PLSC 503 – Spring 2020
Multiplicative Interactions, II

March 24, 2020

```
> ClintonTherm<-read.csv("ClintonTherm.csv")
> summary(ClintonTherm)

    caseid       ClintonTherm    RConserv      ClintonConserv
 Min.   :1001   Min.   :  0    Min.   :1.000   Min.   :1.000
 1st Qu.:1440   1st Qu.: 30    1st Qu.:3.000   1st Qu.:2.000
 Median :1854   Median : 60    Median :4.000   Median :3.000
 Mean   :2001   Mean   : 57    Mean   :4.323   Mean   :2.985
 3rd Qu.:2262   3rd Qu.: 85    3rd Qu.:5.000   3rd Qu.:4.000
 Max.   :3403   Max.   :100    Max.   :7.000   Max.   :7.000
      PID            GOP
 Min.   :1.000   Min.   :0.0000
 1st Qu.:1.000   1st Qu.:0.0000
 Median :2.000   Median :0.0000
 Mean   :2.059   Mean   :0.3161
 3rd Qu.:3.000   3rd Qu.:1.0000
 Max.   :5.000   Max.   :1.0000
```

A Basic Regression

```
> summary(with(ClintonTherm, lm(ClintonTherm~RConserv+GOP)))

Call:
lm(formula = ClintonTherm ~ RConserv + GOP)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  93.4756     2.2278   41.96   <2e-16 ***
RConserv     -6.4866     0.5373  -12.07   <2e-16 ***
GOP         -26.6699     1.6056  -16.61   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.65 on 1294 degrees of freedom
Multiple R-squared: 0.3795,Adjusted R-squared: 0.3786
F-statistic: 395.7 on 2 and 1294 DF,  p-value: < 2.2e-16
```

An Interactive Model

```
> fit1<-with(ClintonTherm, lm(ClintonTherm~RConserv+GOP+
            RConserv*GOP))
> summary(fit1)

Call:
lm(formula = ClintonTherm ~ RConserv + GOP + RConserv * GOP)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  89.9271     2.4866   36.165  < 2e-16 ***
RConserv     -5.5705     0.6085   -9.154  < 2e-16 ***
GOP          -6.4840     6.5690   -0.987  0.32379
RConserv:GOP -4.0581     1.2808   -3.168  0.00157 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.57 on 1293 degrees of freedom
Multiple R-squared: 0.3843,Adjusted R-squared: 0.3829
F-statistic:   269 on 3 and 1293 DF,  p-value: < 2.2e-16
```
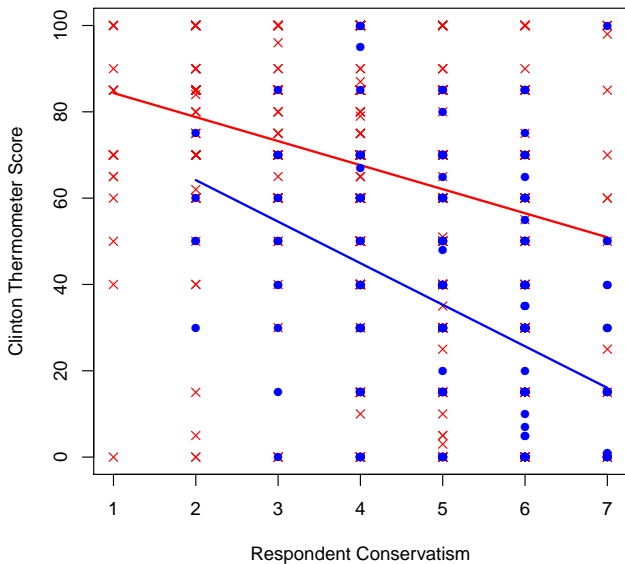
Two Regressions, Sort Of

$$
\begin{aligned}
E(\text{Thermometer} \mid \text{Non-GOP})_i &= 89.9 - 6.5(0) - 5.6(\text{R's Conservatism}_i) \\
&\quad -4.0(0 \times \text{R's Conservatism}_i) \\
&= \mathbf{89.9} - \mathbf{5.6}(\text{R's Conservatism}_i)
\end{aligned}
$$

$$
\begin{aligned}
E(\text{Thermometer} \mid \text{GOP})_i &= [89.9 - 6.5(1)] + [-5.6 - 4.0(1 \times \text{R's Conservatism}_i)] \\
&= \mathbf{83.4} - \mathbf{9.6}(\text{R's Conservatism}_i)
\end{aligned}
$$

