PLSC 503 – Spring 2020
Model Fit and Stupid Regression Tricks

January 23, 2020

Variation in Y

$$\begin{aligned}
\mathrm{Var}(Y) &= \mathrm{Var}(\hat{Y} + \hat{u}) \\
&= \mathrm{Var}(\hat{Y}) + \mathrm{Var}(\hat{u}) + 2\,\mathrm{Cov}(\hat{Y}, \hat{u}) \\
&= \mathrm{Var}(\hat{Y}) + \mathrm{Var}(\hat{u})
\end{aligned}$$

TSS	$=$	**MSS**	$+$	**RSS**
("Total")		("Estimated," or "Model")		("Residual")

Regression, Redux

```
> summary(IMDPT)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  173.277      8.489    20.4   <2e-16 ***
DPTpct        -1.576      0.101   -15.6   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 26.2 on 175 degrees of freedom
Multiple R-Squared: 0.582,Adjusted R-squared: 0.58
F-statistic:  244 on 1 and 175 DF,  p-value: <2e-16

> anova(IMDPT)
Analysis of Variance Table

Response: infantmortalityperK
           Df Sum Sq Mean Sq F value Pr(>F)
DPTpct      1 167423  167423     244 <2e-16 ***
Residuals 175 120033     686
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

Moving Parts

$$
\begin{aligned}
TSS &= \text{total variability in } Y \text{ around its mean} \\
&= \sum (Y_i - \bar{Y})^2 \\
&= 167423 + 120033 \\
&= \mathbf{287456}
\end{aligned}
$$

$$
\begin{aligned}
MSS(\equiv \texttt{DPTpct}) &= \text{model (“explained” or “regression”) sum of squares} \\
&= \sum (\hat{Y}_i - \bar{Y})^2 \\
&= \mathbf{167423}
\end{aligned}
$$

$$
\begin{aligned}
RSS(\equiv \texttt{Residuals}) &= \text{residual (“unexplained” or “error”) sum of squares} \\
&= \sum \hat{u}_i^2 \\
&= \mathbf{120033}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\sigma}^2 &= \frac{RSS}{N-k} \\
&= \frac{\sum \hat{u}_i^2}{N-2} \\
&= \frac{120033}{175} \\
&= \mathbf{686}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\sigma} &= \text{“SEE” (the standard error of the estimate, or } \texttt{Residual standard error}\text{)} \\
&= \sqrt{\hat{\sigma}^2} \\
&= \sqrt{686} \\
&= \mathbf{26.2}
\end{aligned}
$$

R^2 Introduced

$$
\begin{aligned}
R^2 &= \frac{\text{MSS}}{\text{TSS}} \\
&= \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} \\
&= 1 - \frac{\text{RSS}}{\text{TSS}} \\
&= 1 - \frac{\sum \hat{u}_i^2}{\sum(Y_i - \bar{Y})^2}
\end{aligned}
$$

R^2 Is...

R-squared:

- is "the proportion of variance explained"
- $\in [0, 1]$
  - $R^2 = 1.0 \equiv$ a "perfect (linear) fit"'
  - $R^2 = 0 \equiv$ no (linear) $X - Y$ association

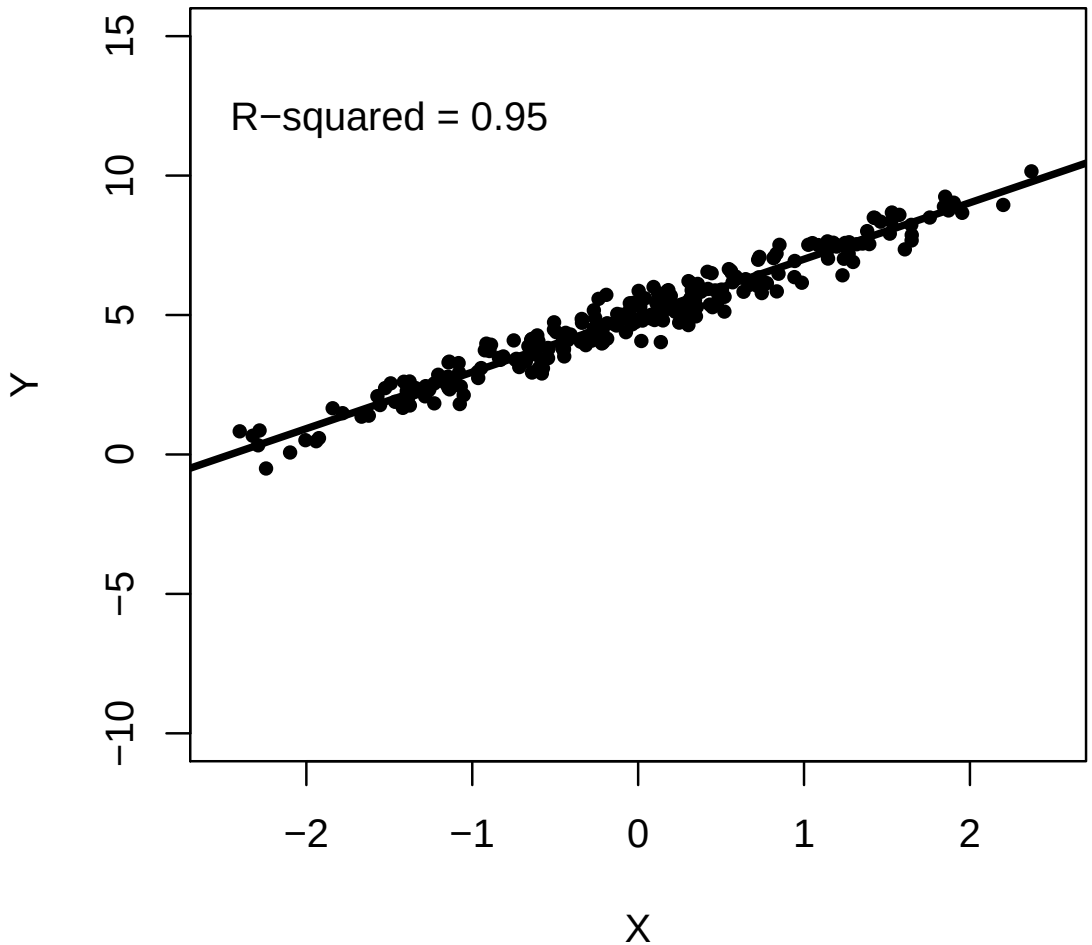
For a single X,

$$\begin{aligned} R^2 &= \hat{\beta}_1^2 \frac{\sum (X_i - \bar{X})^2}{\sum (Y_i - \bar{Y})^2} \\ &= r_{XY}^2 \end{aligned}$$

A (Simulated) Example

