

Factorisation including Quadratics and other expressions KS4 Higher Tier

Non-Calculator

(A) Factorise completely:

1. $2x - 4$ 2. $4x + 8$ 3. $6x - 8$ 4. $10x - 15$ 5. $12 - 6x$ 6. $28 - 14x$

7. $2x - 4y$ 8. $4x + 8y$ 9. $6x - 8y$ 10. $10x - 15y$ 11. $6a - 2ab - 12ac$

12. $6xy - 3x$ 13. $9xy - 6y$ 14. $x^2 - x$ 15. $x^3 - x^2$ 16. $5x^2y + 10xy$

17. $6x^3y^2 - 30x^2y$ 18. $10a^2bc + 20abc^2 + 30ab^2c$ 19. $x^2y^2 - 3xy$

20. $x^2y^2 - xy$

Quadratics and higher powers:

(B) Factorise completely:

1. $x^2 - 4x + 4$ 2. $x^2 - 3x + 2$ 3. $x^2 - 4x + 3$ 4. $x^2 + 5x + 6$

5. $2x^2 - 8x + 8$ 6. $2x^2 - 6x + 4$ 7. $3x^2 - 12x + 9$ 8. $4x^2 + 20x + 24$

9. $x^2 + 4x + 4$ 10. $2x^2 + 8x + 8$ 11. $x^2 + 3x + 2$ 12. $x^2 - 11x + 10$

13. $x^2 - x - 2$ 14. $x^2 + x - 2$ 15. $x^2 + 2x - 3$ 16. $x^2 + 3x - 10$

17. $2x^2 - 9x + 4$ 18. $2x^2 + 9x + 4$ 19. $3x^2 - 16x + 5$ 20. $3x^2 + 16x + 5$

21. $3x^2 - 14x + 15$ 22. $3x^2 + 14x + 15$ 23. $4x^2 + 20x + 25$

24. $4x^2 - 20x + 25$ 25. $6x^2 + 11x + 3$ 26. $6x^2 - 11x + 3$

27. $6x^2 + 17x + 5$ 28. $6x^2 - 13x + 6$ 29. $3x^2 - 3x - 6$

30. $4x^2 + 4x - 8$ 31. $2x^2 + 4x - 6$ 32. $4x^2 + 12x - 40$

33. $6x^2 - 3x - 30$ 34. $8x^2 + 2x - 15$ 35. $8x^2 - 14x - 15$

36. $6x^2 - 7x - 3$

37. $6x^4 - 7x^2 - 3$

38. $6x^4 - 13x^2 + 6$

39. $8x^2 + 2xy - 15y^2$

40. $6x^2 - 11xy + 3y^2$

41. $a^2 + ab + ac + bc$

42. $mn + mc + an + ac$

43. $a^2 + ab - ac - bc$

44. $mn + mc - an - ac$

45. $mn - mc - an + ac$

46. $n^3 + 9n^2 + 18n$

47. $6n^3 + 7n^2 - 5n$

48. $6n^3 - 19n^2 + 15n$

49. $8n^3 - 2n^2 - 21n$

50. $3n^3 + 46n^2 - 32n$

Difference of two squares and more challenging factorization:

(c) Factorise completely:

1. $x^2 - 1$ 2. $x^2 - 4$ 3. $x^2 - 49$ 4. $n^2 - 9$ 5. $a^2 - b^2$

6. $n^2 - 25$ 7. $25 - n^2$ 8. $2n^2 - 50$ 9. $4x^2 - 25$ 10. $8x^2 - 50$

11. $4x^2 - 49$ 12. $9x^2 - 16$ 13. $25 - 4x^2$ 14. $9n^2 - 25$ 15. $4 - 9n^2$

16. $9n^2 - 25m^2$ 17. $x^2 - \frac{1}{4}$ 18. $x^2 - 2\frac{1}{4}$ 19. $\frac{4}{25}x^2 - 1$

$$* 20. n^4 - 1$$

$$* 21. n^4 - m^2$$

$$* 22. n^4 - m^4$$

$$* 23. x^4 - 16n^4$$

$$* 24. 81x^4 - 16$$

$$* 25. x^4 - y^4$$

$$* 26. x^4 - 81$$

$$* 27. x^8 - 256$$

$$* 28. 64n^4 - 4$$

$$* 29. x^6 - x^2$$

$$* 30. x^4 - \frac{16}{81}$$

$$* 31. x^2 - \frac{1}{x^2}$$

Applications:

