^{ij} + d_\rho$$

For $\forall\, a_j^i,\ j \in i_d$

$$t_{ij} \leftarrow t_{ij} + t_\chi + t_\rho + \sum t_k^{ij} + \sum t_l^{ij} + \sum t_m^{ij}$$
$$f_{ij} \leftarrow f_{ij} + f_\chi + f_\rho + \sum f_k^{ij} + \sum f_l^{ij} + \sum f_m^{ij}$$
$$\hat{d}_{ij} \leftarrow \hat{d}_{ij} - d_\chi - d_\rho - \sum d_k^{ij} - \sum d_l^{ij} - \sum d_m^{ij}$$

For $\forall\, a_j^i,\ i \notin \{i_s, i_d\}, j \notin \{i_s, i_d\}$

$$t_{ij} \leftarrow t_{ij} + t_\rho + t_j + \sum t_k^{ij} + \sum t_l^{ij} + \sum t_m^{ij} - t_\rho$$
$$f_{ij} \leftarrow f_{ij} + f_\rho + f_j + \sum f_k^{ij} + \sum f_l^{ij} + \sum f_m^{ij} - f_\rho$$
$$\tilde{d}_{ij} \leftarrow \hat{d}_{ij} - d_\rho - d_j - \sum d_k^{ij} - \sum d_l^{ij} - \sum d_m^{ij} + d_\rho$$

For $\forall\, a_j^i$

$$t_{ij} \leftarrow t_{ij} + \frac{max\{0, \hat{d}_{ij}\}}{S_{ij}^{fe}}$$
$$f_{ij} \leftarrow f_{ij} + f_e \frac{max\{0, \hat{d}_{ij}\}}{S_{ij}^{f_e}}$$

OUTPUT

t_{ij}, f_{ij}

STORE

t_{ij}, f_{ij}

STOP

4. TIME ROBUST FUEL-EFFICIENT PATH

The stochastic nature of delays on the arcs and nodes of the network needs to be more explicitly modeled to ensure that the resulting path is more accurate and representative. The delays on the arcs and nodes follow statistical distribution. Working with the expected values of the delays would fail to uncover the impact of many other combinations of delays on arcs and nodes. To address this issue, a Monte-Carlo simulation-based heuristic approach is proposed where the delays on each link are sampled a priori to find the fuel-efficient path. To ensure most of the combinations of the delays are explored, the exercise of sampling the delays and finding the fuel-efficient path is repeated multiple times. The sampled delays at arcs and nodes do not change during the execution of a shortest path algorithm, that is, the shortest path algorithm finds path through a non-stochastic network. However, the delays on the arcs and nodes of this non-stochastic network differ across runs. In essence, multiple virtual road networks are generated with the same topography but with different combinations of delays on nodes and arcs.

4.1. Path generation pseudo code

NOTATIONS

$N =$	maximum number of simulation runs
$n =$	index of simulation runs, $n = 1, 2, \ldots, N$
$p_n =$	path generated in simulation run n
$r =$	index of uniform random numbers between 0 and 1
$F_\alpha^{-1}(r) =$	random variate for delay at a node-intersection with traffic signal corresponding to r
$F_\beta^{-1}(r) =$	random variate for delay at an arc intersection with traffic signal corresponding to r

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$F_{\gamma}^{-1}(r) =$ random variate for delay at an arc intersection without traffic signal corresponding to r
$F_{\delta}^{-1}(r) =$ random variate for delay at a speed breaker corresponding to r
$\epsilon =$ a value between 0 and 1 indicating the percentage of instances a call-taxi stops at an arc intersections without traffic signals
$f^{p_n} =$ fuel consumption for path n
$t^{p_n} =$ travel time for path n
$d^{p_n} =$ travel distance for path n

The following pseudo-code details the steps involved in finding the fuel efficient path and its corresponding travel time in a road network. The delays on arcs and nodes are found through inverse transform sampling.

INPUT

$V, A, d_{ij}, i_s, i_d, f_e, f_e^{idle}, S_{ij}^{fe}, \rho, \chi, \epsilon, S_l^{ij}, S_m^{ij}$, N

INITIALIZE

$n = 0$

PROCEDURE Monte-Carlo Sampling

$n \leftarrow n+1$

For $\forall\ i,\ i \neq i_s$

Generate r

$\alpha_i \leftarrow F_{\alpha}^{-1}(r)$

$f_i \leftarrow 0.537\frac{S_{ij}^{fe}}{\chi} + \alpha_i f_e^{idle} + 0.13 {S_{ij}^{fe}}^2 + 0.42\frac{S_{ij}^{fe}}{\rho}$

$t_i \leftarrow \frac{S_{ij}^{fe}}{\chi} + \alpha_i + \frac{S_{ij}^{fe}}{\rho}$

$d_i \leftarrow S_{ij}^{fe} t_\chi - \frac{\chi {t_\chi}^2}{2} + \frac{\rho {t_\rho}^2}{2}$

For $\forall\ a_j^i$

For $\forall\ k\ in\ a_j^i$

Generate r

$\beta_k^{ij} \leftarrow F_{\beta}^{-1}(r)$

$f_k^{ij} \leftarrow 0.537\frac{S_{ij}^{fe}}{\chi} + \beta_k^{ij} f_e^{idle} + 0.13 {S_{ij}^{fe}}^2 + 0.42\frac{S_{ij}^{fe}}{\rho}$