Thermometer Scores by Conservatism, GOP and Non-GOP

Interactive Results are (Almost) Identical to Separate Regressions

```
> NonReps<-subset(ClintonTherm,GOP==0)
> summary(with(NonReps, lm(ClintonTherm~RConserv)))

Call:
lm(formula = ClintonTherm ~ RConserv, data = NonReps)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 89.9271     2.4695  36.416   <2e-16 ***
RConserv    -5.5705     0.6043  -9.217   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.41 on 885 degrees of freedom
Multiple R-squared: 0.08759,Adjusted R-squared: 0.08656
F-statistic: 84.96 on 1 and 885 DF,  p-value: < 2.2e-16
```

Interactive Results are (Almost) Identical to Separate Regressions

```
> Reps<-subset(ClintonTherm,GOP==1)
> summary(with(Reps, lm(ClintonTherm~RConserv)))

Call:
lm(formula = ClintonTherm ~ RConserv, data = Reps)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   83.443      6.170  13.524  < 2e-16 ***
RConserv      -9.629      1.144  -8.419 6.52e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 23.92 on 408 degrees of freedom
Multiple R-squared: 0.148,Adjusted R-squared: 0.1459
F-statistic: 70.88 on 1 and 408 DF,  p-value: 6.518e-16
```

Discovering $\hat{\psi}_1$ and $\hat{\psi}_2$

For `RConserv`:

$$
\begin{aligned}
\text{Clinton Thermometer}_i &= \beta_0 + (\beta_1 + \beta_3 \text{GOP}_i)\text{R's Conservatism}_i + \\
&\quad \beta_2 \text{GOP}_i + u_i \\
&= \beta_0 + \psi_{1i}\text{R's Conservatism}_i + \beta_2\text{GOP}_i + u_i.
\end{aligned}
$$

So:

$$
\hat{\psi}_{1i} = \hat{\beta}_1 + \hat{\beta}_3 \times \text{GOP}_i
$$

and

$$
\hat{\sigma}_{\psi_1} = \sqrt{\widehat{\text{Var}(\hat{\beta}_1)} + (\text{GOP})^2\widehat{\text{Var}(\hat{\beta}_3)} + 2(\text{GOP})\widehat{\text{Cov}(\hat{\beta}_1, \hat{\beta}_3)}}.
$$

For GOP:

$$
\begin{aligned}
\text{Clinton Thermometer}_i &= \beta_0 + (\beta_2 + \beta_3 \times \text{R's Conservatism}_i)\text{GOP}_i + \\
&\quad \beta_1(\text{R's Conservatism}_i) + u_i \\
&= \beta_0 + \psi_{2i}\text{GOP}_i + \beta_1(\text{R's Conservatism}_i) + u_i.
\end{aligned}
$$

So:

$$
\hat{\psi}_{2i} = \hat{\beta}_2 + \hat{\beta}_3 \times (\text{R's Conservatism}_i).
$$

and

$$
\hat{\sigma}_{\psi_2} = \sqrt{\widehat{\text{Var}(\hat{\beta}_2)} + (\text{R's Conservatism}_i)^2\widehat{\text{Var}(\hat{\beta}_3)} + 2k\widehat{\text{Cov}(\hat{\beta}_2, \hat{\beta}_3)}}.
$$

```
> Psi1<-fit1$coeff[2]+fit1$coeff[4]

> Psi1
 RConserv
-9.628577

> SPsi1<-sqrt(vcov(fit1)[2,2] + (1)^2*vcov(fit1)[4,4] + 2*1*vcov(fit1)[2,4])
> SPsi1
[1] 1.127016

> Psi1 / SPsi1 # <-- t-statistic
 RConserv
-8.543422
```

Discovering $\hat{\psi}_1$ and $\hat{\psi}_2$

```
> # psi_2 | RConserv = 1
> fit1$coeff[3]+(1 * fit1$coeff[4])
      GOP
-10.54208

> sqrt(vcov(fit1)[3,3] + (1)^2*vcov(fit1)[4,4] + 2*1*vcov(fit1)[3,4])
[1] 5.335847

# Implies t is approximately 2


> # psi_2 | RConserv = 7
> fit1$coeff[3]+(7 * fit1$coeff[4])
      GOP
-34.89045

> sqrt(vcov(fit1)[3,3] + (7)^2*vcov(fit1)[4,4] + 2*7*vcov(fit1)[3,4])
[1] 3.048302

# t is approximately 11
```