```
seed <- 7222009
set.seed(seed)
> X<-rnorm(250)
> Y1<-5+2*X+rnorm(250,mean=0,sd=sqrt(0.2))
> Y2<-5+2*X+rnorm(250,mean=0,sd=sqrt(20))
> fit<-lm(Y1~X)
> summary(fit)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.97712    0.02846  174.86   <2e-16 ***
X            2.02529    0.02785   72.73   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.4491 on 248 degrees of freedom
Multiple R-squared:  0.9552,Adjusted R-squared:  0.955
F-statistic:  5290 on 1 and 248 DF,  p-value: < 2.2e-16
```

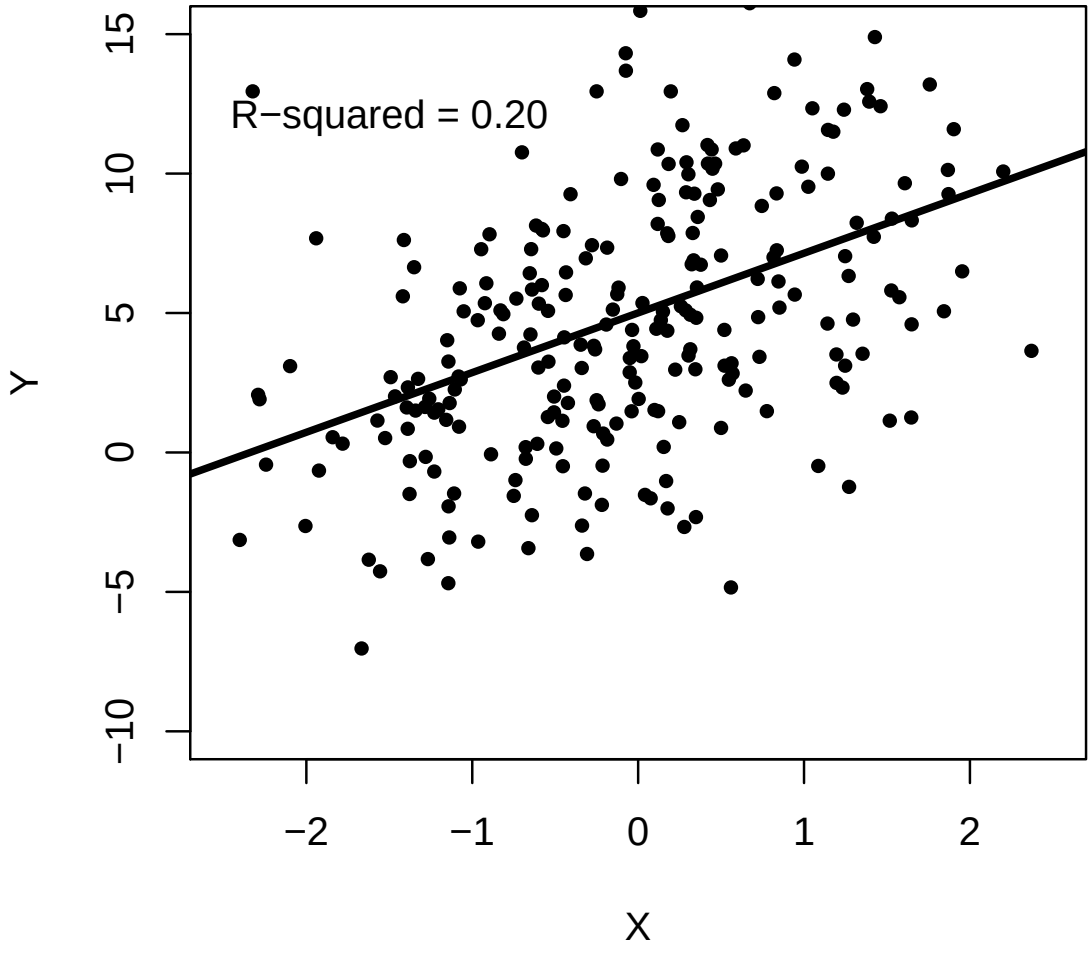
Regression of $Y_i = 5 + 2X_i + u_i$ $(R^2 = 0.95)$



Same Slope/Intercept, Different R^2