Use factorisation to find the value of each of the following:

1. $29^2 - 19^2$ 2. $49^2 - 9^2$ 3. $2012^2 - 2011^2$ 4. $(\frac{9}{16})^2 - (\frac{7}{16})^2$

5. $\frac{71^2 - 29^2}{55^2 - 45^2}$

6. $\sqrt{(39^2 - 36^2)}$

7. $\sqrt{(35^2 - 28^2)}$

8. $\sqrt{(45^2 - 27^2)}$

9. $\frac{3(71^2 - 29^2)}{45^2 - 15^2}$

*10. $10^2 - 9^2 + 8^2 - 7^2 + 6^2 - 5^2 + 4^2 - 3^2 + 2^2 - 1^2 =$

*11. Repeat Q10, from 20^2 down to 1^2 and show that the answers is 210..

Beyond GCSE

(D) Factorise completely

1. $x^3 - 1$

2. $x^3 + 1$

3. $x^3 - 8$

4. $x^3 + 8$

5. $27x^3 - 64$

6. $x^6 - 1$

7. $x^6 + 1$

8. $x^3 + \frac{1}{x^3}$

9. $x^3 - \frac{1}{x^3}$

10. $x^3y^3 - x^3$

ANSWERS/SOLUTIONS

(A) Factorise completely:

$$\begin{array}{llllll} 1. 2x - 4 & 2. 4x + 8 & 3. 6x - 8 & 4. 10x - 15 & 5. 12 - 6x & 6. 28 - 14x \\ = \underline{\underline{2(x-2)}} & = \underline{\underline{4(x+2)}} & = \underline{\underline{2(3x-4)}} & = \underline{\underline{5(2x-3)}} & = \underline{\underline{6(2-x)}} & = \underline{\underline{14(2-x)}} \end{array}$$

$$\begin{array}{llllll} 7. 2x - 4y & 8. 4x + 8y & 9. 6x - 8y & 10. 10x - 15y & 11. 6a - 2ab - 12ac \\ = \underline{\underline{2(x-4y)}} & = \underline{\underline{4(x+2y)}} & = \underline{\underline{2(3x-4y)}} & = \underline{\underline{5(2x-3y)}} & = \underline{\underline{2a(3-2b-6c)}} \end{array}$$

$$\begin{array}{llllll} 12. 6xy - 3x & 13. 9xy - 6y & 14. x^2 - x & 15. x^3 - x^2 & 16. 5x^2y + 10xy \\ = \underline{\underline{3x(2y-1)}} & = \underline{\underline{3y(3x-2)}} & = \underline{\underline{x(x-1)}} & = \underline{\underline{x^2(x-1)}} & = \underline{\underline{5xy(x+2)}} \end{array}$$

$$\begin{array}{lll} 17. 6x^3y^2 - 30x^2y & 18. 10a^2bc + 20abc^2 + 30ab^2c & 19. x^2y^2 - 3xy \\ = \underline{\underline{6x^2y(xy-5)}} & = \underline{\underline{10abc(a+2c+3b)}} & = \underline{\underline{xy(xy-3)}} \end{array}$$

$$20. x^2y^2 - xy = \underline{\underline{xy(xy-1)}}$$

Quadratics and higher powers:

(B) Factorise completely:

$$\begin{array}{llll} 1. x^2 - 4x + 4 & 2. x^2 - 3x + 2 & 3. x^2 - 4x + 3 & 4. x^2 + 5x + 6 \\ = (x-2)(x-2) & = (x-1)(x-2) & = (x-3)(x-1) & = (x+2)(x+3) \\ \text{OR } \underline{\underline{(x-2)^2}} & & & \end{array}$$

