2)

a) $n = 1$

$A = [a_{11}]$

Error $\hat{u}_{11} = \frac{a_{11}}{1+\epsilon}$

$$\hat{u}_{11} = \frac{a_{11}}{1+\theta_1}$$

$$\hat{u}_{11}(1+\theta_1) = a_{11}$$

$$\hat{u}_{11} + \hat{u}_{11}\theta_1 = a_{11}$$

$$\hat{u}_{11} = a_{11} - \hat{u}_{11}\theta_1$$

$$\hat{u}_{11} = a_{11} + \delta a_{11} \quad \& \quad \delta a_{11} = -\hat{u}\theta_1$$

$$\|\delta a_{11}\| = \|\hat{u}_1, \theta_1\|$$

$$\check{L}\check{U} = A + \Delta A$$

$$\check{L} = [1]$$

$$\hat{U} = [\check{u}_{11}]$$

$$|\delta a_{11}| = |\theta_1 \check{u}_{11}|$$

$$= |\theta_1 \check{u}_{11}|$$

$$\le \gamma_1 |\check{u}_{11}| \quad \& \quad |\theta_1| \le \gamma_1$$

$$= \gamma_n \check{u}_{11}$$

$$= \gamma_n |1| |\check{u}_{11}|$$

$$= \gamma_n |\check{L}| |\check{U}|$$

$$|\delta a_{11}| \le \gamma_n |\check{L}| |\check{U}|$$

$$|\Delta A| \le \gamma_n |\check{L}| |\check{U}|$$

b) induction

$$A_{00} + \Delta A_{00} = \check{L}_{00} \check{U}_{00}$$

$$|\Delta A_{00}| \leq \gamma_n |\check{L}_{00}| |\check{U}_{00}|$$

$$A \in R^{(n+1) (n+1)}$$

$$A = \left(\begin{array}{c|c} A_{00} & a_{01} \\ \hline a_{10}^T & \alpha_{11} \end{array}\right)$$

$\to$ $$|A_{00} + \Delta A_{00} = \check{L}_{00} \check{U}_{00}$$

$$|\Delta A_{00}| \leq \gamma_n |\check{L}_{00}| |\check{U}_{00}|$$

$$\boxed{|\Delta A_{00}| \leq \gamma_{n+1} |\check{L}_{00}| |\check{U}_{00}|}$$ Coul

$\to$ Using Theorem 6.4.1.3

$$y = (L + \Delta L)\breve{x}$$

$$|\Delta L| \leq \max(\gamma\gamma_{n+1})|L|$$

$$y = L\breve{x} + \Delta L\breve{x}$$

$$|L\breve{x} - y| = |\Delta L\breve{x}|$$

$$|\Delta L\breve{x}| = |\Delta L||\breve{x}|$$

$$\leq \gamma_n |L| |\breve{x}|$$

$$L_{00}\breve{v}_{01} = \breve{a}_{01} - \Delta \breve{L}_{00}\breve{v}_{01}$$

setting $\delta a_{01} = -\Delta L_{00}\breve{v}_{01}$

$$|\delta a_{a01}| = |\Delta\breve{L}_{00}||\breve{v}_{01}|$$

$$\leq \gamma_{n+1}|\breve{L}_{00}||\breve{v}_{01}|$$

eq2 —

$$\boxed{\delta a_{01} \leq \gamma_{n+1}|\breve{L}_{00}||\breve{v}_{01}|}$$

$\rightarrow \quad u_{11} = \alpha_{11} - \check{l}_{10}^{T} \check{u}_{01}$

for given $u = y - Ax$.

$$\check{u} = y - Ax + \delta u$$

where $\delta u \leq \gamma_n |A||x| +$

$$\gamma_1 |\check{u}|$$

$$\leq \gamma_n (|A||x| + |\check{u}|)$$

$$\delta u = \delta y$$

$$\hat{u} = y - Ax + \delta y$$

$$\bar{\delta y} \leq \gamma_n (|A||x| + |\hat{u}|)$$

$$u_{11} = \alpha_{11} - \check{l}_{10}^{T} \check{u}_{01}$$

Eq.-3 $\boxed{\check{u}_{11} = \alpha_{11} - \check{l}_{10}^{T} \check{u}_{01} + \delta\alpha_{11}}$