```
> fit2<-lm(Y2~X)
> summary(fit2)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.0048     0.2757  18.151  < 2e-16 ***
X             2.1402     0.2697   7.934 7.29e-14 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 4.351 on 248 degrees of freedom
Multiple R-squared:  0.2024,Adjusted R-squared:  0.1992
F-statistic: 62.95 on 1 and 248 DF,  p-value: 7.288e-14
```

Regression of $Y_i = 5 + 2X_i + u_i$ $(R^2 = 0.20)$



R^2 is Also an *Estimate*...

Luskin: Population analogue "P^2":

$$P^2 = 1 - \frac{\sigma^2}{\sigma_Y^2}$$

Then $\hat{P}^2 = R^2$ has variance:

$$\widehat{\text{Var}(R^2)} = \frac{4R^2(1-R^2)^2(N-k)^2}{(N^2-1)(N+3)}$$

and standard error:

$$\widehat{\text{s.e.}(R^2)} = \sqrt{\frac{4R^2(1-R^2)^2(N-k)^2}{(N^2-1)(N+3)}}.$$

Adjusted R^2

$$R^2_{adj.} = 1 - \frac{(1 - R^2)(N - c)}{(N - k)}$$

where $c = 1$ if there is a constant in the model and $c = 0$ otherwise.

$R^2_{adj.}$:

- $R^2_{adj.} \to R^2$ as $N \to \infty$
- $R^2_{adj.}$ can be > 1, or < 0...
- $R^2_{adj.}$ increases with model "fit," but
- The extent of that increase is discounted by a factor proportional to the number of covariates.

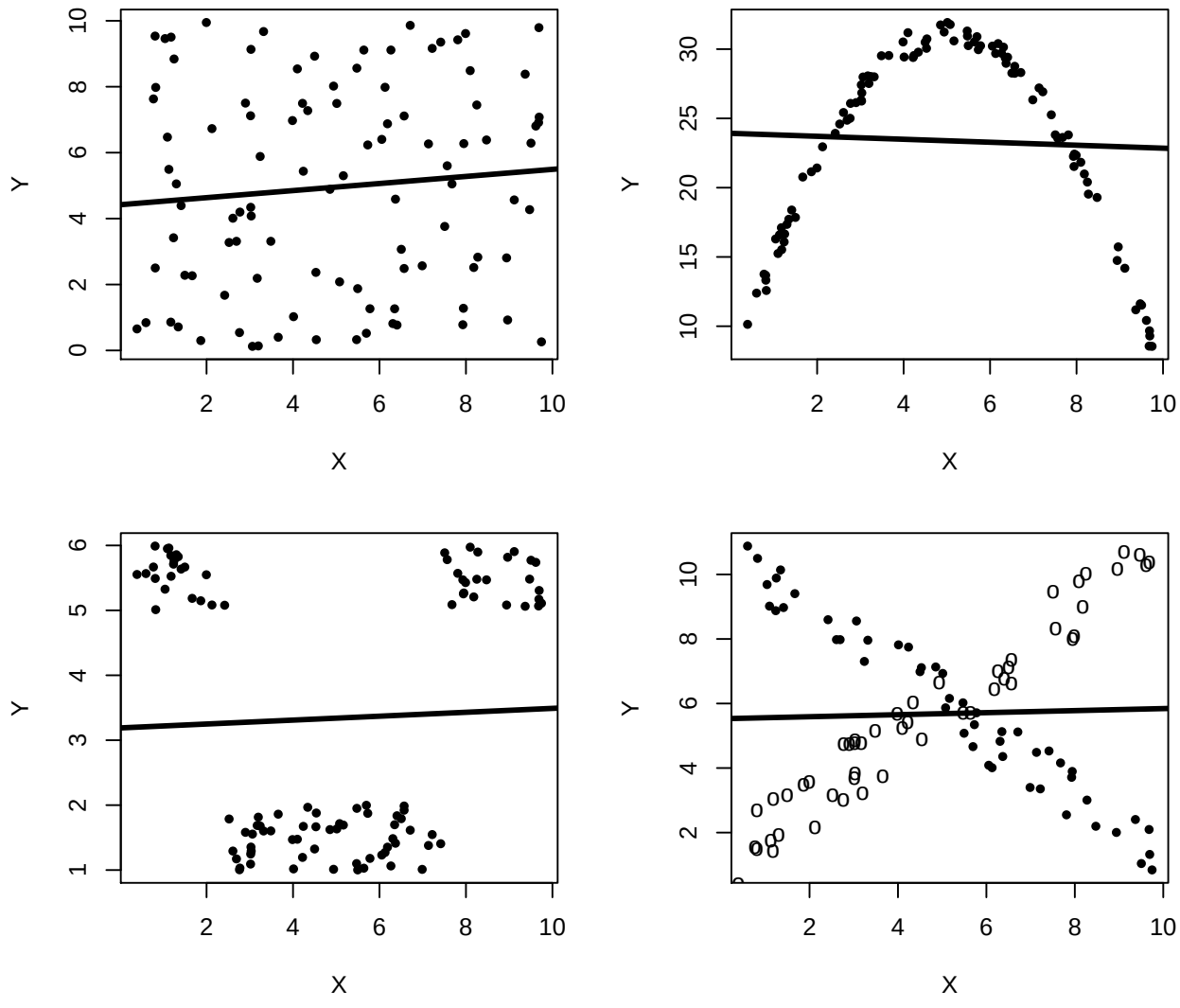
R^2 Alternatives

- Standard Error of the Estimate:

$$\text{SEE} = \sqrt{\frac{\text{RSS}}{N-k}}$$

- F-tests (later...)
- ROC / AUC (later...)
- Graphical methods

Caution: Different Ways to get $R^2 \approx 0$



Stupid Regression Tricks

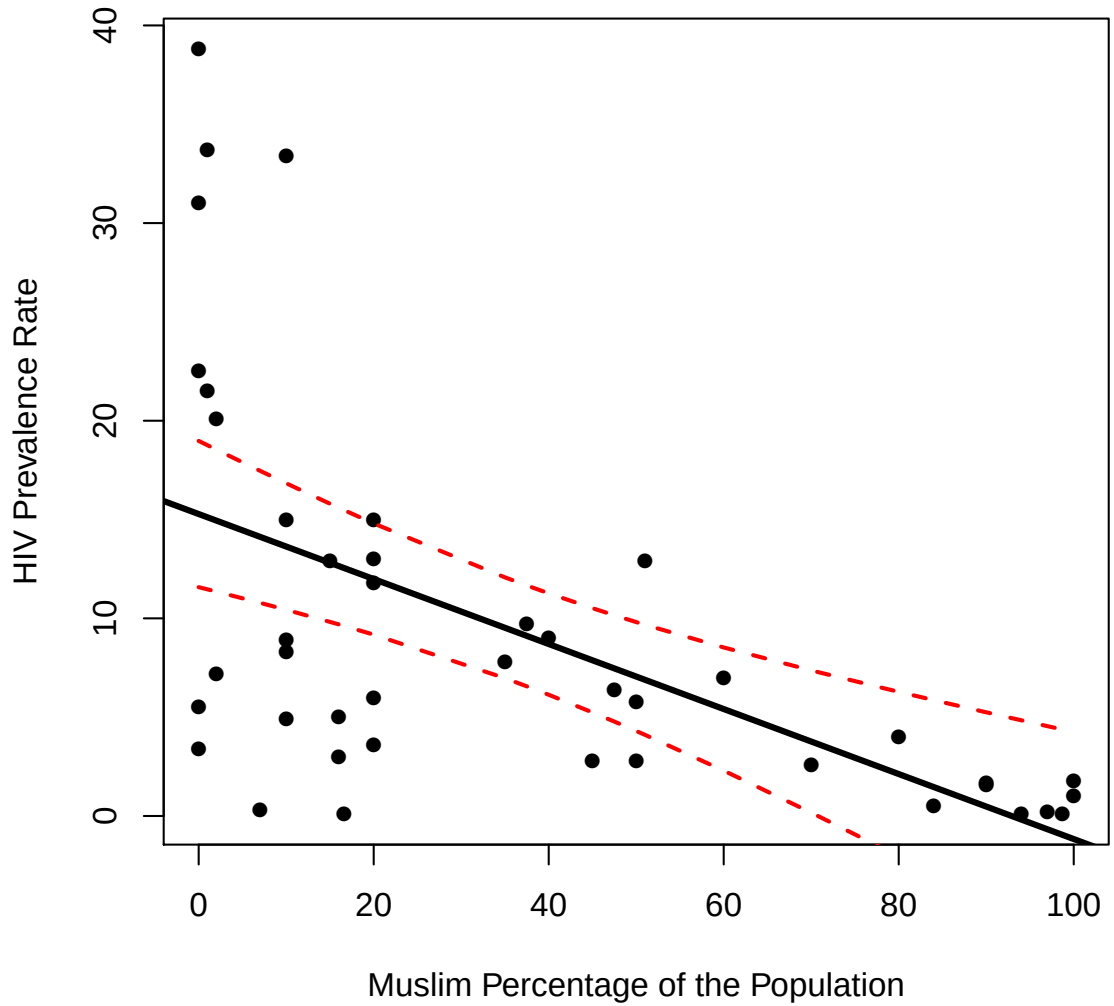
Africa (2001) Data

