USN | | | | | | | | | | | **21MATCS/IS31**

First Semester B.E. Semester End Examination, January-March 2022-23

Discrete Mathematical Structures and Numerical methods

Time: 3 Hours **Max. Marks: 100**

Instructions:	*1.*	*From Part A answer any 5 questions each Question Carries 6 Marks.*
	2.	*From Part B answer any one full question from each unit and each Question Carries 10 Marks.*
	3.	*From Part C answer any one full question and each Question Carries 20 Marks.*

<table>
<tr><th colspan="6">PART A</th></tr>
<tr><th colspan="2">Answer any Five.</th><th>L</th><th>CO</th><th>PO</th><th>M</th></tr>
<tr><td>1</td><td>Use the laws of logics to simplify the compound proposition $\neg(p\wedge q)\vee[\neg(q\leftrightarrow p)]$ and $(p\wedge q)\vee[\neg(p\leftrightarrow(\neg q))]$.</td><td>(L2)</td><td>1</td><td>1</td><td>(6)</td></tr>
<tr><td>2</td><td>Write the converse, inverse and contrapositive along with their truth value for the following conditional
“If x is a positive integer, then x^2 is also an integer”</td><td>(L2)</td><td>(1)</td><td>(1)</td><td>(6)</td></tr>
<tr><td>3</td><td>Let $A=\{1,2,3,4\}$ and R be the relation on A defined by xRy if and only if both x & y are even or odd numbers. Is R an equivalence relation? Write zero-one matrix and draw diagraph of R.</td><td>(L2)</td><td>(2)</td><td>(1)</td><td>(6)</td></tr>
<tr><td>4</td><td>Using Lagrange’s interpolation formula, calculate the profit in the year 2000 from the following data.
<table>
<tr><td>**Year**</td><td>1997</td><td>1999</td><td>2001</td><td>2002</td></tr>
<tr><td>**Profit in Lakhs (Rupees)**</td><td>43</td><td>65</td><td>159</td><td>248</td></tr>
</table></td><td>(L2)</td><td>(4)</td><td>(1)</td><td>(6)</td></tr>
<tr><td>5</td><td>Employ the Trapezoidal rule to evaluate $\int_0^1 \frac{1}{1+x}dx$. Compare this solution with analytical solution.</td><td>(L2)</td><td>(4)</td><td>(1)</td><td>(6)</td></tr>
<tr><td>6</td><td>Apply the Newton-Raphson method to determine the one of the real roots of the equation $x^3-4x-9=0$.</td><td>(L2)</td><td>(5)</td><td>(1)</td><td>(6)</td></tr>
<tr><td>7</td><td>Use the Taylor’s series method to find the value of y at $x=0.1$ and correct to four decimal places for $y'=e^{2x}-y,\ y(0)=1$.</td><td>(L2)</td><td>(5)</td><td>(1)</td><td>(6)</td></tr>
<tr><th colspan="6">PART B</th></tr>
<tr><td></td><td></td><th>UNIT-I</th><th>L</th><th>CO</th><th>PO</th><th>M</th></tr>
<tr><td>8</td><td>a)</td><td colspan="5">Apply the rule of inferences to test the validity of the following argument.
If I will study discrete math, then I will study computer science.
If I will study protein structures, then I will study biochemistry.
I will study discrete math or I will study protein structures.
∴I will study computer science or biochemistry.</td></tr>
<tr><td></td><td></td><td></td><td>(L3)</td><td>(1)</td><td>(1)</td><td>(6)</td></tr>
<tr><td></td><td>b)</td><td colspan="5">Justify (i) conditional and its contrapositive, (ii) converse and inverse are logically equivalent.</td></tr>
<tr><td></td><td></td><td></td><td>(L3)</td><td>(1)</td><td>(1)</td><td>(4)</td></tr>
<tr><td></td><td></td><th>OR</th><td></td><td></td><td></td><td></td></tr>
<tr><td>9</td><td></td><td colspan="5">Consider the following open statements:
$p(x): x>0$; $q(x): x$ is an even; $r(x): x$ is a perfect square; $s(x): x$ is divisible by 3; $t(x): x$ is divisible by 7.
Write down the following quantified statements in symbolic form.</td></tr>
</table>

Note: L (Level),CO (Course Outcome), PO (Programme Outcome), M (Marks)

(i) At least one integer is even,
(ii) There exists a positive integer that is even.
(iii) Some even integers are divisible by 3.
(iv) Every integer is either even or odd.
(v) If x is an even and perfect square, then x is divisible by 3.
Write down the following symbolic statements in words.
(vi) $\forall x, [r(x) \to p(x)]$.
(vii) $\exists x, [s(x) \wedge (\neg q(x))]$.
(viii) $\forall x, \neg r(x)$.
(ix) $\forall x, [r(x) \vee t(x)]$.