$t_k^{ij} \leftarrow \frac{S_{ij}^{fe}}{\chi} + \beta_k^{ij} + \frac{S_{ij}^{fe}}{\rho}$

$d_k^{ij} \leftarrow S_{ij}^{fe} t_\chi - \frac{\chi {t_\chi}^2}{2} + \frac{\rho {t_\rho}^2}{2}$

For $\forall\ l\ in\ a_j^i$

Generate r

$\gamma_l^{ij} \leftarrow F_{\gamma}^{-1}(r)$

If $\epsilon \leq 1 - r$

$S_l^{ij} \leftarrow 0$

$f_l^{ij} \leftarrow 0.537\frac{S_{ij}^{fe}}{\chi} + \gamma_l^{ij} f_e^{idle} + 0.13 {S_{ij}^{fe}}^2 + 0.42\frac{S_{ij}^{fe}}{\rho}$

Else

$$f_l^{ij} \leftarrow 0.537\frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\chi} + \gamma_l^{ij} S_l^{ij} \times \left(0.118 - 0.00306 S_l^{ij}\right) + 0.13\left({S_{ij}^{fe}}^2 - {S_l^{ij}}^2\right) + 0.42\frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\rho}$$

$$t_l^{ij} \leftarrow \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\chi} + \gamma_l^{ij} + \frac{\left(S_{ij}^{fe} - S_l^{ij}\right)}{\rho}$$

$$d_l^{ij} \leftarrow S_{ij}^{fe}\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\chi}\right) - \frac{1}{2}\chi\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\chi}\right)^2 + S_l^{ij}\gamma_l^{ij} + S_l^{ij}\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\rho}\right) + \frac{1}{2}\rho\left(\frac{S_{ij}^{fe} - S_l^{ij}}{\rho}\right)^2$$

DOI: 10.1002/atr

For $\forall\, m \; in \; a_j^i$

Generate r

$\delta_m^{ij} \leftarrow F_\delta^{-1}(r)$

$$f_m^{ij} \leftarrow 0.537 \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\chi} + \delta_m^{ij} S_m^{ij} \times \left(0.118 - 0.00306 S_m^{ij}\right) + 0.13\left({S_{ij}^{fe}}^2 - {S_m^{ij}}^2\right) + 0.42 \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\rho}$$

$$t_m^{ij} \leftarrow \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\chi} + \delta_m^{ij} + \frac{\left(S_{ij}^{fe} - S_m^{ij}\right)}{\rho}$$

$$d_m^{ij} \leftarrow S_{ij}^{f_e}\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\chi}\right) - \frac{1}{2}\chi\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\chi}\right)^2 + S_m^{ij}\delta_m^{ij} + S_m^{ij}\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\rho}\right) + \frac{1}{2}\rho\left(\frac{S_{ij}^{fe} - S_m^{ij}}{\rho}\right)^2$$

OUTPUT

$t_\rho, t_\chi, t_j, t_k^{ij}, t_l^{ij}, t_m^{ij}, f_\rho, f_\chi, f_j, f_k^{ij}, f_l^{ij}, f_m^{ij}, d_\rho, d_\chi, d_j, d_k^{ij}, d_l^{ij}, d_m^{ij}$

STORE

$t_\rho, t_\chi, t_j, t_k^{ij}, t_l^{ij}, t_m^{ij}, f_\rho, f_\chi, f_j, f_k^{ij}, f_l^{ij}, f_m^{ij}, d_\rho, d_\chi, d_j, d_k^{ij}, d_l^{ij}, d_m^{ij}$

For $\forall\, a_j^i$

$t_{ij} \leftarrow 0$

$f_{ij} \leftarrow 0$

$\hat{d}_{ij} \leftarrow d_{ij}$

PROCEDURE Reconciliation

INPUT

$d_{ij}, i_s, i_d, t_\rho, t_\chi, t_j, t_k^{ij}, t_l^{ij}, t_m^{ij}, f_\rho, f_\chi, f_j, f_k^{ij}, f_l^{ij}, f_m^{ij}, d_\rho, d_\chi, d_j, d_k^{ij}, d_l^{ij}, d_m^{ij}$

OUTPUT

t_{ij}, f_{ij}

STORE

t_{ij}, f_{ij}

PROCEDURE Dijkstra Algorithm

INPUT

$V, A, d_{ij}, i_s, i_d, f_{ij}, t_{ij}$

Objective

Minimize f^{p_n}

OUTPUT

$p_n, f^{p_n}, t^{p_n}, d^{p_n}$

STORE

$p_n, f^{p_n}, t^{p_n}, d^{p_n}$

If $n = N$

Stop

Else

Go to PROCEDURE Monte-Carlo Sampling

Consider a completely connected road network with four nodes. The Dijkstra's algorithm [62] in the pseudo-code finds the path from the source to destination that has the minimal fuel consumption. For this path, the corresponding travel time and travel distance are also recorded. It is possible that more than one path shares the same sequence of nodes and has the same distance but varies in the fuel consumption and travel time. This variation is expected and needed to obtain a path that is fuel-efficient as well as robust on travel time. Consider a four-node illustrative road network shown in Figure 12. An illustrative output of the previous pseudo-code for the four-node road network is tabulated in Table II. The path p_n indicates the nodes and their sequence from the source to the destination.