$$\begin{array}{llll} 5. 2x^2 - 8x + 8 & 6. 2x^2 - 6x + 4 & 7. 3x^2 - 12x + 9 & 8. 4x^2 + 20x + 24 \\ = 2(x^2 - 4x + 4) & = 2(x^2 - 3x + 2) & = 3(x^2 - 4x + 3) & = 4(x^2 + 5x + 6) \\ = 2(x-2)(x-2) & = \underline{\underline{2(x-1)(x-2)}} & = \underline{\underline{3(x-1)(x-3)}} & = \underline{\underline{4(x+2)(x+3)}} \\ \text{OR } \underline{\underline{2(x-2)^2}} & & & \end{array}$$

$$\begin{array}{llll}
 9. x^2 + 4x + 4 & 10. 2x^2 + 8x + 8 & 11. x^2 + 3x + 2 & 12. x^2 - 11x + 10 \\
 = (x+2)(x+2) & = 2(x^2 + 4x + 4) & = (x+1)(x+2) & = (x-1)(x-10) \\
 \text{OR } = (x+2)^2 & = 2(x+2)(x+2) & & \\
 & \text{OR } 2(x+2)^2 & &
 \end{array}$$

$$\begin{array}{llll}
 13. x^2 - x - 2 & 14. x^2 + x - 2 & 15. x^2 + 2x - 3 & 16. x^2 + 3x - 10 \\
 = (x+1)(x-2) & = (x-1)(x+2) & = (x-1)(x+3) & = (x-2)(x+5)
 \end{array}$$

$$\begin{array}{llll}
 17. 2x^2 - 9x + 4 & 18. 2x^2 + 9x + 4 & 19. 3x^2 - 16x + 5 & 20. 3x^2 + 16x + 5 \\
 = (2x-1)(x-4) & = (2x+1)(x+4) & = (3x-1)(x-5) & = (3x+1)(x+5)
 \end{array}$$

$$\begin{array}{l}
 21. 3x^2 - 14x + 15 \\
 = (3x-5)(x-3)
 \end{array}$$

$$\begin{array}{l}
 22. 3x^2 + 14x + 15 \\
 = (3x+5)(x+3)
 \end{array}$$

$$\begin{array}{l}
 23. 4x^2 + 20x + 25 \\
 = (2x+5)(2x+5) \\
 \text{OR } (2x+5)^2
 \end{array}$$

$$\begin{array}{l}
 24. 4x^2 - 20x + 25 \\
 = (2x-5)(2x-5) \\
 \text{OR } (2x-5)^2
 \end{array}$$

$$\begin{array}{l}
 25. 6x^2 + 11x + 3 \\
 = (3x+1)(2x+3)
 \end{array}$$

$$\begin{array}{l}
 26. 6x^2 - 11x + 3 \\
 = (3x-1)(2x-3)
 \end{array}$$

$$\begin{array}{l}
 27. 6x^2 + 17x + 5 \\
 = (3x+1)(2x+5)
 \end{array}$$

$$\begin{array}{l}
 28. 6x^2 - 13x + 6 \\
 = (3x-2)(2x-3)
 \end{array}$$

$$\begin{array}{l}
 29. 3x^2 - 3x - 6 \\
 = 3(x^2 - x - 2) \\
 = 3(x+1)(x-2)
 \end{array}$$

$$\begin{array}{l}
 30. 4x^2 + 4x - 8 \\
 = 4(x^2 + x - 2) \\
 = 4(x-1)(x+2)
 \end{array}$$

$$\begin{array}{l}
 31. 2x^2 + 4x - 6 \\
 = 2(x^2 + 2x - 3) \\
 = 2(x-1)(x+3)
 \end{array}$$

$$\begin{array}{l}
 32. 4x^2 + 12x - 40 \\
 = 4(x^2 + 3x - 10) \\
 = 4(x-2)(x+5)
 \end{array}$$

$$\begin{array}{l}
 33. 6x^2 - 3x - 30 \\
 = 3(2x^2 - x - 10) \\
 = 3(2x-5)(x+2)
 \end{array}$$

