2(c) compute x_0, x_1, x_2

$$\begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_0 & 1 & r_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix}^T \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} L_{00}^T & \lambda_0 e_L & 0 \\ 0 & 1 & 0 \\ 0 & r_{12} e_F & U_{22}^T \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$L_{00}^T x_0 + \lambda_{10} e_L x_1 = 0$$

$$x_1 = 1$$

$$v_{12} e_F x_1 + U_{22}^T x_2 = 0$$

$$\Rightarrow \quad L_{00}^T x_0 = -\lambda_{10} e_2 \quad ①$$

$$U_{22}^T x_2 = -v_{12} e_F \quad ②$$

$$x_1 = 1$$

$$\lambda_{10} e_L = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \lambda_{10} \end{pmatrix}$$

$$v_{12} e_F = \begin{pmatrix} v_{12} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

If we observe closely

L_{00}, U_{22} have special bidiagonal

structure

If A is a $n \times n$ matrix

$$m(L_{00}) + m(U_{22}) = n-1$$

$$l_{ii+1} = 0$$

hence all entries vectors be solved

$$O(n)$$