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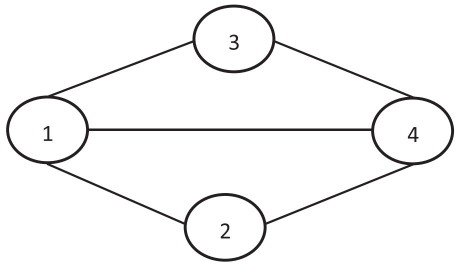

Figure 12. Illustrative road network.

Table II. Illustrative output of the path generation pseudo code.

n	p_n	f^{p_n}	t^{p_n}	d^{p_n}
1	1-2-4	10	30	15
2	1-4	9	28	13
3	1-3-4	14	42	14
4	1-2-4	12	40	15
5	1-4	11	36	13

4.2. Multi-criteria selection of a path

Finding the fuel-efficient path helps save cost for a call-taxi but reaching a pick-up point, and a drop-off point at the shortest possible time helps in achieving customer satisfaction. Given that the delays on each link are stochastic, the obtained fuel-efficient path need not be actually time robust, that is, the travel time along the fuel-efficient path could vary during actual travel. The Monte-Carlo simulation runs generate multiple travel times for a fuel-efficient path because of the stochastic nature of delays along the path. The travel time reliability of the fuel-efficient path is captured using the coefficient of variation of these travel times. The goal is to find an approach that assigns multi-criteria score for a path based on fuel consumption, travel time along the path, and its reliability. A procedure is developed to select a path among the multiple paths generated based on the following factors:

a. the probability that the path is fuel-efficient;
b. the average travel time of the path; and
c. the coefficient of variation of travel time of the path.

Two paths p_n and p_n' are said to be unique if there is at least one difference in the set of nodes or their sequence in the path from the source to destination. Similarly, two paths p_n and p_n' are said to be identical if there is no difference in the set of nodes and their sequence in the path from the source to destination. It should be noted that two identical paths need not have the same fuel consumption or the travel time.

NOTATIONS

$P =$ set containing unique paths generated, $|P| \leq N$
$p_u =$ index of unique paths in P
$p_u^c =$ number of paths sharing the same set of nodes, arcs and their sequence with p_u
$q_u =$ index of paths identical to the path p_u among the N paths generated, $q_u = 1, \ldots, p_u^c$
$\theta^{p_u} =$ proportion of paths among the N paths that are identical to p_u
$\vartheta^{p_u} =$ average fuel consumed for paths identical to p_u among the N paths
$\mu^{p_u} =$ average travel time of paths identical to p_u among the N paths
$\omega^{p_u} =$ coefficient of variation of travel times of paths identical to p_u among the N paths
$\overline{\mu}^{p_u} =$ adjusted average travel time of paths identical to p_u among the N paths
$\overline{\overline{\mu}}^{p_u} =$ normalized value of $\overline{\mu}^{p_u}$
$\tau^{p_u} =$ combined score for path p_u

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The Monte-Carlo sampling of the road network delay timings results in multiple paths generated between origin and destination. A unique path, p_u, could have been generated multiple times, each with different travel time and fuel consumption. The number of instances of occurrence of each unique path is calculated as taking the count of the number of paths sharing the same set of nodes and their sequence.

$$p_u^c = \sum\nolimits_{n, p_n \equiv p_u} 1 \tag{21}$$

Each unique path might have different numbers of occurring instances through the Monte-Carlo sampling, and the likelihood that it is fuel efficient or the probability that it is fuel efficient is obtained as the fraction of instances that generated it.

$$\theta^{p_u} = \frac{p_u^c}{N} \tag{22}$$

The metric θ^{p_u} needs to be maximized as any savings in the fuel consumption help in reducing the cost of running call-taxis.

The fuel consumption and travel time of each instance of a unique path need not be the same, and hence, their average values are taken as representatives for the unique path.

$$\vartheta^{p_u} = \frac{\sum_{q_u} f^{q_u}}{p_u^c} \tag{23}$$

$$\mu^{p_u} = \frac{\sum_{q_u} t^{q_u}}{p_u^c} \tag{24}$$

The metric μ^{p_u} indicates the quickness of a unique path or how fast a call-taxi could reach a pick-up or a drop-off point. This metric needs to be minimized as a faster travel time would yield better customer service. However, there is a risk that the actual travel time would be different from the average travel time, and this is measured using coefficient of variation.

$$\omega^{p_u} = \frac{\sqrt{\frac{\sum_{q_u} \left(t^{q_u} - \mu^{p_u}\right)^2}{p_u^c - 1}}}{\mu^{p_u}} \tag{25}$$

The risk of deviating from the average travel time of a unique path is modeled by increasing the travel time based on coefficient of variation.

$$\overline{\mu}^{p_u} = \frac{\mu^{p_u}}{\left(1 - \omega^{p_u}\right)^2} \tag{26}$$

The resulting adjusted average travel time of unique path $\overline{\mu}^{p_u}$ is used as an estimate for the risk of variation of individual path travel times that were used to obtain the average.

