# Operations Research III: Theory

# Quiz for Week 6 (Lagrange Duality and the KKT Condition)

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1. Consider the following nonlinear program

$$\begin{aligned} \min \quad & (x_1-4)^2+(x_2-2)^2 \\ \text{s.t.} \quad & 2x_1+x_2 \leq 6. \end{aligned}$$

Let $f(x_1,x_2)=(x_1-4)^2+(x_2-2)^2$ be the objective function. What are the leading principal minors of the Hessian matrix of $f(x_1,x_2)$?

(a) 0 and 2.
(b) 0 and 4.
(c) 2 and 2.
(d) 2 and 4.
(e) None of the above.

2. Continue from the previous question. For the Lagrangian

$$\mathcal{L}(x_1,x_2|\lambda)=(x_1-4)^2+(x_2-2)^2+\lambda(6-2x_1-x_2),$$

which of the following statements is correct?

(a) We should have $\lambda \geq 0$.
(b) We should have $\lambda \leq 0$.
(c) We should have $\lambda > 0$.
(d) We should have $\lambda < 0$.
(e) None of the above.

3. Continue from the previous question. According to the first-order condition of the Lagrangian, which of the following is a necessary condition for an optimal solution?

(a) $x_1+2x_2=0$.
(b) $x_1-2x_2=0$.
(c) $2x_1-x_2=0$.
(d) $2x_1+2x_2=0$.
(e) None of the above.

4. Continue from the previous question. Which of the following is a local optimal solution to the nonlinear program?

(a) $(4,2)$.
(b) $(\frac{12}{5},\frac{6}{5})$.
(c) $(3,0)$.
(d) $(0,6)$.
(e) None of the above.

5. For the following statements, select all that are correct:

(a) For a linear program, linear programming duality and Lagrange duality is equivalent (i.e., the dual programs obtained through the two ways are identical).

(b) For an unconstrained nonlinear program, the KKT condition is equivalent to the first-order condition.

(c) For an unconstrained nonlinear program, the KKT condition is necessary and sufficient.

(d) For a constrained nonlinear program, Lagrange duality provides a tight bound.

(e) None of the above.