```
> temp<-getURL("https://raw.githubusercontent.com/PrisonRodeo/PLSC503-2020-git/master/Data/africa2001.csv")
> africa<-read.csv(text=temp, header=TRUE)
> summary(africa)

     ccode          cabbr                         country     population           popthou
 Min.   :404   AGO    : 1   Angola                    : 1   Min.   :   470000   Min.   :   470
 1st Qu.:452   BDI    : 1   Benin                     : 1   1st Qu.:  3446000   1st Qu.:  3446
 Median :510   BEN    : 1   Botswana                  : 1   Median :  9662000   Median :  9662
 Mean   :510   BWA    : 1   Burundi                   : 1   Mean   : 17388558   Mean   : 17390
 3rd Qu.:556   CAF    : 1   Cameroon                  : 1   3rd Qu.: 19150000   3rd Qu.: 19189
 Max.   :651   CIV    : 1   Central African Republic: 1   Max.   :117000000   Max.   :116929
               (Other):37   (Other)                   :37
     popden            polity          gdppppd          tradegdp           war             adrate
 Min.   :0.0022   Min.   :-9.000   Min.   : 0.500   Min.   :  4.03   Min.   :0.000   Min.   : 0.10
 1st Qu.:0.0134   1st Qu.:-4.500   1st Qu.: 0.855   1st Qu.:  7.64   1st Qu.:0.000   1st Qu.: 2.70
 Median :0.0357   Median : 0.000   Median : 1.200   Median : 13.56   Median :0.000   Median : 6.00
 Mean   :0.0643   Mean   : 0.512   Mean   : 2.159   Mean   : 30.49   Mean   :0.116   Mean   : 9.37
 3rd Qu.:0.0683   3rd Qu.: 5.500   3rd Qu.: 2.040   3rd Qu.: 30.01   3rd Qu.:0.000   3rd Qu.:12.90
 Max.   :0.5740   Max.   :10.000   Max.   :10.800   Max.   :272.69   Max.   :1.000   Max.   :38.80

   healthexp              subsaharan    muslperc        literacy     internalwar      intensity
 Min.   :2.00   Not Sub-Saharan: 6   Min.   :  0.0   Min.   :17.0   Min.   :0.000   Min.   :0.000
 1st Qu.:3.45   Sub-Saharan    :37   1st Qu.: 10.0   1st Qu.:43.0   1st Qu.:0.000   1st Qu.:0.000
 Median :4.40                       Median : 20.0   Median :61.0   Median :0.000   Median :0.000
 Mean   :4.60                       Mean   : 36.0   Mean   :60.1   Mean   :0.302   Mean   :0.581
 3rd Qu.:5.80                       3rd Qu.: 55.5   3rd Qu.:78.5   3rd Qu.:1.000   3rd Qu.:1.000
 Max.   :8.60                       Max.   :100.0   Max.   :89.0   Max.   :1.000   Max.   :3.000
```

A Simple Regression

```
> fit<-with(africa, lm(adrate~muslperc))
> summary(fit)

Call:
lm(formula = adrate ~ muslperc)

Residuals:
    Min      1Q  Median      3Q     Max
-13.828  -5.206   0.279   2.022  23.521

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  15.2787     1.8322    8.34 0.00000000023 ***
muslperc     -0.1644     0.0369   -4.45 0.00006390853 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

