2(b)

(i) Show $\phi_1 = \delta + \epsilon_1 - \alpha_{11}$

$$A = LDL^T$$

$$\begin{pmatrix} A_{00} & \alpha_{10} e_L & 0 \\ \alpha_{10} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ 0 & \alpha_{21} e_F & A_{22} \end{pmatrix} =$$

$$\left(\begin{array}{c|c|c} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10} e_L & 0 \\ \lambda_{10} e_L^T D_{00} L_{00}^T & \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \delta_1 & \delta_1 \lambda_{21} e_F^T \\ 0 & \lambda_{21} e_F \delta_1 & \lambda_{21} e_F \delta_1 \lambda_{21} e_F^T + L_{22} D_{22} L_{22}^T \end{array} \right.$$

$$\boxed{\alpha_{11} = \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \delta_1}$$ ①

$A = U \Sigma U^T$

$$= \begin{pmatrix} U_{00} \Sigma_{00} U_{00}^T + v_{01} e_L \epsilon_1 v_{01} e_L^T & v_{01} \epsilon_1 & 0 \\ \epsilon_1 v_{01} e_L^T & \epsilon_1 + v_{12} e_F^T \Sigma_{22} v_{12} e_F & \cdots \\ 0 & U_{22} \Sigma_{22} v_{12} e_F & \vdots \end{pmatrix}$$

$$\boxed{\alpha_{11} = \epsilon_1 + v_{12} e_F^T \Sigma_{22} v_{12} e_F} \quad (2)$$

using factorization

$$\boxed{\alpha_{11} = t_0 e_L^T P_{00} t_0 e_L + \phi_1 + v_{12} e_F^T \Sigma_{22} v_{12} e_F}$$

using (1) & (2)

$$\alpha_{11} = \alpha_{11} - \delta_1 + \phi_1 + \alpha_{11} - \epsilon_1$$

$$\phi_1 = \delta_1 + \epsilon_1 - \alpha_{11}$$

(ii) Cost of twisted factorization

$\phi_1 = \delta_1 + c_1 - 2_{1,1}$

$O(1)$

(iii) Cost of all twisted factorization for $n \times n$ (n diagonal elements)

$O(n)$