An Easier Way: `linearHypothesis()`

```
> library(car)
> linearHypothesis(fit1,"RConserv+RConserv:GOP")
Linear hypothesis test

Hypothesis:
RConserv  + RConserv:GOP = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + GOP + RConserv * GOP

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   1294 758714
2   1293 718173  1     40541  72.99 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1    1

> # Note: Same as t-test:
> sqrt(72.99)
[1] 8.543419
```

An Easier Way: `linearHypothesis()`

```
> # psi_2 | RConserv = 7:
> linearHypothesis(fit1,"GOP+7*RConserv:GOP")
Linear hypothesis test

Hypothesis:
GOP  + 7 RConserv:GOP = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + GOP + RConserv * GOP

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   1294 790938
2   1293 718173  1     72766 131.01 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```
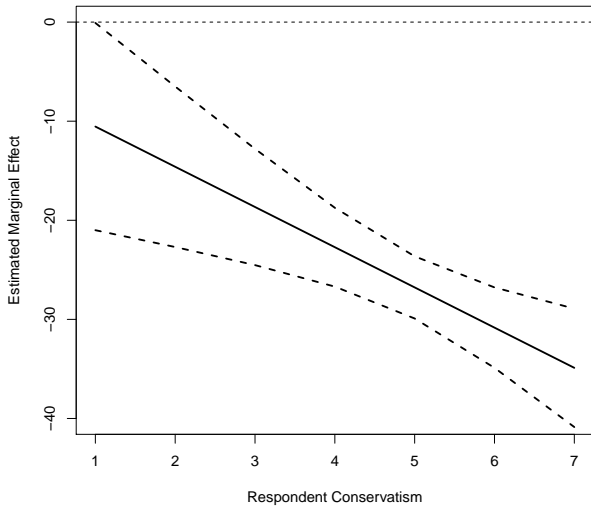
```
> ConsSim<-seq(1,7,1)
> psis<-fit1$coeff[3]+(ConsSim * fit1$coeff[4])
> psis.ses<-sqrt(vcov(fit1)[3,3] +
  (ConsSim)^2*vcov(fit1)[4,4] + 2*ConsSim*vcov(fit1)[3,4])

> plot(ConsSim,psis,t="l",lwd=2,xlab="Respondent Conservatism",
  ylab="Estimated Marginal Effect",ylim=c(-40,0))
> lines(ConsSim,psis+(1.96*psis.ses),lty=2,lwd=2)
> lines(ConsSim,psis-(1.96*psis.ses),lty=2,lwd=2)
> abline(h=0,lwd=1,lty=2)
```

Interacting Two Continuous Covariates

```
> fit2<-with(ClintonTherm,
+         lm(ClintonTherm~RConserv+ClintonConserv+RConserv*ClintonConserv))
> summary(fit2)

Call:
lm(formula = ClintonTherm ~ RConserv + ClintonConserv + RConserv *
    ClintonConserv)

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)             119.3515     5.1634  23.115  < 2e-16 ***
RConserv                -19.5673     1.0362 -18.884  < 2e-16 ***
ClintonConserv           -7.9311     1.6477  -4.813 1.66e-06 ***
RConserv:ClintonConserv   3.6293     0.3394  10.695  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 22.03 on 1293 degrees of freedom
Multiple R-squared: 0.4619,Adjusted R-squared: 0.4606
F-statistic:    370 on 3 and 1293 DF,  p-value: < 2.2e-16
```

```
> fit2$coef[2]+(1*fit2$coef[4])
 RConserv
-15.93803

> sqrt(vcov(fit2)[2,2] + (1)^2*vcov(fit2)[4,4] + 2*1*vcov(fit2)[2,4])
[1] 0.7439696


> linearHypothesis(fit2,"RConserv+1*RConserv:ClintonConserv")
Linear hypothesis test

Hypothesis:
RConserv  + RConserv:ClintonConserv = 0

Model 1: restricted model
Model 2: ClintonTherm ~ RConserv + ClintonConserv + RConserv * ClintonConserv

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   1294 850442
2   1293 627658  1    222784 458.94 < 2.2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

```
> # psi_1 | ClintonConserv = mean
> fit2$coef[2]+((mean(ClintonTherm$ClintonConserv))*fit2$coef[4])

 RConserv
-8.735424

> sqrt(vcov(fit2)[2,2] + (mean(ClintonTherm$ClintonConserv)^2*vcov(fit2)[4,4] +
+                         2*(mean(ClintonTherm$ClintonConserv))*vcov(fit2)[2,4]))

[1] 0.4507971

> pt(((fit2$coef[2]+(2.985*fit2$coef[4])) / sqrt(vcov(fit2)[2,2] +
+      (2.985)^2*vcov(fit2)[4,4] + 2*2.985*vcov(fit2)[2,4])),df=1293)

    RConserv
6.483788e-74

> # psi_2 | RConserv = 1
> fit2$coef[3]+(1*fit2$coef[4])

ClintonConserv
     -4.301803

> # psi_2 | RConserv = 6
> fit2$coef[3]+(6*fit2$coef[4])

ClintonConserv
      13.84463
```