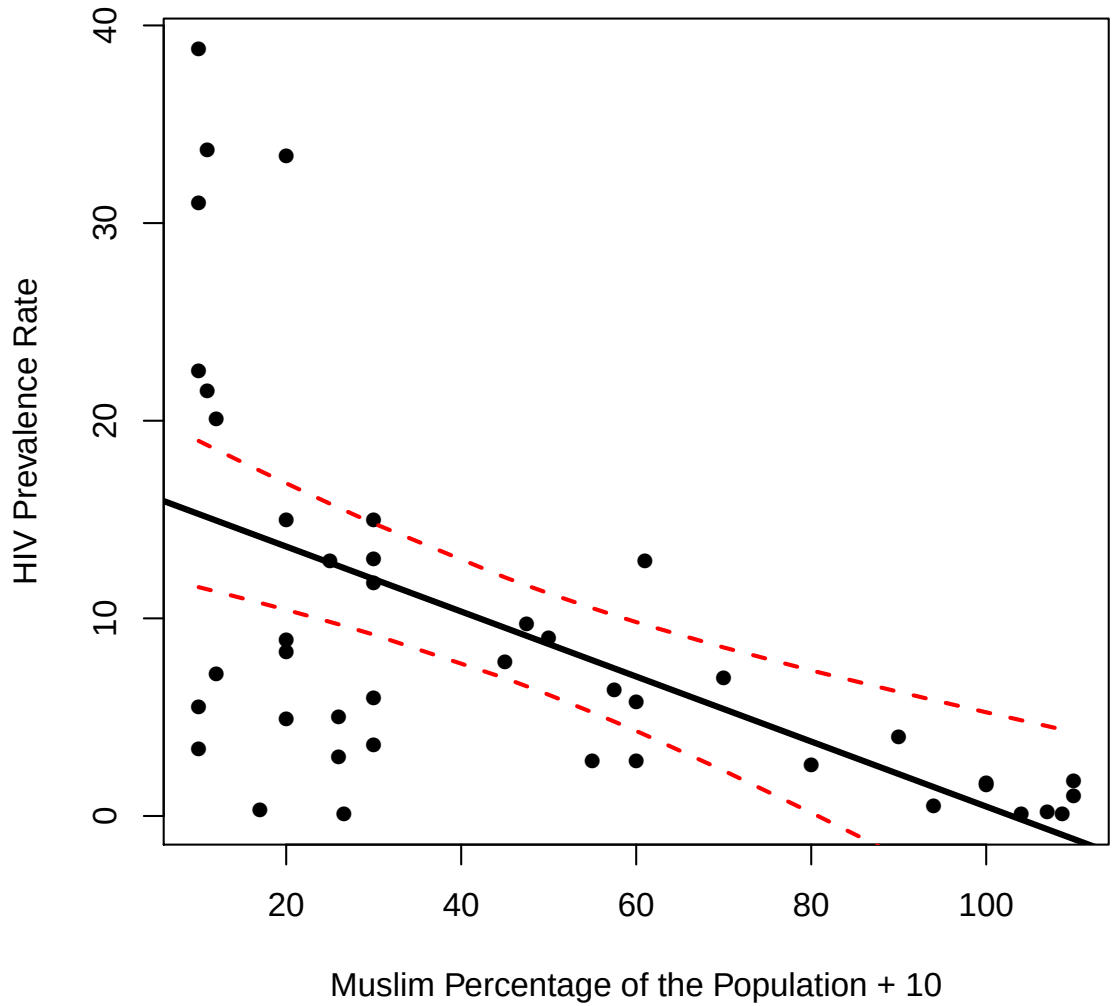
Scatterplot of HIV/AIDS Rates on Muslim Population Percentage, Africa 2001



Adding a Constant to X

```
> africa$muslplusten<-africa$muslperc+10
> fit2<-with(africa, lm(adrate~muslplusten,data=africa))
> summary(fit2)

Call:
lm(formula = adrate ~ muslplusten, data = africa)

Residuals:
    Min      1Q  Median      3Q     Max
-13.828  -5.206   0.279   2.022  23.521

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  16.9232     2.1152    8.00 0.00000000066 ***
muslplusten  -0.1644     0.0369   -4.45 0.00006390853 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

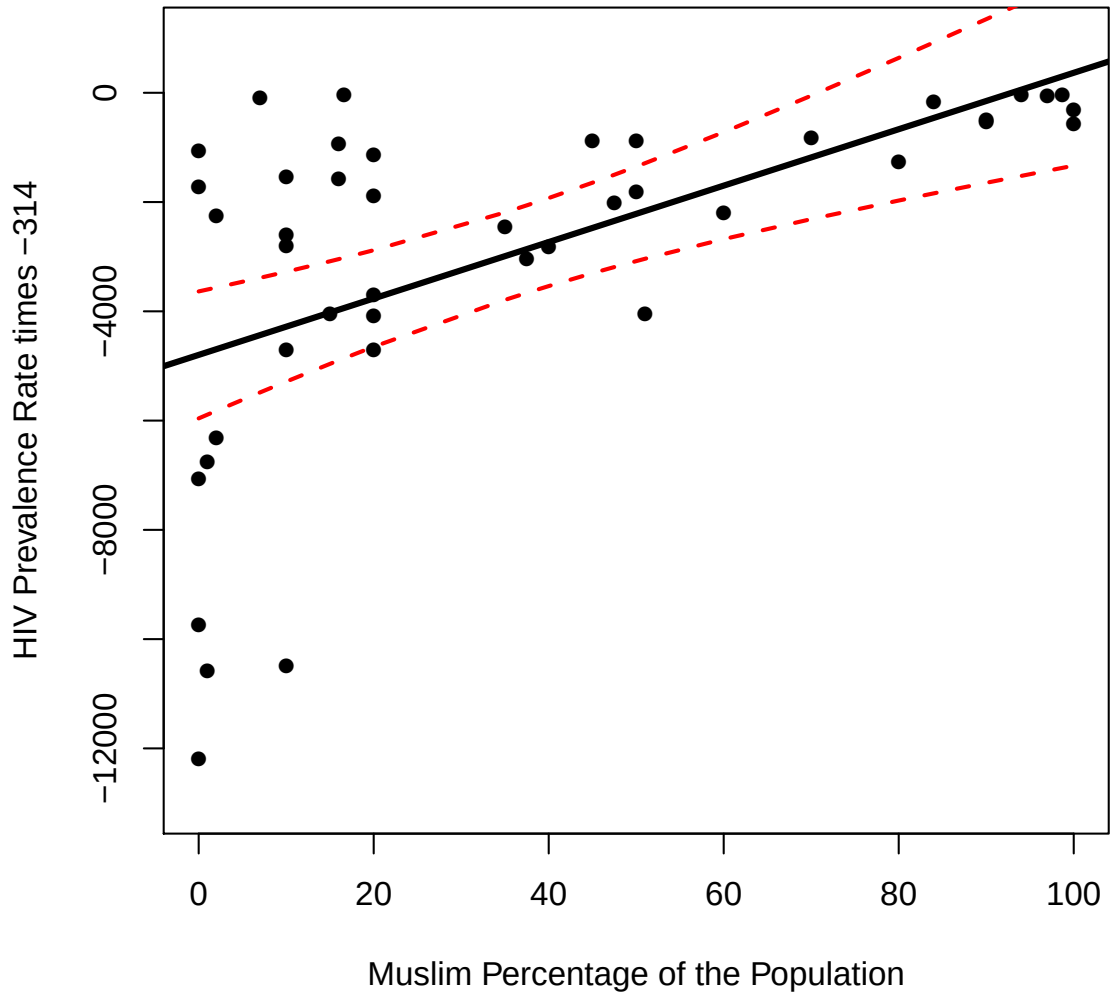
Scatterplot of HIV/AIDS Rates on Rescaled Muslim Population Percentage



Multiplying Y by a Constant

