

# An Improved E-Min-Max Localization Algorithm in Wireless Sensor Networks

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**Abstract**—Min-Max localization algorithm is usually used by wireless sensor nodes because of the reason that it considers few conditions and have a coarse localization which can determine a blind node within a certain range. In this paper, we propose improved Extended Min-Max (improved E-Min-Max) algorithm for localization in wireless sensor networks (WSNs). Experimental results show that improved E-Min-Max algorithm outperforms E-Min-Max algorithm in the localization accuracy.

**Keywords**—WSNs; anchor; Min-Max; Localization

## I. INTRODUCTION

Location information is very important for many WSNs applications such as environmental monitoring, battlefield surveillance, disaster relief, medical diagnostics, site security, and so on [1-5]. Most of those applications use the same principles: based on the measurement of the distance between the target and beacon nodes is estimated and then the position of the target is calculated with a positioning algorithm [6-9]. The main difference between the variety of existing localization algorithms is the handling of distance measurement errors. The simplest way to estimate the distance of an unknown node to a reachable anchor node is through the received signal strength (RSS). Since most existing wireless radios are able to measure RSS, the RSS-based relative location estimation plays an important role of localization in practice. Although time-of-arrival (TOA), time-different-of-arrival (TDOA), and angle-of-arrival (AOA) have a small measurement error, those ranging techniques require extra hardware on nodes to make them capable of measuring distances.

In this paper we propose a novel Extended Min-Max localization algorithm. Due to the proposed algorithm, the error distribution must be known which should be easy to achieve for static deployments of nodes. Especially, the proposed algorithm shows a big improvement on the position error for outdoor localization because it weakens the effect of the path loss and noises. Real experiments show that the proposed localization algorithm is better than other Min-Max localization algorithms in terms of the localization accuracy.

The main difference to other localization algorithms is that we improve the weight model of the vertice. With the improvement of the weight model of the vertices, we reduce the localization error. We present a deep discussion about the localization error of our algorithm and compare the results to other algorithms through experiments.

The rest of this paper is organized as follows. We present the related work and the original Min-Max algorithm and Extended Min-Max algorithm in Section II. An improved Extended Min-Max localization algorithm is proposed in Section III. The performance of the proposed algorithm is presented in Section IV. Finally, our conclusions and future work is presented in Section V.

## II. RELATED WORK

There are several measurement techniques for localization in wireless sensor networks. Range based methods are common and efficient tools for measuring the distance between the target and anchor nodes, such as received signal strength (RSS), time-of-arrival (TOA), time-difference-of-arrival (TDOA) and time-of-flight (TOF). TOF measures the round-trip time of packet and averages the result together to reduce the impact of time-varying errors. It is a promising solution for its low cost and feasible for the capacity of real-time application [1].

Range-based localization algorithms are designed to reduce the ranging error, and then reduce the localization error [1-2]. There are exact and approximate range-based localization algorithms. The exact methods provide the exact coordinate of the blind node (BN) while there is no error in the distance determination. For example, Range-based Least Square (R-LS) and Linear Least Square (L-LS) are two common and accurate ways for localization. However, the approximate algorithms can provide a coarse position. For example, Grid-scan method divides the target field into several cells, and select a cell as the position of the blind node by using the voting method. Note that the main advantages of the approximate algorithms are their low complexity, and the robustness against the error. In fact, geometry relationship obtains a relative position of the blind node rather than the actual position. So, the optimization of the measured distance

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can directly reduce the localization error because the measured distance can be affected by other factors, such as noises, the pass loss, multipath effects and reflections of the received signals and so on.

Recent papers on Min-Max algorithm have proposed improvements on the basic algorithm to enhance its performance for specific scenarios [10-11]. The Min-Max algorithm is an effective and simple method for localization. In [10], an Extended Min-Max algorithm (E-Min-Max) is proposed. The Membership Degree Min-Max (MD-Min-Max) localization algorithm as a precise and simple iteration algorithm for indoor localization is proposed in [11]. Real experiments show that the MD-Min-Max localization algorithm performs very well in short-range scenarios. In this paper we study the Min-Max algorithm for improving localization accuracy in WSNs.