Among the N simulated road network delays, the metrics θ^{p_u} and ϑ^{p_u} helps in identifying the fuel efficient path. We note that θ^{p_u} is a better metric than ϑ^{p_u} as it helps in identifying a path that is most likely fuel efficient. Moreover, ϑ^{p_u} could be biased based on p_u^c. Hence, for the analysis purpose, we shall be using θ^{p_u} to identify the fuel-efficient path.

The values of θ^{p_u} and $\overline{\mu}^{p_u}$ might not be aligned, that is, a unique path with the highest θ^{p_u} need not have the lowest $\overline{\mu}^{p_u}$. A combined score is proposed for each unique path p_u based on θ^{p_u} and the

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Table III. Finding the multi-criteria score for the paths.

p_u	p_u^c	θ^{p_u}	ϑ^{p_u}	μ^{p_u}	ω^{p_u}	$\overline{\mu}^{p_u}$	$\overline{\overline{\mu}}^{p_u}$	τ^{p_u}
1-2-4	2	0.4	11	35	0.202	54.961	0.764	0.306
1-4	2	0.4	10	32	0.177	47.244	0.888	0.356
1-3-4	1	0.2	14	42	0	42	1	0.2

normalized values of $\overline{\mu}^{p_u}$ (between 0 and 1) under the assumption that both these metrics are of equal importance.

$$\overline{\overline{\mu}}^{p_u} = \frac{\min_{p_u} \overline{\mu}^{p_u}}{\overline{\mu}^{p_u}} \tag{27}$$

The combined score is obtained by taking the product of θ^{p_u} and $\overline{\mu}^{p_u}$, and a unique path with the highest combined score represents time robust fuel-efficient path in the stochastic road network.

$$\tau^{p_u} = \theta^{p_u} \times \overline{\mu}^{p_u} \tag{28}$$

The combined score, τ^{p_u}, helps in selecting a path that has lower fuel consumption as well as robust travel time. Table III illustrates the procedure described in this section for the paths in Table II.

5. COMPUTATIONAL ANALYSIS

A road network based on a real-world city road network topology is created with 44 nodes and 68 arcs (Figure 13) for the purpose of computational analysis and demonstration of the proposed approach. The arc lengths and obstacles in each arc in this road network are hypothetically generated resulting in a route distance of 276.28 km, 44 node-intersections with signals, 109 arc intersections with signals, 172 arc intersections without signals, and 142 speed breakers.

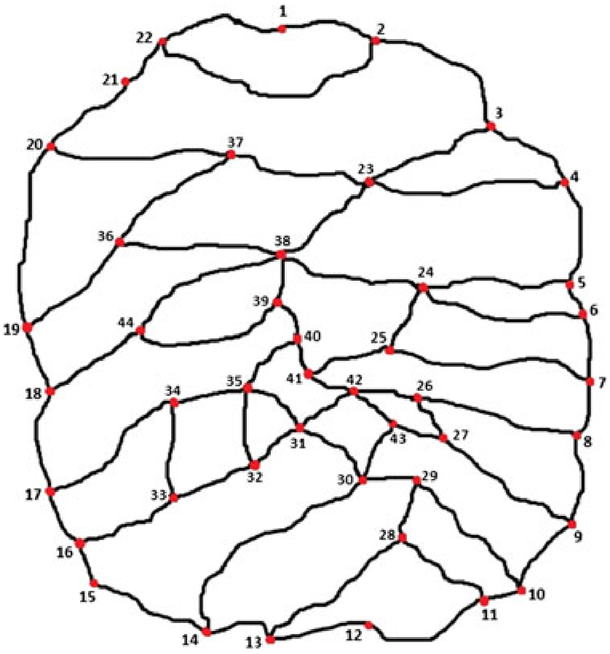

Figure 13. Road network for computational study.

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The ideal fuel consumption of a call-taxi considered is 0.926 mL/second, and this would be achieved when it travels at a speed of 16.67 m/second. The ideal fuel consumption value is an average of the manufacturer specified values of some of the most commonly used cars for call-taxis in India, which is 18 km/L when driven at a speed of 60 km/hour. The delays at the four obstacle types follow a uniform distribution with parameters $U(0, 120)$ seconds for node intersections with traffic signals, $U(0, 60)$ seconds for arc intersections with traffic signals, $U(0, 20)$ seconds for arc intersections without traffic signals, and $U(5, 15)$ seconds for speed breakers. The slowdown speeds at arc intersections without traffic signals and speed breakers are 5 and 15 km/hour respectively with call-taxis stopping at arc-intersections without traffic signals for 75% of the time. The obstacle delay distributions and slow down speeds are based on observations made at few obstacles of each type in the city road network. The acceleration rate of call-taxi is 0.65 m/second^2, and its deceleration rate is 0.82 m/second^2, which are based on the values given in Ferreira [41]. It should be noted that the proposed methodology is fully parameterized, which makes it amicable to be applied to any road network and for any car type by inputting relevant values to the parameters.

