Operations Research III: Theory

Quiz for Week 7 (Case Study: Regression Models and Support Vector Machine)

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1. Let
$$f(\alpha,\beta)=\sum_{i=1}^{n}\Big[y_i-(\alpha+\beta^{\mathrm{T}}x^i)\Big]^2+\lambda\sum_{j=1}^{p}\beta_j^2$$
be the objective function of ridge regression. Which of the following is $\frac{\partial f(\alpha,\beta)}{\partial\beta_j}$?

   (a) $2\sum_{i=1}^{n}[y_i-(\alpha+\beta^{\mathrm{T}}x^i)]x_j^i+2\lambda$.
   (b) $-2\sum_{i=1}^{n}[y_i-(\alpha+\beta^{\mathrm{T}}x^i)]x_j^i+2\lambda$.
   (c) $2\sum_{i=1}^{n}[y_i-(\alpha+\beta^{\mathrm{T}}x^i)]x_j^i+2\lambda\beta_j$.
   (d) $-2\sum_{i=1}^{n}[y_i-(\alpha+\beta^{\mathrm{T}}x^i)]x_j^i+2\lambda\beta_j$.
   (e) None of the above.

2. What is the outcome of projecting the vector $(1,1)$ onto the vector $(2,1)$?

   (a) $(2,1)$.
   (b) $(\frac{6}{\sqrt{5}},\frac{3}{\sqrt{5}})$.
   (c) $(\frac{6}{5},\frac{3}{5})$.
   (d) $(\frac{4}{5},\frac{2}{5})$.
   (e) None of the above.

3. Consider three data points $(2,1)$, $(4,2)$, and $(1,3)$, where the first two belong to class 1 (say, success) and the last one belongs to class 2 (say, failure). The SVM problem to classify these three points can be formulated as
$$\begin{aligned}\min_{\alpha,\beta_1,\beta_2}\quad & \frac{1}{2}(\beta_1^2+\beta_2^2)\\ \text{s.t.}\quad & \alpha+4\beta_1+2\beta_2\geq 1\\ & \alpha+2\beta_1+\beta_2\geq 1\\ & \alpha+\beta_1+3\beta_2\leq -1.\end{aligned}$$
   Which of the following is an optimal solution to the above nonlinear program?

   (a) $(\alpha,\beta_1,\beta_2)=(1,0.4,0.8)$.
   (b) $(\alpha,\beta_1,\beta_2)=(1,0.4,-0.8)$.
   (c) $(\alpha,\beta_1,\beta_2)=(1,-0.4,0.8)$.
   (d) $(\alpha,\beta_1,\beta_2)=(-1,0.4,-0.8)$.
   (e) None of the above.

4. Continue from the previous question. For such an instance, SVM gives us a line to separate the two classes perfectly, and that line may be depicted on a two-dimensional figure with the three data points. What is the intercept of this line?

   (a) 1.

(b) $\frac{5}{4}$.

(c) $\frac{3}{2}$.

(d) -1.

(e) None of the above.

Note for the instructing team only: The line is $x_2 = \frac{-\alpha-\beta_1 x_1}{\beta_2} = \frac{5}{4} + \frac{1}{2}x_1$. The figure is in Figure 1.

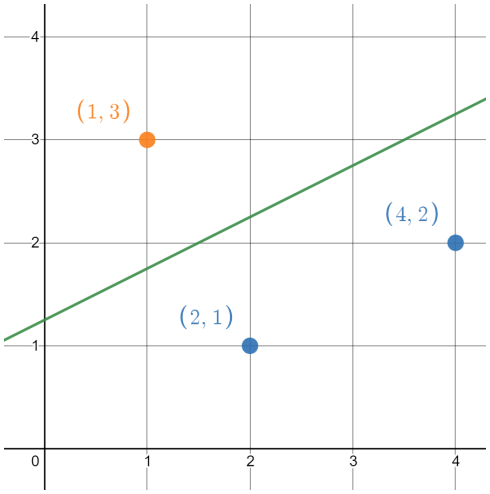

Figure 1: Visualization of the classification result

5. For the following statements, select all that are correct:

   (a) The primal SVM problem disallowing violation is a convex program.

   (b) The primal SVM problem disallowing violation may be infeasible.

   (c) The primal SVM problem allowing violation is a convex program.

   (d) The primal SVM problem allowing violation may be infeasible.

   (e) The dual SVM problem is a convex program.