2.a

KCT738
Final ALAA

Given a symmetric Tri diagonal matrix
derive $A = UDU^T$

$$A \rightarrow \begin{pmatrix} A_{FF} & \alpha_{FM} e_L & 0 \\ & \alpha_{mm} & \alpha_{mL} e_F^T \\ & & A_{LL} \end{pmatrix}$$

$$= \begin{pmatrix} \ddots & & & \\ & & \alpha_{mF} & \\ & a_{mf} & a_{mm} & \alpha_{Lm} \\ & & \alpha_{Lm} & \ddots \end{pmatrix}$$

e_L & e_F are standard basis vectors

$$\alpha_{mF} e_L = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \alpha_{mF} \end{pmatrix}$$

$$\alpha_{m\cancel{F}} e_F^T = \alpha_{mL} (1, 0 \cdots 0)$$

$$= (\alpha_{mL} \; 0 \cdots 0)$$

we repartition

$$
\begin{pmatrix}
A_{FF} & \alpha_{Fm} e_L & 0 \\
 & \alpha_{mm} & \alpha_{mL} e_F^T \\
 & & A_{LL}
\end{pmatrix}
$$

$$
\begin{pmatrix}
A_{00} & \alpha_{01} e_L & 0 & 0 \\
* & \alpha_{11} & \alpha_{12} & 0 \\
* & * & \alpha_{22} & \alpha_{23} e_F^T \\
* & * & * & A_{33}
\end{pmatrix}
$$

$$
U \rightarrow \begin{pmatrix}
U_{00} & u_{01} \\
0 & 1
\end{pmatrix}
$$

$$
D \rightarrow \begin{pmatrix}
D_{00} & 0 \\
0 & \delta_1
\end{pmatrix}
$$

$$UDU^T = \left(\begin{array}{c|c} V_{00} D_{00} V_{00}^T + \delta_1 v_{01} v_{01}^T & v_{01} \delta_1 \\ * & \delta_1 \end{array} \right)$$

$$A = UDU^T$$

$$\begin{pmatrix} A_{00} & \alpha_{01} e_L & 0 & 0 \\ * & \alpha_{11} & \alpha_{12} & 0 \\ * & * & \alpha_{22} & \alpha_{23} e_F^T \\ * & * & * & A_{33} \end{pmatrix}$$

$$= \left(\begin{array}{c|c} V_{00} D_{00} V_{00}^T & v_{01} \delta_1 \\ \hline * & \delta_1 \end{array} \right)$$

The algorithm overwrites the upper triangular part of A

$$\alpha_{22} := \delta_1 = \alpha_{22}$$

$$\begin{pmatrix} 0 \\ \alpha_{12} \end{pmatrix} = \upsilon_{01} \delta_1$$

$$\Rightarrow \begin{pmatrix} \bar{0} \\ \alpha_{12} \end{pmatrix} := \frac{1}{\alpha_{22}} \begin{pmatrix} 0 \\ \alpha_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_{12}/\alpha_{22} \end{pmatrix}$$

$$\alpha_{12} := \frac{\alpha_{12}}{\alpha_{22}}$$

$$A_{00} := A_{00} - \begin{pmatrix} 0 \\ \alpha_{12} \end{pmatrix} \begin{pmatrix} 0 \\ \alpha_{12} \end{pmatrix}^T$$

$$:= \left(\begin{array}{c|c} A_{00} & \alpha_{01} e_L \\ \hline * & \alpha_{11} - \alpha_{12}^2 \end{array}\right)$$

Algo:-

Partition $A \Rightarrow \begin{pmatrix} A_{FF} & \alpha_{Fm} e_L^T & 0 \\ * & \alpha_{mm} & \alpha_{mL} e_F^T \\ * & * & A_{LL} \end{pmatrix}$

where $A_{LL} = 0 \times 0$

While $m(A_{LL}) < m(A)$ do

repartition

$$\left(\begin{array}{c|c|c} A_{FF} & \alpha_{Fm} e_L & 0 \\ \hline * & \alpha_{mm} & \alpha_{mL} e_F^T \\ \hline * & * & A_{LL} \end{array}\right) \rightarrow$$

$$\begin{pmatrix} A_{00} & \alpha_{01} e_L & 0 & 0 \\ * & \alpha_{11} & \alpha_{12} & 0 \\ * & * & \alpha_{22} & \alpha_{23} e_F^T \\ * & * & * & A_{33} \end{pmatrix}$$

where:

$$\alpha_{11} := \alpha_{11} - \alpha_{12}^2$$

$$\alpha_{12} := \frac{\alpha_{12}}{\alpha_{22}}$$

Continue with

$$\begin{pmatrix} A_{FF} & \alpha_{FM} e_L & 0 \\ * & \alpha_{MM} & \alpha_{ML} e_F^T \\ * & * & A_{LL} \end{pmatrix}$$

End while