$$\begin{array}{l}
 34. 8x^2 + 2x - 15 \\
 = (4x-5)(2x+3)
 \end{array}$$

$$\begin{array}{l}
 35. 8x^2 - 14x - 15 \\
 = (4x+3)(2x-5)
 \end{array}$$

$$36. 6x^2 - 7x - 3 \\ = \underline{(3x+1)(2x-3)}$$

$$37. 6x^4 - 7x^2 - 3 \\ = \underline{(3x^2+1)(2x^2-3)}$$

$$38. 6x^4 - 13x^2 + 6 \\ = \underline{(3x^2-2)(2x^2-3)}$$

$$39. 8x^2 + 2xy - 15y^2 \\ = \underline{(4x-5y)(2x+3y)}$$

$$40. 6x^2 - 11xy + 3y^2 \\ = \underline{(3x-y)(2x-3y)}$$

$$41. a^2 + ab + ac + bc \\ = a(a+b) + c(a+b) \\ = \underline{(a+b)(a+c)}$$

$$42. mn + mc + an + ac \\ = m(n+c) + a(n+c) \\ = \underline{(n+c)(m+a)}$$

$$43. a^2 + ab - ac - bc \\ = a(a+b) - c(a+b) \\ = \underline{(a+b)(a-c)}$$

$$44. mn + mc - an - ac \\ = m(n+c) - a(n+c) \\ = \underline{(n+c)(m-a)}$$

$$45. mn - mc - an + ac \\ = m(n-c) - a(n-c) \\ = \underline{(n-c)(m-a)}$$

$$46. n^3 + 9n^2 + 18n \\ = n(n^2 + 9n + 18) \\ = \underline{n(n+3)(n+6)}$$

$$47. 6n^3 + 7n^2 - 5n \\ = n(6n^2 + 7n - 5) \\ = \underline{n(3n+5)(2n-1)}$$

$$48. 6n^3 - 19n^2 + 15n \\ = n(6n^2 - 19n + 15) \\ = \underline{n(3n-5)(2n-3)}$$

$$49. 8n^3 - 2n^2 - 21n \\ = n(8n^2 - 2n - 21) \\ = \underline{n(4n-7)(2n+3)}$$

$$50. 3n^3 + 46n^2 - 32n \\ = n(3n^2 + 46n - 32) \\ = \underline{n(3n-2)(n+16)}$$

Difference of two squares and more challenging factorization:

(c) Factorise completely:

$$\begin{array}{lllll}
 1. \ x^2 - 1 & 2. \ x^2 - 4 & 3. \ x^2 - 49 & 4. \ n^2 - 9 & 5. \ a^2 - b^2 \\
 = \underline{(x+1)(x-1)} & = \underline{(x+2)(x-2)} & = \underline{(x+7)(x-7)} & = \underline{(x+3)(x-3)} & = \underline{(a+b)(a-b)}
 \end{array}$$

$$\begin{array}{lllll}
 6. \ n^2 - 25 & 7. \ 25 - n^2 & 8. \ 2n^2 - 50 & 9. \ 4x^2 - 25 & 10. \ 8x^2 - 50 \\
 = \underline{(n+5)(n-5)} & = \underline{(5+n)(5-n)} & = \underline{2(n^2-25)} & = \underline{(2x+5)(2x-5)} & = \underline{2(4x^2-25)} \\
 & & = \underline{2(n+5)(n-5)} & & = \underline{2(2x+5)(2x-5)}
 \end{array}$$

$$\begin{array}{lllll}
 11. \ 4x^2 - 49 & 12. \ 9x^2 - 16 & 13. \ 25 - 4x^2 & 14. \ 9n^2 - 25 & 15. \ 4 - 9n^2 \\
 = \underline{(2x+7)(2x-7)} & = \underline{(3x+4)(3x-4)} & = \underline{(5+2x)(5-2x)} & = \underline{(3n+5)(3n-5)} & = \underline{(2+3n)(2-3n)}
 \end{array}$$

$$\begin{array}{llll}
 16. \ 9n^2 - 25m^2 & 17. \ x^2 - \frac{1}{4} & 18. \ x^2 - 2\frac{1}{4} & 19. \ \frac{4}{25}x^2 - 1 \\
 = \underline{(3n+5m)(3n-5m)} & = \underline{(x+\frac{1}{2})(x-\frac{1}{2})} & = x^2 - \frac{9}{4} & = \underline{(\frac{2}{5}x+1)(\frac{2}{5}x-1)} \\
 & & = \underline{(x+\frac{3}{2})(x-\frac{3}{2})} &
 \end{array}$$