The number of runs required, N, for Monte-Carlo sampling is decided by experimenting its value from 1000 to 20, 000. It was found that for an O-D pair, the output did not significantly vary between a 1000 and a 20, 000 run sampling although the difference in the computation time was significantly large. Hence, the Monte-Carlo sampling of the road network obstacle delays was carried out 1000 times (N= 1000) while finding the paths between an origin and destination. This implies that for a given combination of O-D pair and vehicle fuel economy, the road network delays were sampled 1000 times, and for each sample, the fuel efficient path is found from the origin to the destination based on the pseudo-code mentioned in Section 4.1. The procedure described in Section 4.2 is then applied to identify unique paths and assign multi-criteria score to them. The solution procedure is modeled in C programming language and executed in a desktop computer with Intel Core i7 3.4-GHz processor and 8-GB RAM.

A judgmental sampling approach is used to identify 77 O-D pairs from 946 O-D pairs in the road network for which the proposed solution approach is applied to find the fuel efficient time robust paths. The O-D pairs were sampled on the basis of having scope for alternate unique paths getting generated. Table IV lists the frequency distribution of number of unique paths for O-D pair and the specific O-D pairs from the sample for each class interval.

The output corresponding to select O-D pair for each class interval of unique paths in Table IV is present in Table V. It took less than a second to generate the output for an O-D pair, which makes the proposed approach practically feasible to be developed into a smartphone application or included as an additional feature in GPS system for call-taxis. The bolded value of τ^{p_u} indicates the path having the highest combined score among the multiple unique paths for an O-D pair. For instances where there is just one unique path, the value of τ^{p_u} is always 1. However, the values of other columns in the tables help in judging the fuel efficiency as well as the time robustness of a unique path.

The computational study revealed few insights on the interrelationship between travel distance, travel time and fuel consumption along the fuel-efficient path for the 77 O-D pairs:

Table IV. Frequency distribution of unique paths for the 77 O-D pairs.

Unique paths	Count	O-D pairs
1	36	1-38, 4-17, 4-19, 5-42, 6-18, 6-20, 7-38, 8-17, 9-36, 11-29, 11-36, 14-11, 16-5, 16-34, 18-4, 18-8, 18-24, 18-38, 19-8, 22-3, 24-3, 27-35, 27-42, 30-42, 31-40, 35-8, 35-42, 36-23, 38-30, 38-44, 40-31, 42-9, 43-26, 44-9, 44-24, 44-34
2	23	9-19, 10-34, 12-38, 16-35, 16-40, 17-41, 18-2, 19-39, 24-26, 25-6, 25-26, 26-13, 27-32, 34-32, 34-36, 35-43, 37-38, 41-6, 41-8, 41-38, 42-12, 43-8, 43-17
3	15	1-12, 3-14, 4-33, 12-38, 20-7, 20-9, 20-38, 27-24, 32-41, 33-3, 33-26, 33-36, 33-42, 36-27, 37-7
4	2	16-41, 32-6
5	1	33-24

O-D, origin–destination.

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Table V. Computational results for select origin–destination pairs.

i_s	i_d	p_u	θ^{p_u}	ϑ^{p_u}	μ^{p_u}	ω^{p_u}	$\overline{\mu}^{p_u}$	$\overline{\overline{\mu}}^{p_u}$	τ^{p_u}
16	5	16-17-18-44-38-24-5	1	5.89	4008.29	0.0246	4213.65	1	**1**
41	38	41-40-39-38	0.314	2.39	1598.52	0.0398	1734.10	1	0.314
		41-25-24-38	0.686	2.39	1774.78	0.0379	1917.61	0.904	**0.620**
33	26	33-34-35-31-42-26	0.518	3.68	3247.26	0.0288	3443.06	0.932	**0.483**
		33-32-31-42-26	0.391	3.68	3021.74	0.0298	3210.55	1	0.391
		33-32-35-31-42-26	0.091	3.74	3672.43	0.0278	3885.69	0.826	0.075
32	6	32-35-40-39-38-24-6	0.777	5.88	4169.43	0.0250	4386.37	0.981	**0.763**
		32-31-42-26-8-7-6	0.177	5.94	4772.96	0.0203	4972.85	0.866	0.153
		32-31-42-41-25-24-6	0.009	5.93	4178.64	0.0149	4306.87	1	0.009
		32-35-31-42-26-8-7-6	0.037	5.99	5446.45	0.0191	5660.66	0.760	0.028
33	24	33-34-35-40-39-38-24	0.675	5.23	3594.81	0.0261	3790.28	1	**0.675**
		33-32-35-40-39-38-24	0.252	5.26	4017.71	0.0249	4225.52	0.896	0.226
		33-32-31-42-41-25-24	0.01	5.36	4060.95	0.0284	4302.34	0.881	0.008
		33-34-17-18-44-38-24	0.058	5.28	3863.57	0.0288	4096.69	0.925	0.054
		33-34-35-31-42-41-25-24	0.005	5.39	4358.08	0.0168	4508.77	0.841	0.004