Marginal Effect Plot, II

```
> psis2<-fit2$coef[3]+(ConsSim*fit2$coef[4])
> psis2.ses<-sqrt(vcov(fit2)[3,3] + (ConsSim)^2*vcov(fit2)[4,4]
  + 2*ConsSim*vcov(fit2)[3,4])

> plot(ConsSim,psis2,t="l",lwd=2,xlab="Respondent's
  Conservatism",ylab="Marginal Effect of Clinton's
  Conservatism",ylim=c(-10,20))
> lines(ConsSim,psis2+(1.96*psis2.ses),lty=2,lwd=2)
> lines(ConsSim,psis2-(1.96*psis2.ses),lty=2,lwd=2)
> abline(h=0,lty=2,lwd=1,col="red")
```
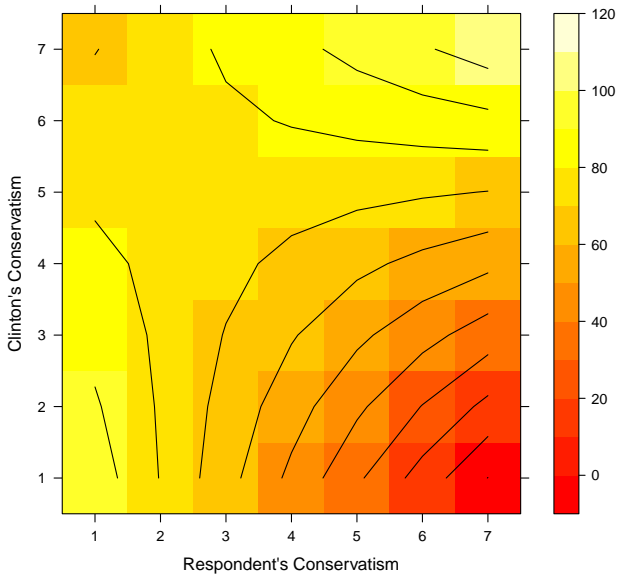
Respondent's Conservatism

˜ Conservatism
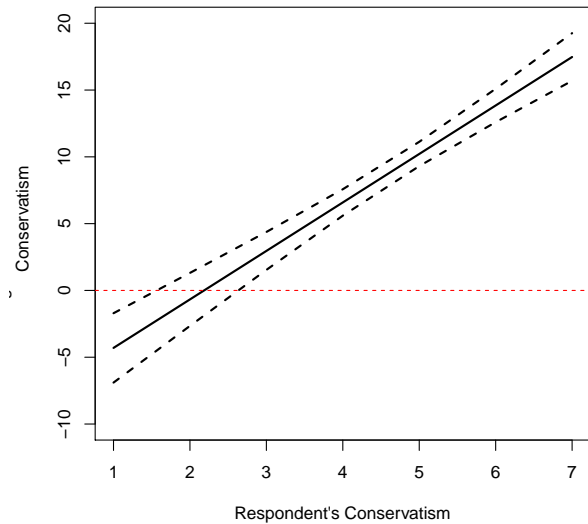
Predicted Values: A Contour Plot

```
> library(lattice)
> grid<-expand.grid(RConserv=seq(1,7,1),
  ClintonConserv=seq(1,7,1))
> hats<-predict(fit2,newdata=grid)

> levelplot(hats~grid$RConserv*grid$ClintonConserv,
  contour=TRUE,
  cuts=12,pretty=TRUE,xlab="Respondent's Conservatism",
  ylab="Clinton's Conservatism",
  col.regions=heat.colors)
```

Predicted Values: A Wireframe Plot

```
> trellis.par.set("axis.line",list(col="transparent"))

> wireframe(hats~grid$RConserv*grid$ClintonConserv,
  drape=TRUE,
  xlab=list("Respondent's Conservatism",rot=30),
  ylab=list("Clinton's Conservatism",
  rot=-40),zlab=list("Predictions",rot=90),
  scales=list(arrows=FALSE,col="black"),
  zoom=0.85,pretty=TRUE),
  col.regions=colorRampPalette(c("blue","red"))(100))
```