Operations Research III: Theory

Quiz for Week 1 (Course Overview)

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1. Consider the following linear program

$$\begin{aligned} \max \quad & x_1 + 3x_2 \\ \text{s.t.} \quad & -x_1 + x_2 \leq 3 \\ & -x_1 + 2x_2 \leq 8 \\ & 3x_1 + x_2 \leq 18 \\ & x_i \geq 0 \quad \forall i = 1, 2. \end{aligned}$$

   What is an optimal solution to this linear program?

   (a) $(4, 6)$.
   (b) $(6, 0)$.
   (c) $(0, 3)$.
   (d) $(0, 0)$.
   (e) None of the above.

2. Continue from the previous question. Let the slack variables for constraints 1, 2, and 3 be x_3, x_4, and x_5, respectively. Consider the basis $B = (x_2, x_4, x_5)$. What is the corresponding basic feasible solution?

   (a) There is no corresponding basic feasible solution.
   (b) $(0, 0, 3, 8, 18)$.
   (c) $(0, 3, 2, 0, 15)$.
   (d) $(0, 2, 3, 0, 15)$.
   (e) None of the above.

3. Continue from the previous question. Let $N = (x_1, x_3)$ be the set of nonbasic variables. What is the matrix A_N?

   (a) $\begin{bmatrix} -1 & 1 \\ -1 & 2 \\ 3 & 1 \end{bmatrix}$.
   (b) $\begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 3 & 0 \end{bmatrix}$.
   (c) $\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 3 & 0 \end{bmatrix}$.
   (d) $\begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 4 & -1 \end{bmatrix}$.
   (e) None of the above.

4. Continue from the previous question. What is the transpose of the reduced cost vector $\bar{c}_N^T$?

(a) $\begin{bmatrix} 1 & 0 \end{bmatrix}$.

(b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$.

(c) $\begin{bmatrix} -4 & 3 \end{bmatrix}$.

(d) $\begin{bmatrix} 3 & -4 \end{bmatrix}$.

(e) None of the above.

5. Continue from the previous question. According to the reduced costs and ratio test, which variable will be the leaving variable at this basis?

(a) x_1.

(b) x_2.

(c) x_4.

(d) x_5.

(e) None of the above.