a. More than 46% of the O-D pairs generate just one unique path that is fuel efficient and time robust. In such instances, there is no decision involved in picking up the best path and $\overline{\mu}^{p_u}$ provides an estimate of the expected travel time after correcting for its variation and the corresponding fuel consumption, ϑ^{p_u}.
b. More than 53% of the O-D pairs generated at least one alternate path that is fuel efficient and time robust. Given that the two unique paths need not be of same length, it validates the fact that in a congested road network, a shortest distance path need not always be a shortest time path. The combined score, τ^{p_u}, helps in identifying the path that is not only fuel efficient but also has a higher likelihood of arriving at the destination in μ^{p_u} minutes.
c. For an O-D pair with alternate unique paths, there could be no or marginal increase in fuel consumption for a significant increase in travel time between the paths.
d. The coefficient of variation of travel times across all unique paths among the 77 O-D pairs is less than 3%. However, the coefficient of variation of average travel times of alternate unique paths for an O-D pair is higher. This indicates the robustness of travel times of each unique path as well as the distinction in the travel times between unique paths for an O-D pair.
e. For O-D pairs with at least one alternate unique path, there is a significant difference in the value of θ^{p_u} between the highest and the second highest. However, this cannot be generalized as it depends a lot on the number and type of obstacles on each alternate path as well as the duration of delay on each of them.
f. The Monte-Carlo sampling approach helps in incorporating stochastic nature of travel time and resulting fuel consumption, thereby providing a more realistic estimate of the expected fuel consumption and travel time for the unique paths for an O-D pair.

We note that the proposed approach is fully parameterized, which makes it amicable for customization and real world application with actual calibrated inputs.

6. CONCLUSION

The ability to reach a location at the predicted time and at the same time minimize the fuel consumption associated with it is of great importance for call-taxi service providers. This not only helps them increase the customer base and increase revenue but also reduce the cost associated with it, which are critical to make the call-taxi system profitable. Obstacles on a city road network make it difficult to find the fuel efficient time robust path for a call-taxi as it results in stochastic travel times.

A Monte-Carlo simulation based approach is developed for sampling obstacle delays in a stochastic road network. The Dijkstra's algorithm is run for each combination of sampled delay in the road network to find the path from the origin and destination for the objective of minimizing the fuel consumption. A multi-criteria score is assigned for each unique path between an origin destination pair

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based on the probability that the path is fuel-efficient, the average travel time of the path and the coefficient of variation of the travel times of the path.

The methodology developed in this paper has a direct application in the navigation system used by call-taxi drivers, which could further be customized based on live traffic updates on each node and arc to periodically update the expected delays at various obstacles. In practice, where there are many unquantifiable variables that cause disruption to the originally obtained path of a call-taxi, the developed approach can be implemented dynamically by re-computing the time-robust fuel-efficient path for a call-taxi at regular time intervals and on need basis, re-route the call-taxi to its destination along the new best path. The parameterization of the developed approach makes it practical to be used for any car type and road network by inputting relevant values to the parameters.

The Monte-Carlo simulation approach described in this paper can be extended and applied to other related problems such as solid transportation problem, traveling salesman problem, vehicle routing problem, empty vehicle repositioning problem, vehicle fleet sizing problem, etc., where the travel times are stochastic in nature. A scope for future work would be to explore modeling of delays at the obstacles in a city road network described in this paper as fuzzy numbers and accordingly develop a method to calculate fuel consumption. The resulting variant of the multi-criteria fuzzy shortest path problem could be solved using heuristics, meta-heuristics or algorithmic solution approaches. Another related problem of interest would be to look into the aspect of call-taxi revenue generation and customer service. The presence of a call-taxi in the vicinity of a demand region at the right time would increase its chance of getting assigned to a customer in that region, thereby increasing the revenue opportunities and enhancing customer service. This problem can be researched more to develop solution approaches for fleet sizing and empty repositioning of call-taxis under stochastic travel times in a city road network.

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REFERENCES

1. Devanathan V. Madurai Hails Call Taxi Service. http://timesofindia.indiatimes.com/city/madurai/Madurai-hails-Call-Taxi-Service/articleshow/19332833.cms [13 March 2015].
2. Aarhaug J, Skollerud K. Taxi: different solutions in different segments. *Transportation Research Procedia* 2014; **1**: 276–283.
3. Dial RB. Autonomous dial-a-ride transit introductory overview. *Transportation Research Part C* 1995; **3**(5): 261–275.
4. Jung J, Jayakrishnan R, Park JY. Design and Modeling of Real-Time Shared-Taxi Dispatch Algorithms. Proceedings of Transportation Research Board 92nd Annual Meeting 2013.
5. MacieJewski M, Bischoff J. Large-scale microscopic simulation of taxi services. *Procedia Computer Science* 2015; **52**: 358–364.
6. Lammoglia A, Faye RM, Josselin D. A Dynamic Cooperation Modeling for Improving Taxi Fleet Efficiency. Proceedings of the AGILE'2012 International Conference on Geographic Information Science 2012; 163-168.
7. Salanova JF, Estrada M, Aifadopoulou G, Mitsakis E. A review of modeling of taxi services. *Procedia Social and Behavioral Sciences* 2011; **20**: 150–161.
8. Salanova JF, Romeu ME, Amat C. Aggregated modeling of urban taxi services. *Procedia—Social and Behavioral Sciences* 2014; **160**: 352–361.
9. Wong RCP, Szeto WY, Wong SC. A cell-based logit-opportunity taxi customer-search model. *Transportation Research Part C* 2014; **48**: 84–96.
10. Wang B, Zhao L, Pang Y, Zhang D, Yang X. Analysis of passenger's choice between shuttle bus and illegal taxi. *Procedia—Social and Behavioral Science* 2013; **96**: 1948–1960.
11. Yang H, Wong SC. A network model for urban taxi services. *Transportation Research Part B* 1998; **32**(4): 235–246.
12. Kim H, Yang I, Choi K. An agent-based simulation model for analyzing the impact of asymmetric passenger demand on taxi service. *KSCE Journal of Civil Engineering* 2011; **15**(1): 187–195.
13. Grau JMS, Romeu MAE. Agent based modeling for simulating taxi services. *Procedia Computer Science* 2015; **52**: 902–907.
14. Xiang Z, Chu C, Chen H. The study of a dynamic dial-a-ride problem under time-dependent and stochastic environments. *European Journal of Operational Research* 2008; **185**: 534–551.