```
> africa$screwyrate<-africa$adrate*(-314)
> fit3<-with(africa, lm(screwyrate~muslperc))
> summary(fit3)

Call:
lm(formula = screwyrate ~ muslperc)

Residuals:
   Min     1Q Median     3Q    Max
 -7386   -635    -88   1635   4342

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  -4797.5      575.3   -8.34 0.00000000023 ***
muslperc        51.6       11.6    4.45 0.00006390853 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 2600 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

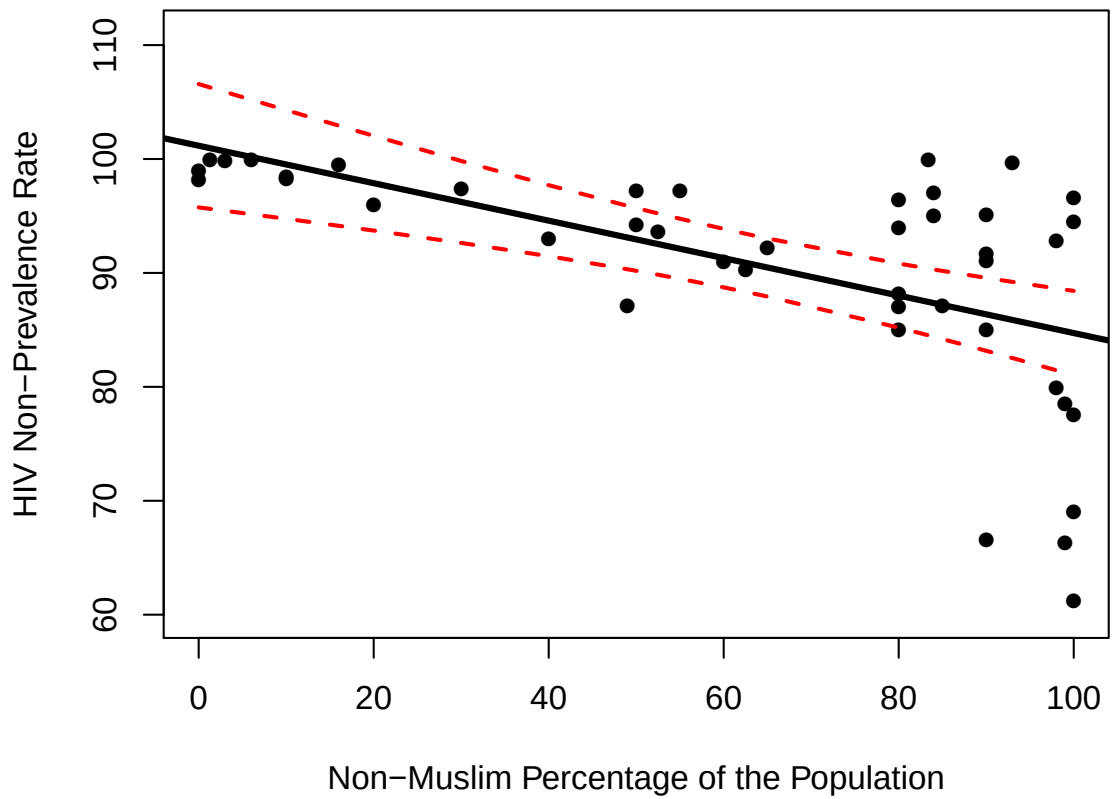
Scatterplot of Rescaled HIV/AIDS Rates on Muslim Population Percentage