(L3) (1) (1) (10)

UNIT-II

L CO PO M

10 State and prove fundamental theorem on equivalence relation.

(L3) (2) (1) (10)

OR

11 On the set Z of all integers, a relation R is defined by aRb if and only if $a-b$ is divisible by 3. Verify that R is an equivalence relation . Determine the partition induced by the relation R by applying the fundamental theorem on equivalence relation.

(L3) (2) (1) (10)

UNIT - III

L CO PO M

12 Apply generating function method to solve the following non-homogenous linear recurrence relation with the given initial condition.

$a_n = a_{n-1} + n, \forall n \geq 1,\ a_0 = 1.$

(L3) (3) (1) (10)

OR

13 Apply characteristic root method to solve the following homogenous linear recurrence relation with the given initial condition.

$u_n = u_{n-1} + u_{n-2}, \forall n \geq 2,\ u_1 = 1, u_2 = 3.$

(L3) (3) (1) (10)

UNIT - IV

L CO PO M

14 A farmer engaged in farming mango trees observes and collects the following data regarding the tree's height on particular days.

No. of Days	5	10	15	20	25
Height of Trees (in MM)	10	18	24	28	30

Generate the growth function of the mango tree and hence estimate the height of the mango tree in 18 days by choosing the apropitiate interpolation method.

(L3) (4) (1) (10)

OR

15 a) Apply Newton's divided difference formula to estimate the approximate velocity of a car at time t=8 seconds for the following data.

Time t (in seconds)	5	10	15	20
Velocity v (in m/sec^2)	10	18	24	28

(L3) (4) (1) (6)

b) Apply the Simpson's 1/3rd rule to evaluate the value of $\int_0^6 \frac{1}{x+e^x}dx$ dividing the interval equally 6 parts.

(L3) (4) (1) (4)

UNIT-V

L CO PO M

16	a)	Employ Regula-falsi method to approximate the root of the equation $xe^{x} - 2 = 0$ and correct it to 3 decimal places.				
			(L3)	**(5)**	**(1)**	**(5)**
	b)	Use the Euler's method to find the value of y at $x = 0.1$ and correct 4 decimal places for $y' = xy + 1,\ y(0) = 1$ taking $h = 0.02$.				
			(L3)	**(5)**	**(1)**	**(5)**
		OR				
17		Apply Runge-Kutta method to find $y(0.2)$ given that $y' = \frac{y - x}{y + x},\ y(0) = 1$ taking $h = 0.1$.				
			(L3)	**(5)**	**(1)**	**(10)**
PART C			**L**	**CO**	**PO**	**M**
18	a)	Construct the any partial ordering relation of your choice from the set consisting of five elements. And also draw its Hasse diagram for this relation.				
			(L4)	**(2)**	**(1)**	**(10)**
	b)	Let $A = \{a, b, c, d, e\}$ and the relation R on set A defined as $R = \{(a,a),(a,c),(a,e),(b,d),(c,a),(d,c),(d,d),(e,a),(e,c)\}$. (i) Find the reflexive and symmetric closure of the this relation R. (ii) Apply the Warshall's algorithm to compute the transitive closure of R.				
			(L4)	**(2)**	**(1)**	**(10)**
		OR				
19	a)	A deposit of 100,000 rupees made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year. (i) Construct a recurrence relation for p_n, where p_n is the amount in the account at the end of n years if no money is withdrawn. (ii) How much is in the account after n years, if no money has been withdrawn?				
			(L4)	**(3)**	**(1)**	**(10)**
	b)	Suppose that the number of bacteria in a colony triples every hour. (i) Set up a recurrence relation for the number of bacteria after n hour have elapsed. (ii) Solve this recurrence relation and hence find the how many bacteria will be in colony in 10 hours if initially there were 100 bacteria.				
			(L4)	**(3)**	**(1)**	**(10)**

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