15. Schilde M, Doerner KF, Hartl RF. Integrating stochastic time-dependent travel speed in solution methods for the dynamic dial-a-ride problem. *European Journal of Operational Research* 2014; **238**: 18–30.
16. Patire AD, Wright M, Prodhomme B, Bayen AM. How much GPS data do we need? *Transportation Research Part C* 2015; **58**: 325–342.
17. Ehmke JF, Meisel S, Mattfeld DC. Floating car based travel times for city logistics. *Transportation Research Part C* 2012; **21**: 338–352.
18. Zhan X, Hasan S, Ukkusuri SV, Kamga C. Urban link travel time estimation using large scale taxi data with partial information. *Transportation Research Part C* 2013; **33**: 37–49.
19. Rahmani M, Jenelius E, Koutsopoulos HN. Non-parametric estimation of route travel time distributions from low frequency floating car data. *Transportation Research Part C* 2015; **58**: 343–362.
20. Azaron A, Kianfar F. Dynamic shortest paths in stochastic dynamic networks: ship routing problem. *European Journal of Operational Research* 2003; **144**: 138–156.
21. Peer SK, Sharma DK. Finding the shortest path in stochastic networks. *Computers and Mathematics with Applications* 2007; **53**: 729–740.
22. Cheng J, Lisser A. Maximum probability shortest path problem. *Discrete Applied Mathematics* 2015; **192**: 40–48.
23. Kosuch S, Lisser A. Stochastic shortest path problem with delay excess penalty. *Electronics Notes in Discrete Mathematics* 2010; **36**: 511–518.
24. Cheng J, Lisser A, Letournel M. Distributionally robust stochastic shortest path problem. *Electronic Notes in Discrete Mathematics* 2013; **41**: 511–518.
25. Lin Y-K. Time version of the shortest path problem in a stochastic network. *Journal of Computational and Applied Mathematics* 2009; **228**: 150–157.
26. Siddiqi UF, Shiraishi Y, Dahb M, Sait SM. A memory efficient stochastic evolution based algorithm for the multi-objective shortest path problem. *Applied Soft Computing* 2014; **14**: 653–662.
27. Farhanchi M, Hassanzadeh R, Mahdavi R, Mahdavi-Amiri N. A modified Ant colony system for finding the expected shortest path in networks with variable Arc lengths and probabilistic nodes. *Applied Soft Computing* 2014; **21**: 491–500.
28. Fu L, Rilett LR. Expected shortest paths in dynamic and stochastic traffic networks. *Transportation Research Part B* 1998; **32**(7): 499–516.
29. Ji X, Iwamura K, Shao Z. New models for shortest path problem with fuzzy arc lengths. *Applied Mathematical Computing* 2007; **31**: 259–269.
30. Nielsen LR, Andersen KA, Pretolani D. Ranking paths in stochastic time-dependent networks. *European Journal of Operational Research* 2014; **236**: 903–914.
31. Pan Y, Sun L, Ge M. Finding reliable shortest path in stochastic time dependent network. *Procedia—Social and Behavioral Sciences* 2013; **96**: 451–460.
32. Gendreau M, Ghiani G, Guerriero E. Time-dependent routing problems: a review. *Computers & Operations Research* 2015; **64**: 189–197.
33. Dekker R, Bloemhof J, Mallidis I. Operations research for green logistics. *European Journal of Operational Research* 2012; **219**: 671–679.
34. Prahara E, Lubis HA-RS, Sjafruddin A. Development of instantaneous car fuel consumption model. *Journal of Eastern Asian Society for Transport Studies* 1999; **3**(1): 123–138.
35. Yao E, Song Y. Study on eco-route planning algorithm and environmental impact assessment. *Journal of Intelligent Transportation Systems* 2013; **17**(1): 42–53.
36. Mensing F, Bideaux E, Trigui R, Ribet J, Jeanneret B. Eco-driving: an economic or ecologic driving style? *Transportation Research Part C* 2014; **38**: 110–121.
37. Kluge S, Santa C, Dangl S, et al. On the computation of the energy-optimal route dependent on the traffic load in Ingolstadt. *Transportation Research Part C* 2013; **36**: 97–115.
38. Catay B, Yildirim UM. Finding the Greenest Path on Time-Dependent Road Networks. 26th European Conference on Operational Research, Rome, 1-4 July 2013.
39. Ben-Chaim M, Shmerling E, Kuperman A. Analytic modeling of vehicle fuel consumption. *Energies* 2013; **6**: 117–127.
40. Kang M-W, Shariat S, Jha MK. New highway geometric design methods for minimizing vehicular fuel consumption and improving safety. *Transportation Research Part C* 2013; **31**: 99–111.
41. Ferreira LJA. Car Fuel Consumption in Urban Traffic. The Results of a Survey in Leeds using Instrumented Vehicles. http://eprints.whiterose.ac.uk/2376/ [2 June 2015].
42. Avoid High Speeds. http://www.nrcan.gc.ca/energy/efficiency/transportation/cars-light-trucks/fuel-efficient-driving-techniques/7513 [24 March 2015].
43. Rouwendal J. An economic analysis of fuel use per kilometer by private cars. *Journal of Transport Economies and Policy* 1996; **30**(1): 3–14.
44. Suzdaleva E, Nagy I. Data-based speed limit restricting eco-driving system. *Transportation Research Part C* 2014; **44**: 253–264.
45. Jamson AH, Hibberd DL, Merat N. Interface design considerations for an in-vehicle eco-driving assistance system. *Transportation Research Part C* 2015; **58**: 642–656.
46. Jamson SL, Hibberd DL, Jamson AH. Driver's ability to learn eco-driving skills; effects on fuel efficient and safe driving behaviour. *Transportation Research Part C* 2015; **58**: 657–668.