Reversing the scales of X and Y

```
> africa$nonmuslimpct <- 100 - africa$muslperc
> africa$noninfected <- 100 - africa$adrate
> fit4<-lm(noninfected~nonmuslimpct,data=africa)
> summary(fit4)

Call:
lm(formula = noninfected ~ nonmuslimpct, data = africa)

Residuals:
    Min      1Q  Median      3Q     Max
-23.521  -2.022  -0.279   5.206  13.828

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  101.1660     2.6808   37.74  < 2e-16 ***
nonmuslimpct  -0.1644     0.0369   -4.45 0.000064 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

Scatterplot of HIV/AIDS Non-Infection Rates on Non-Muslim Population Percentage



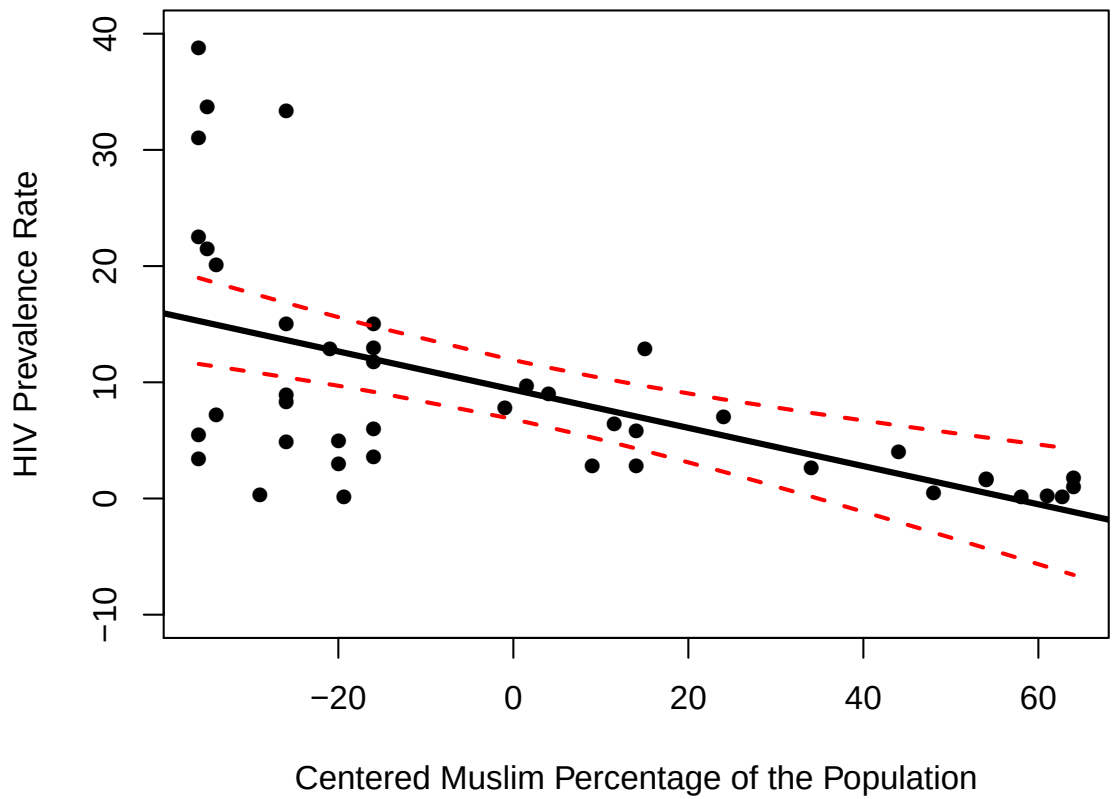
Linear Transformations

- Adding (subtracting) a positive constant to X shifts the X-axis to the left (right).
- Adding (subtracting) a positive constant to Y shifts the Y-axis downwards (upwards).
- Multiplying X (Y) times a positive constant greater than 1.0 stretches the X (Y) axis.
- Multiplying X (Y) times a positive constant less than 1.0 shrinks the X (Y) axis.
- Multiplying X (Y) times a negative constant inverts the X (Y) axis, and stretches / shrinks it as above.

Use: "Centering" a Variable

```
> africa$muslcenter<-africa$muslperc - mean(africa$muslperc, na.rm=TRUE)
> fit5<-lm(adrate~muslcenter,data=africa)
> summary(fit5)

Call:
lm(formula = adrate ~ muslcenter, data = africa)

Residuals:
    Min      1Q  Median      3Q     Max
-13.828  -5.206   0.279   2.022  23.521

Coefficients:
            Estimate Std. Error t value     Pr(>|t|)
(Intercept)   9.3651     1.2622    7.42 0.0000000042 ***
muslcenter   -0.1644     0.0369   -4.45 0.0000639085 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

Scatterplot of HIV/AIDS Infection Rates on (Centered) Muslim Population Percentage



