

Instructions:

'droptol' set to $1e-3$ (very low)
and x defined as a random vector,

Method of Steepest Descent (no preconditioning) ~ 18000

Preconditioned Method of Steepest Descent (incomplete Cholesky) - 70

CG (no preconditioning) ~ 240

PCG (incomplete Cholesky) ~ 20

Notes and Observations:

- 1) The number of iterations in the Method of Steepest is consistent. A is well-conditioned, Preconditioning it with a matrix M that resembles A brings it closer to resembling an Identity matrix which is an example of a well-conditioned matrix. The smaller the value of 'droptol', the more M resembles A . Using a 'droptol' of $1e-3$, the preconditioned Method of Steepest Descent reduced the number of iterations from 18000 to 70, which is less than 260 times the number of iterations taken by the Method of Steepest Descent without preconditioning.
- 2) Conjugate Gradient Descent benefits from using A -conjugate search directions such that the next search direction is the minimum in the span of all the previous search directions, instead of restricting x to be on the union of lines defined by $x_{(j)} + \alpha p_{(j)}$ (where $j=0, \dots, k$). Hence, it takes 230 iterations which is less than 75 times the number of iterations taken by the Method of Steepest Descent. However, it doesn't outperform the preconditioned Method of Steepest Descent, which shows how the conditioning of a linear system A has a big impact on the number of iterations to converge to a solution.
- 3) Preconditioning the Conjugate Gradient Method accelerates its performance as expected and brings the number of iterations to convergence to 20, less than half the number of iterations taken by the preconditioned Method of Steepest Descent which is expected since it leverages a better way to pick the next search direction (as discussed above).