DOI: 10.1002/atr

47. Gilman E, Keskinarkaus A, Tamminen S, Pirttikangas S, Roning J, Riekki J. Personalized assistance for fuel-efficient driving. *Transportation Research Part C* 2015; **58**: 681–705.
48. Zhao X, Wu Y, Rong J, Zhang Y. Development of a driver simulator based eco-driving support system. *Transportation Research Part C* 2015; **58**: 631–641.
49. Shokri F, Chu M-Y, Mokhtarian HR, Rahmat RAOK, Ismail A. Best-route based on fuel economy. *European Journal of Scientific Research* 2009; **32**(2): 177–186.
50. Motavalli J. Ford Eco-route: The Most Fuel-efficient Path from A to B. http://wheels.blogs.nytimes.com/2010/04/16/ford-eco-route-the-most-fuel-efficient-path-from-a-to-b/?_r=0 [13 March 2015].
51. Hamacher HW, Ruzika S, Tjandra SA. Algorithms for time-dependent bicriteria shortest path problems. *Discrete Optimization* 2006; **3**: 238–254.
52. Opasanon S, Miller-Hooks E. Multicriteria adaptive paths in stochastic, time-varying networks. *Discrete Optimization* 2006; **173**: 72–91.
53. Mohamed C, Bassem J, Taicir L. A genetic algorithms to solve the bi-criteria shortest path problem. *Electronic Notes in Discrete Mathematics* 2010; **36**: 851–858.
54. Liu L, Mu H, Yang X, He R, Li Y. An oriented spanning tree based genetic algorithm for multi-criteria shortest path problems. *Applied Soft Computing* 2012; **12**: 506–515.
55. Siddiqi UF, Shiraishi Y, Dahb M, Sait SM. A memory efficient stochastic evolution based algorithm for the multi-objective shortest path problem. *Applied Soft Computing* 2014; **14**: 653–662.
56. Pulido FJ, Mandow L, Cruz JLPDL. Multiobjective shortest path problems with lexicographic goal-based preferences. *European Journal of Operational Research* 2014; **239**: 89–101.
57. Duque D, Lozano L, Medaglia AL. An exact method for biobjective shortest path problem for large-scale road networks. *European Journal of Operational Research* 2015; **242**: 788–797.
58. Godwin T. Finding the Robust Quickest Paths for Call-Taxis in a City Road Network. Proceedings of the 2015 IEEE International Conference on Industrial Engineering and Engineering Management, Singapore, December 6-9: 180–184.
59. Banks J, Carson JS II, Nelson BL, Nicol DM. *Discrete-event System Simulation* (Fifth edn). Pearson: New Delhi, 2013.
60. Parida P, Gangopadya S. Estimation of Fuel Loss During Idling of Vehicles at Signalized Intersections at Delhi. http://www.irc.org.in/ENU/knowledge/archive/Technical%20Papers%20for%20Irc%20Journals/Estimation%20of%20Fuel%20Loss%20During%20Idling%20of%20Vehicles%20at%20Signalised%20Intersections%20in%20Delhi.pdf [25 March 2015].
61. Neha B, Sushil P, Saishav P. Traffic congestion and fuel wastage due to idling vehicles at crossroads. *National Journal of Community Medicine* 2010; **1**(1): 41–43.
62. Dijkstra EW. A note on two problems in connection with graphs. *Numerical Mathematics* 1959; **1**: 269–271.

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