$$\begin{aligned}
 * 20. n^4 - 1 \\
 &= (n^2+1)(n^2-1) \\
 &= \underline{(n^2+1)(n+1)(n-1)}
 \end{aligned}$$

$$\begin{aligned}
 * 21. n^4 - m^2 \\
 &= \underline{(n^2+m)(n^2-m)}
 \end{aligned}$$

$$\begin{aligned}
 * 22. n^4 - m^4 \\
 &= (n^2+m^2)(n^2-m^2) \\
 &= \underline{(n^2+m^2)(n+m)(n-m)}
 \end{aligned}$$

$$\begin{aligned}
 * 23. x^4 - 16n^4 \\
 &= (x^2+4n^2)(x^2-4n^2) \\
 &= \underline{(x^2+4n^2)(x+2n)(x-2n)}
 \end{aligned}$$

$$\begin{aligned}
 * 24. 81x^4 - 16 \\
 &= (9x^2+4)(9x^2-4) \\
 &= \underline{(9x^2+4)(3x+2)(3x-2)}
 \end{aligned}$$

$$* 25. x^4 - y^4$$

$$\begin{aligned}
 * 26. x^4 - 81 \\
 &= (x^2+9)(x^2-9) \\
 &= \underline{(x^2+9)(x+3)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow \underline{(x^2+y^2)(x^2-y^2)} \\
 &= \underline{(x^2+y^2)(x+y)(x-y)}
 \end{aligned}$$

$$\begin{aligned}
 * 27. x^8 - 256 \\
 &= x^8 - 2^8 \\
 &= (x^4+2^4)(x^4-2^4) \\
 &= (x^4+16)(x^2+2^2)(x^2-2^2) \\
 &= (x^4+16)(x^2+4)(x^2-4) \\
 &= \underline{(x^4+16)(x^2+4)(x+2)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 * 28. 64n^4 - 4 \\
 &= 4(16n^4-1) \\
 &= 4(4n^2+1)(4n^2-1) \\
 &= \underline{4(4n^2+1)(2n+1)(2n-1)}
 \end{aligned}$$

$$* 29. x^6 - x^2$$

$$* 30. x^4 - \frac{16}{81}$$

$$\begin{aligned}
 &= x^2(x^4-1) \\
 &= x^2(x^2+1)(x^2-1) \\
 &= \underline{x^2(x^2+1)(x+1)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 * 31. x^2 - \frac{1}{x^2} \\
 &= \underline{(x + \frac{1}{x})(x - \frac{1}{x})}
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow \underline{(x^2 + \frac{4}{9})(x^2 - \frac{4}{9})} \\
 &= \underline{(x^2 + \frac{4}{9})(x + \frac{2}{3})(x - \frac{2}{3})}
 \end{aligned}$$

Applications:

Use factorisation to find the value of each of the following:

$$\begin{aligned} 1. \quad 29^2 - 19^2 \\ &= (29+19)(29-19) \\ &= (48)(10) \\ &= \underline{480} \end{aligned}$$

$$\begin{aligned} 2. \quad 49^2 - 9^2 \\ &= (49+9)(49-9) \\ &= (58)(40) \\ &= \underline{2320} \end{aligned}$$

$$\begin{aligned} 3. \quad 2012^2 - 2011^2 \\ &= (2012+2011)(2012-2011) \\ &= (4023)(1) \\ &= \underline{4023} \end{aligned}$$

$$\begin{aligned} 4. \quad \left(\frac{9}{16}\right)^2 - \left(\frac{7}{16}\right)^2 \\ &= \left(\frac{9}{16} + \frac{7}{16}\right)\left(\frac{9}{16} - \frac{7}{16}\right) \\ &= \left(\frac{16}{16}\right)\left(\frac{2}{16}\right) = (1)\left(\frac{1}{8}\right) \\ &= \underline{\frac{1}{8}} \end{aligned}$$