#### A. Min-Max Algorithm

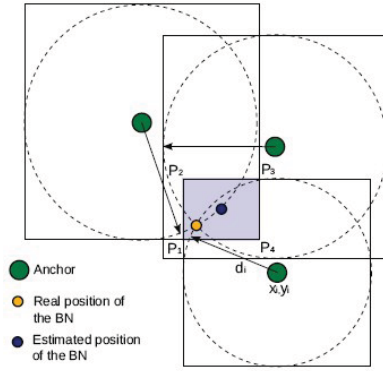


Fig.1 Min-Max Algorithm

The Min-Max algorithm, known as Bounding Box algorithm is a simple and straightforward localization algorithm. As shown in Fig.1, each side of the bounding box is two times the measured distance  $d_i$  between the BN and the AN. The bounding boxes are used to delimit a region where the position of the BN will be estimated. This region is a square whose vertices are placed at the following coordinates:

$$\begin{aligned} P_1 &= [\max(x_i - d_i), \max(y_i - d_i)] \\ P_2 &= [\max(x_i - d_i), \min(y_i + d_i)] \\ P_3 &= [\min(x_i + d_i), \min(y_i + d_i)] \\ P_4 &= [\min(x_i + d_i), \max(y_i - d_i)] \end{aligned} \quad (1)$$

Where  $i = 1, 2, \dots, n$ ,  $\min()$  and  $\max()$  are the minimum and maximum function, respectively.

So, the estimated position of the BN is approximated by calculating the geometric average of the vertices of the square, as shown in Eq. (2).

$$\begin{aligned} x_{es} &= \frac{\min(x_i + d_i) + \max(x_i - d_i)}{2} \\ y_{es} &= \frac{\min(y_i + d_i) + \max(y_i - d_i)}{2} \end{aligned} \quad (2)$$

Generally, the intersection region is formed by the intersections of these bounding boxes as shown in Fig.1. However, it is possible that there is no intersection region between these bounding boxes. This is because the estimated distance is too short compared to a real distance. In such scenarios, Min-Max localization algorithm can calculate a coarse position of the BN without problems.

Although Min-Max localization algorithm is a simple, it can produce high localization error even when having small distance measurement error. Due to multipath effect, most of the measured distances are larger than actual distances, which is especially common in indoor scenarios. Even if the measured distance is imprecise, the target is within the intersection region or close to the intersection region. Therefore, a more reasonable solution should be found to confirm the position of BN in the intersection region.

#### B. Extended Min-Max (E-Min-Max)

Min-Max localization algorithm is a simple, but it can produce high localization error. Therefore, Extended Min-Max localization algorithm is proposed in [10]. In the original paper E-Min-Max localization algorithm is presented with four different weights ( $W_1, W_2, W_3, W_4$ ) which are expressed as:

$$W_1(j) = \frac{1}{\sum_{i=1}^n |D_{i,j} - d_i|} \quad (3)$$

$$W_2(j) = \frac{1}{\sum_{i=1}^n (D_{i,j} - d_i)^2} \quad (4)$$

$$W_3(j) = \frac{1}{\sum_{i=1}^n |M_{i,j} - d_i|} \quad (5)$$

$$W_4(j) = \frac{1}{\sum_{i=1}^n |D_{i,j}^2 - d_i^2|} \quad (6)$$

Where  $D_{i,j}$  and  $M_{i,j}$  are the Euclidean distance and the Manhattan distance between anchor  $i$  and vertex  $j$  of the intersection region, respectively.

$$D_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (7)$$

$$M_{i,j} = |x_i - x_j| + |y_i - y_j| \quad (8)$$

So, the position of the blind node is estimated by calculating the weighted centroid with the weights and the coordinates of the vertices as shown in Eq. (9).

$$(x_{es}, y_{es}) = \left( \frac{\sum_{j=1}^4 W_a(j) \cdot x_j}{\sum_{j=1}^4 W_a(j)}, \frac{\sum_{j=1}^4 W_a(j) \cdot y_j}{\sum_{j=1}^4 W_a(j)} \right) \quad (9)$$

Compared to the original Min-Max localization algorithm, E-Min-Max localization algorithm increases algorithm complexity to estimate the weights for the vertices, but it reduces the localization error.

Generally, most of distance estimation errors are extremely large due to non-line-of-sight (NLOS) propagation and the pass loss even if close to the actual target position. Thus, E-Min-Max cannot improve the localization accuracy in some cases. In this paper, we will consider the optimization of the measured distance for this problem, thus the localization error can be reduced.

