Operations Research III: Theory

Quiz for Week 2 (Linear Programming Duality)

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

1. Consider the following linear program

$$\begin{aligned} \max \quad & 4x_1 - 2x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 + x_2 \leq 10 \\ & x_2 + x_3 \geq -3 \\ & x_1 + 3x_2 - 3x_3 = 14 \\ & x_1 \geq 0, x_2 \leq 0, x_3 \text{ urs.} \end{aligned}$$

   Let y_1, y_2, and y_3 be the dual variables of the dual linear program, where y_i corresponds to the ith primal constraint. What is the dual constraint corresponding to the primal variable x_2?

   (a) $y_1 + y_2 + 3y_3 \leq -2$.
   (b) $y_1 + y_2 + 3y_3 \geq 2$.
   (c) $y_1 + y_2 + 3y_3 \leq 2$.
   (d) $y_1 + y_2 + 3y_3 \geq -2$.
   (e) None of the above.

2. Continue from the previous question. Let $\bar{x}$ and $\bar{y}$ be primal and dual feasible solutions, respectively, where $\bar{y}_i$ corresponds to the ith primal constraint. Which of the following always hold for $\bar{x}$ and $\bar{y}$?

   (a) $4\bar{x}_1 - 2\bar{x}_2 + \bar{x}_3 \geq 10\bar{y}_1 - 3\bar{y}_2 + 14\bar{y}_3$.
   (b) $4\bar{x}_1 - 2\bar{x}_2 + \bar{x}_3 = 10\bar{y}_1 - 3\bar{y}_2 + 14\bar{y}_3$.
   (c) $4\bar{x}_1 - 2\bar{x}_2 + \bar{x}_3 \leq 10\bar{y}_1 - 3\bar{y}_2 + 14\bar{y}_3$.
   (d) All of the above.
   (e) None of the above.

   Note for the instructing team only: Weak duality.

3. Continue from the previous question. Which of the following always hold for $\bar{x}$ and $\bar{y}$?

   (a) $(10 - 2\bar{x}_1 - \bar{x}_2)\bar{y}_1 = 0$.
   (b) $(-3 - \bar{x}_2 - \bar{x}_3)\bar{y}_2 = 0$.
   (c) $(14 - \bar{x}_1 - 3\bar{x}_2 + 3\bar{x}_3)\bar{y}_3 = 0$.
   (d) All of the above.
   (e) None of the above.

   Note for the instructing team only: Complementary slackness holds only for optimal solutions. It holds for the third constraint because $14 - \bar{x}_1 - 3\bar{x}_2 + 3\bar{x}_3 = 0$ for any primal feasible solution $\bar{x}$.

4. Continue from the previous question. Let x^* and y^* be primal and dual optimal solutions, respectively, where y_i^* corresponds to the ith primal constraint. Which of the following always hold for x^* and y^*?

   (a) $y_1^* \geq 0$.
   (b) $y_2^* \geq 0$.

(c) $y_3^* \geq 0$.

(d) All of the above.

(e) None of the above.

5. Continue from the previous question. Which of the following statements are correct?

(a) $4x_1^* - 2x_2^* + x_3^* = 10y_1^* - 3y_2^* + 14y_3^*$.

(b) The shadow price of the first primal constraint is y_1^*.

(c) $(14 - x_1^* - 3x_2^* + 3x_3^*)y_3^* = 0$.

(d) All of the above.

(e) None of the above.