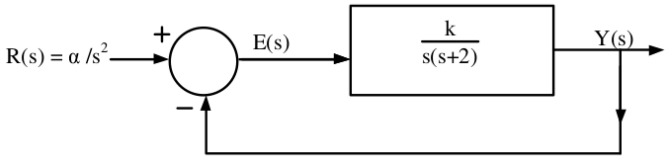
EE23BTECH11024 - G.Karthik Yadav*

GATE 2023 EC 41

1. A Closed loop systen is shown in the figure where $k > 0$ and $\alpha > 0$.
The Steady State error due to a ramp input $(R(s) = \alpha s^{-2})$ is given by (GATE 2023 EC 41)



1) $\frac{2\alpha}{k}$
2) $\frac{\alpha}{k}$
3) $\frac{\alpha}{2k}$
4) $\frac{\alpha}{4k}$

Solution:

Symbol	Parameters	value
$R(s)$	Laplace transform Ramp input signal r(t)	αs^{-2}
$G(s)$	Open Loop transfer function	$\frac{Y(s)}{E(s)} = \frac{k}{s(s+2)}$
$Y(s)$	Laplace transform of the output signal y(t)	?
$E(s)$	Laplace transform of the error signal e(t)	R(s) - Y(s)
e_s	Steady State Error	?

TABLE I
INPUT PARAMETERS

from table I „Open loop transfer function $G(s)$

$$G(s) = \frac{Y(s)}{E(s)} \tag{1}$$

$$= \frac{Y(s)}{R(s) - Y(s)} \tag{2}$$

$$Y(s) = \frac{R(s)\,G(s)}{1 + G(s)} \tag{3}$$

from eq (1) and eq (3)

$$G(s) = \frac{k}{s(s+2)} \tag{4}$$

$$Y(s) = \frac{\alpha k s^{-2}}{k + s(s+2)} \tag{5}$$

$$E(s) = R(s) - Y(s) \tag{6}$$

$$E(s) = \frac{R(s)}{1 + G(s)} \tag{7}$$

By Taking Inverse Laplace Transform of eq (4) and eq (5)

$$g(t) = \frac{k\left(1 - e^{-2t}\right)}{2} u(t) \tag{8}$$

$$e_s = \lim_{s \to 0} sE(s) \tag{9}$$

$$= \lim_{s \to 0} s \frac{R(s)}{1 + G(s)} \tag{10}$$

$$= \lim_{s \to 0} \frac{\alpha(s+2)}{s(s+2) + k} \tag{11}$$

$$e_s = \frac{2\alpha}{k} \tag{12}$$

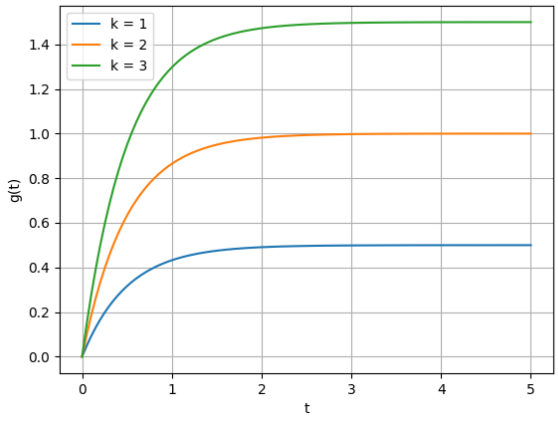


Fig. 1. Plot of $g(t)$ vs t