### III. IMPROVED E-MIN-MAX LOCALIZATION ALGORITHM

From Fig.1, we can know that the actual location of the unknown node is off the center of the intersection region. The actual position of the unknown node is in the center of the intersection region in exceptional cases only. The E-Min-Max algorithm estimates the location of the unknown node which is not in the center of the intersection region. In fact, the estimated position of the unknown node can be located in any point inside the intersection region. Moreover, some distance estimation errors are extremely large due to non-line-of-sight (NLOS) propagation, which results in large residues even if close to the actual target position. Thus, E-Min-Max algorithm cannot reduce the localization error in some cases. In order to reduce the localization error, we further investigate the weight  $W_3$  in Equation (1), two improved weights are proposed as follows

$$W_5(j) = \frac{1}{\sum_{i=1}^n (M_{i,j} - d_i)^2} \quad (10)$$

$$W_6(j) = \frac{1}{\sum_{i=1}^n |M_{i,j}^2 - d_i^2|} \quad (11)$$

Finally, the position of the unknown node is estimated by calculating the weighted centroid with the weights and the coordinates of the vertices as in Equation (9).

### IV. PERFORMANCE EVALUATION

#### A. Wireless Channel Propagation Model

In order to establish the propagation channel, we suppose that the RSS obeys the log-distance shadowing path loss model because the log-distance shadowing path-loss model is more suitable for actual environment. This propagation model is expressed by

$$RSS = PL0 - 10\beta \log_{10}(d / d_0) + N_\alpha \quad (12)$$

Where  $d$  is the real distance between nodes,  $PL0$  is the received signal strength at reference distance  $d_0$  ( $d_0 = 1$  m),  $\beta$  is the path loss exponent, and  $N_\alpha$  is a zero-mean Gaussian noise with standard deviation  $\alpha$ .

#### B. Experiment

In this part, we analyse the performance of the studied localization schemes through a real experiment. In Fig. 2, a wireless node is based on a MSP430 microprocessor and equipped with an IEEE 802.15.4 compliant Chipcon CC2500 radio module. All nodes are deployed in a 100m x 100m area

of the lawn in Anhui University. Four anchors are respectively placed in four corners, an unknown node is placed in 20 different positions.

In this experiment, in order to obtain the position of an unknown node with the improved E-Min-Max algorithm, we must compute  $PL0$  (the received signal strength at reference distance  $d_0 = 1$  m) and  $\beta$  (the path loss exponent) and the distance between any two anchor nodes. Next we introduce how to get these parameters and the localization error of the improved E-Min-Max algorithm in our experiment.

Firstly, any anchor node measures the RSS values from other anchor nodes, and calculates the distance between any two anchor nodes according to the coordinate of anchor nodes. Then, all anchor nodes calculate  $PL0$  according to the RSS values, we choose the average value of all  $PL0$  as  $PL0$ . According to Eq. (12), the path loss exponent can be calculated. Hence, the measured distance between an unknown node and anchor node can be obtained according to Eq. (12) and the RSS values of the unknown node received from anchor nodes. Finally, the localization error of the improved E-Min-Max algorithm can be calculated by the measured distance between an unknown node and anchor nodes.



Fig. 2 Sensor node

TABLE 1 LOCALIZATION ERRORS

the weights	Localization error (meters)
W1	14.5
W2	12.5
W3	13.2
W4	12.1
W5	9.9
W6	6.7

TABLE 1 shows the localization errors of six localization algorithms in the lawn. Clearly, the improved E-Min-Max algorithms is lower than the other Min-Max algorithms in localization error.

### V. CONCLUSIONS

Although E-Min-Max localization algorithm has improved the performance of Min-Max localization algorithm, it still needs to be enhanced in localization accuracy. Hence, in this paper we propose an improved E-Min-Max localization algorithm for reducing localization error. We improve the

weight model  $W_3$  in the proposed algorithm, and the improved weight model are  $W_5$  and  $W_6$ . Experimental results demonstrated that the proposed localization algorithm is superior to E-Min-Max localization algorithm in the localization accuracy. According to the experimental results, the weight models  $W_5$  and  $W_6$  are more reasonable than the weight models  $W_1$ ,  $W_2$ ,  $W_3$  and  $W_4$ . Although improved E-Min-Max localization algorithm with  $W_5$  and  $W_6$  reduces the localization error, but the localization error is still very big. Hence, we will continue to work on improvement of the localization accuracy in the future work.

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