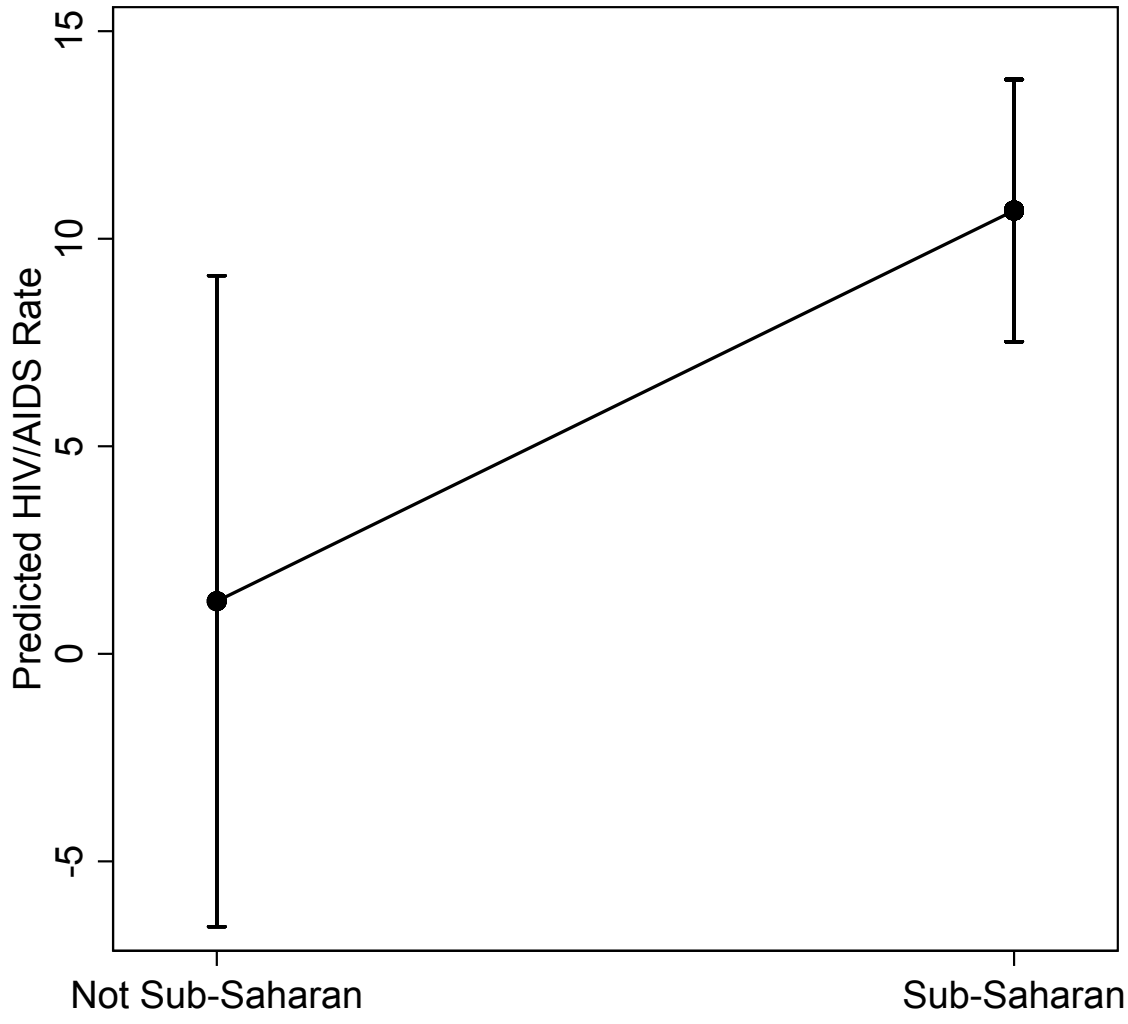
Use: Rescaling X for Interpretability

```
> fit6<-lm(adrate~population,data=africa)
> summary(fit6)

                Estimate    Std. Error t value Pr(>|t|)
(Intercept) 10.5883163475  1.9140361989    5.53 0.000002 ***
population  -0.0000000703  0.0000000671   -1.05      0.3
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared:  0.0261,Adjusted R-squared:  0.00234
F-statistic:  1.1 on 1 and 41 DF,  p-value: 0.301


> africa$popmil<-africa$population / 1000000
> fit7<-lm(adrate~popmil,data=africa)
> summary(fit7)

            Estimate Std. Error t value Pr(>|t|)
(Intercept)  10.5883     1.9140    5.53 0.000002 ***
popmil       -0.0703     0.0671   -1.05      0.3
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 9.95 on 41 degrees of freedom
Multiple R-squared:  0.0261,Adjusted R-squared:  0.00234
F-statistic:  1.1 on 1 and 41 DF,  p-value: 0.301
```

Dichotomous Xs: Bivariate Regression $\equiv$ t-test

```
> fit8<-lm(adrate~subsaharan,data=africa)
> summary(fit8)

Residuals:
   Min     1Q Median     3Q    Max
-10.58  -6.23  -1.78   2.22  28.12

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)              1.27       3.88    0.33     0.75
subsaharanSub-Saharan    9.41       4.19    2.25     0.03 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 9.51 on 41 degrees of freedom
Multiple R-squared:  0.11,Adjusted R-squared:  0.088
F-statistic: 5.05 on 1 and 41 DF,  p-value: 0.03


> with(africa,
+      t.test(adrate~subsaharan, var.equal=TRUE))

Two Sample t-test

data:  adrate by subsaharan
t = -2.2, df = 41, p-value = 0.03
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -17.8659  -0.9576
sample estimates:
mean in group Not Sub-Saharan     mean in group Sub-Saharan
                        1.267                        10.678
```

Expected Values of HIV/AIDS Infection Rates in Saharan and Sub-Saharan Africa



The results:

```
> fit<-lm(adrate~muslperc, data=africa)
> summary.lm(fit)

Call:
lm(formula = adrate ~ muslperc, data = africa)

Residuals:
    Min      1Q  Median      3Q     Max
-13.828  -5.206   0.279   2.022  23.521

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept)  15.2787     1.8322    8.34 0.00000000023 ***
muslperc     -0.1644     0.0369   -4.45 0.00006390853 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.28 on 41 degrees of freedom
Multiple R-squared:  0.326,Adjusted R-squared:  0.31
F-statistic: 19.8 on 1 and 41 DF,  p-value: 0.0000639
```

The table:

Table: OLS Regression Model of HIV/AIDS Rates in Africa, 2001

Variables	Model I
(Constant)	15.28
	(1.83)
Muslim Percentage of the Population	-0.164*
	(0.037)
Adjusted R^2	0.31

Note: $N = 43$. Cell entries are coefficient estimates; numbers in parentheses are estimated standard errors. Asterisks indicate $p < .05$ (one-tailed). See text for details.

Another Table (using default-y `stargazer`)

Table: OLS Regression Model of HIV/AIDS Rates in Africa, 2001

	Model I
(Constant)	15.28^{***}
	(1.83)
Muslim Percentage of the Population	-0.16^{***}
	(0.04)
Observations	43
R^2	0.33
Adjusted R^2	0.31
Residual Std. Error	8.28 (df = 41)
F Statistic	19.83^{***} (df = 1; 41)
Note:	*p<0.1; **p<0.05; ***p<0.01

Some Guidelines ("Rules"?)

Tables:

- *Use column headings descriptively.*
- *Use multiple rows / columns rather than multiple tables.*
- *Learn about significant digits, and don't report more than 4-5 of them.*
- *Use a figure to replace a table when you can.*
- *Be aware of norms about *s.*

Figures:

- *Report the scale of axes, and label them.*
- *Use as much "space" as you need, but no more.*
- *Use color sparingly.*