Operations Research III: Theory

Quiz for Week 3 (Sensitivity Analysis and Dual Simplex)

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. Consider the following linear program

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leq 4 \\ & x_1 + 2x_2 \leq 6 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

   Let s_1 and s_2 be the slack variables for the first and second constraints in the standard form, respectively. Consider the basis $B = (x_1, x_2)$ and $N = (x_3, s_1, s_2)$ as the set of nonbasic variables. Which of the following is the correct values of $c_B^T A_B^{-1} A_N - c_N^T$?

   (a) $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$.
   (b) $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$.
   (c) $\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}$.
   (d) $\begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$.
   (e) None of the above.

2. Suppose that a new variable x_4 is added, and the new linear program is

$$\begin{aligned} \max \quad & 2x_1 + 3x_2 + 4x_4 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 - x_4 \leq 4 \\ & x_1 + 2x_2 - 2x_4 \leq 6 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

   Calculate the reduced cost of x_4 to determine whether it should be entered. Which of the following statement(s) is (are) correct? Check all correct answers.

   (a) The reduced cost of x_4 is nonnegative.
   (b) The reduced cost of x_4 is negative.
   (c) The new program is infeasible.
   (d) The new program is unbounded.
   (e) The original optimal basis does not change after x_4 is added.

   Note only for the instructing team: We have

$$c_B^T A_B^{-1} A_j - c_j = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} - 4 = -7.$$

3. Continue from Question 1. Suppose that a new constraint $x_2 + x_3 \leq 1$ is added. Let s_3 is the slack for the new constraint. Regarding a new basis $B' = (x_1, x_2, s_3)$, calculate $A_{B'}^{-1} b$. Which of the following statement is correct?

   (a) The first constraint is violated by B'.
   (b) The second constraint is violated by B'.
   (c) The third constraint is violated by B'.

(d) B' is feasible.

(e) None of the above.

Note only for the instructing team: We have

$$A_{B'}^{-1}b = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix},$$

4. Continue from the previous question. Let N' be the set of nonbasic variables associated with B'. We have

$$A_{B'}^{-1}A_{N'} = \begin{bmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ ? & 1 & -1 \end{bmatrix}.$$

What is the correct value at the position of the question mark?

(a) 0.

(b) 1.

(c) -1.

(d) -2.

(e) None of the above.

Note only for the instructing team: It should be 2.

5. Continue from the previous question. Which variable should be the entering variable?

(a) x_3.

(b) s_1.

(c) s_2.

(d) s_3.

(e) None of the above.