$$\begin{aligned} 5. \quad \frac{71^2 - 29^2}{55^2 - 45^2} \\ &= \frac{(71+29)(71-29)}{(55+45)(55-45)} \\ &= \frac{(100)(42)}{(100)(10)} \\ &= \underline{4.2} \end{aligned}$$

$$\begin{aligned} 6. \quad \sqrt{(39^2 - 36^2)} \\ &= \sqrt{(39+36)(39-36)} \\ &= \sqrt{(75)(3)} \\ &= \sqrt{25 \times 3 \times 3} \\ &= 5 \times 3 = \underline{15} \end{aligned}$$

$$\begin{aligned} 7. \quad \sqrt{(35^2 - 28^2)} \\ &= \sqrt{(35+28)(35-28)} \\ &= \sqrt{(63)(7)} \\ &= \sqrt{9 \times 7 \times 7} = 3 \times 7 \\ &= \underline{21} \end{aligned}$$

$$\begin{aligned} 8. \quad \sqrt{(45^2 - 27^2)} \\ &= \sqrt{(45+27)(45-27)} \\ &= \sqrt{(72)(18)} = \sqrt{9 \times 8 \times 2 \times 9} \\ &= \sqrt{9 \times 9 \times 16} = 9 \times 4 = \underline{36} \end{aligned}$$

$$\begin{aligned} 9. \quad \frac{3(71^2 - 29^2)}{45^2 - 15^2} &= \frac{3(71-29)(71+29)}{(45+15)(45-15)} \\ &= \frac{3(42)(100)}{(60)(30)} = \underline{7} \end{aligned}$$

$$\begin{aligned} *10. \quad 10^2 - 9^2 + 8^2 - 7^2 + 6^2 - 5^2 + 4^2 - 3^2 + 2^2 - 1^2 &= \text{(Pair off)} \\ &= (10+9)(10-9) + (8+7)(8-7) + (6+5)(6-5) + (4+3)(4-3) + (2+1)(2-1) \\ &= (10+9)(1) + (8+7)(1) + (6+5)(1) + (4+3)(1) + (2+1)(1) \\ &= 10+9+8+7+6+5+4+3+2+1 = \frac{11 \times 10}{2} = \underline{55} \end{aligned}$$

*11. Repeat Q10, from 20^2 down to 1^2 .

$$\text{Ans.} = \frac{21 \times 20}{2} = \underline{210}$$

Note

$$\text{Let } S = 10+9+8+\dots+2+1$$

$$S = 1+2+3+\dots+9+10$$

$$2S = 11+11+11+\dots+11+11 \leftarrow 10 \text{ lots of } 11$$

$$2S = 11 \times 10$$

$$S = \frac{11 \times 10}{2} = \underline{55}$$

Beyond GCSE

(D) Factorise completely

$$1. x^3 - 1 = \underline{(x-1)(x^2+x+1)}$$

$$2. x^3 + 1 = \underline{(x+1)(x^2-x+1)}$$

$$3. x^3 - 8 = \underline{(x-2)(x^2+2x+4)}$$

$$4. x^3 + 8 = \underline{(x+2)(x^2-2x+4)}$$

$$5. 27x^3 - 64 = \underline{(3x-4)(9x^2+12x+16)}$$

$$\begin{aligned} 6. x^6 - 1 &= (x^3+1)(x^3-1) \\ &= (x+1)(x^2-x+1)(x-1)(x^2+x-1) \\ &= \underline{(x+1)(x-1)(x^2-x+1)(x^2+x-1)} \end{aligned}$$

$$\begin{aligned} 7. x^6 + 1 &= (x^2)^3 + 1 \\ &= \underline{(x^2-1)(x^4+x^2+1)} = \underline{(x+1)(x-1)(x^4+x^2+1)} \end{aligned}$$

$$8. x^3 + \frac{1}{x^3} = \underline{\left(x + \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)}$$

$$9. x^3 - \frac{1}{x^3} = \underline{\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)}$$

$$\begin{aligned} 10. x^3y^3 - x^3 &= x^3(y^3-1) \\ &= \underline{x^3(y-1)(y^2+y+1)} \end{aligned}$$

I hope you find this useful and challenging. Please check all answers and let me know if you find any errors. If you would prefer the word version, send me a message and I will upload it. Thank you.