$$\rightarrow \lambda_{10}^{T} \breve{v}_{00} = a_{10}^{T}$$

$$\breve{v}_{00}^{T} \lambda_{10} = a_{10}$$

$$(\breve{v}_{10}^{T} + \Delta \breve{v}_{00}^{T}) \breve{\lambda}_{10} = a_{10}$$

$$\breve{v}_{10}^{T} \breve{\lambda}_{10} + \Delta \breve{v}_{00}^{T} \breve{\lambda}_{10} = a_{10}$$

$$a_{10}^{T} = \breve{\lambda}_{10}^{T} \breve{v}_{00} + \breve{\lambda}_{10}^{T} \Delta \breve{v}_{00}$$

$$\breve{\lambda}_{10}^{T} \breve{v}_{00} = a_{10}^{T} \; \delta a_{10}^{T}$$

$$|\delta a_{10}^{T}| = |\breve{\lambda}_{10}^{T} \Delta \breve{v}_{00}|$$

$$\leq (|\breve{\lambda}_{10}^{T}| \, |\gamma_n| \, |\breve{v}_{00}|)$$

$$= \gamma_n (|\breve{\lambda}_{10}^{T}| \, |\breve{v}_{00}|)$$

$$\boxed{|\delta a_{10}^{T}| \leq \gamma_{n+1} (|\breve{\lambda}_{10}^{T}| \, |v_{00}|)}$$

using Eqn - 1, 2, 3, 4

$$\left(\begin{array}{c|c} A_{00} + \Delta A_{00} & a_{01} + \delta a_{01} \\ \hline a_{10}^{T} + \delta a_{10}^{T} & \alpha_{11} + \delta \alpha_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} \breve{L}_{00} \breve{U}_{00} & \breve{L}_{00} \breve{u}_{01} \\ \hline \breve{l}_{10}^{T} \breve{U}_{00} & \breve{l}_{10}^{T} \breve{u}_{01} + \breve{\upsilon}_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} \breve{L}_{00} & 0 \\ \hline \breve{l}_{10}^{T} & 1 \end{array}\right) \left(\begin{array}{c|c} \breve{U}_{00} & \breve{u}_{01} \\ \hline 0 & \breve{\upsilon}_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} \breve{L}_{00} \breve{U}_{00} + 0 & \breve{L}_{00} \breve{u}_{01} + 0 \\ \hline \breve{l}_{10}^{T} \breve{U}_{00} + 0 & \breve{l}_{10}^{T} \breve{u}_{01} + \breve{\upsilon}_{11} \end{array}\right)$$

which implies

$$\left(\begin{array}{c|c} A_{00} & a_{01} \\ \hline a_{10}^{T} & \alpha_{11} \end{array}\right) + \left(\begin{array}{c|c} \Delta A_{00} & \delta a_{01} \\ \hline \delta a_{10}^{T} & \delta \alpha_{11} \end{array}\right)$$

$$= \left(\begin{array}{c|c} \breve{L}_{00} & 0 \\ \hline \breve{l}_{10}^{T} & 1 \end{array}\right) \left(\begin{array}{c|c} \breve{U}_{00} & \breve{u}_{01} \\ \hline 0 & \breve{\upsilon}_{11} \end{array}\right)$$

$$\begin{pmatrix} \|\Delta A_{00}\| & \|\delta a_{01}\| \\ \|\delta a_{10}^T\| & \delta \alpha_{11} \end{pmatrix} \le$$

$$\gamma_{n+1} \left| \begin{array}{c|c} \check{L}_{00} \check{U}_{00} & \check{L}_{00} \check{u}_{01} \\ \hline \check{l}_{10}^T \check{U}_{00} & \check{l}_{10}^T \check{u}_{01} + \check{\upsilon}_{11} \end{array} \right|$$

$$\le \gamma_{n+1} \left| \left(\begin{array}{c|c} \check{L}_{00} & 0 \\ \hline \check{l}_{10}^T & 1 \end{array} \right) \right| \left| \left(\begin{array}{c|c} \check{U}_{00} & \check{u}_{01} \\ \hline 0 & \check{\upsilon}_{11} \end{array} \right) \right|$$

The above result holds true for all a