

1. The plot of inductor current is shown below:

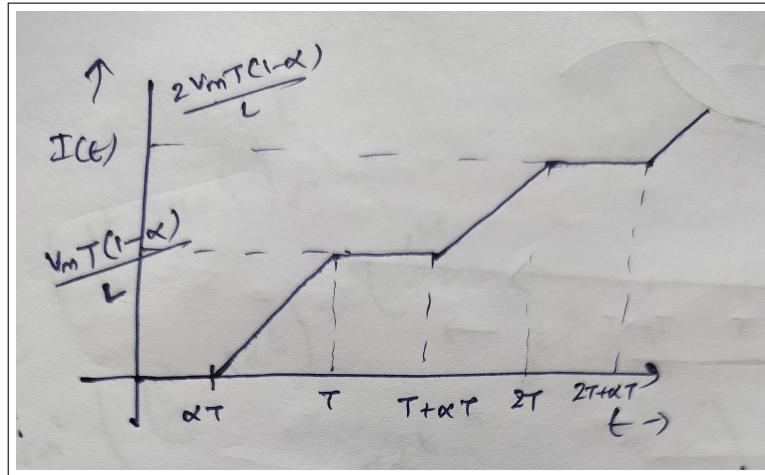


Figure 1: Current through the inductor

2. The average and RMS values of the applied voltage is:

$$V_{avg} = V_m(1 - \alpha)$$

$$V_{rms} = V_m \sqrt{1 - \alpha}$$

As shown in Fig. 1, the inductor current is **not periodic**, hence its average values from 0 to T and T to $2T$ are as follows:

$$I_{avg} = \frac{V_m T}{2L} (1 - \alpha)^2 \quad \text{from } 0 \text{ to } T$$

$$I_{avg} = \frac{V_m T}{2L} (3 - 4\alpha + \alpha^2) \quad \text{from } T \text{ to } 2T$$

3. The plot of the instantaneous power of the inductor is shown below:

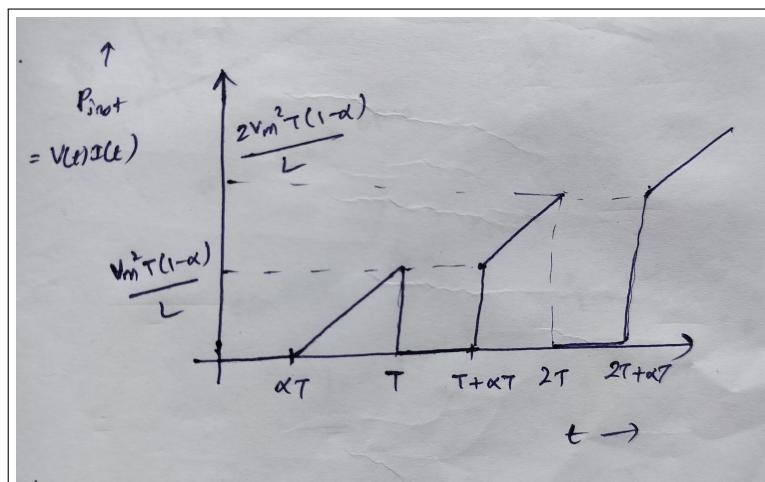


Figure 2: P_{inst} of the inductor

As shown in Fig. 2, the instantaneous power of the inductor is **not periodic**.

4. The expression for energy $e(t)$ stored in the inductor for the first cycle:

$$e(t) = \begin{cases} 0 & 0 < t < \alpha T \\ \frac{V_m^2}{2L}(t - \alpha T)^2 & \alpha T \leq t < T \end{cases}$$

5. The plot of the capacitor voltage is shown below:

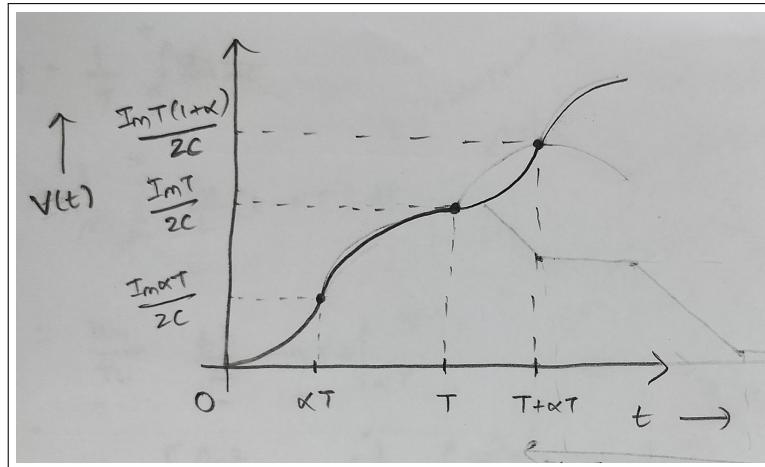


Figure 3: Voltage across the capacitor

6. As shown in Fig. 3, the voltage across the capacitor is **not periodic**. The expressions for the average and the RMS values of the current as well as the average values of voltage across capacitor are :

$$I_{avg} = \frac{I_m}{2} \quad V_{avg} = \frac{I_m T}{6C} (2 - \alpha) \quad \text{from } 0 \text{ to } T$$

$$I_{rms} = \frac{I_m}{\sqrt{3}} \quad V_{avg} = \frac{I_m T}{6C} (5 - \alpha) \quad \text{from } T \text{ to } 2T$$

7. The plot of the instantaneous power of the capacitor is shown below:

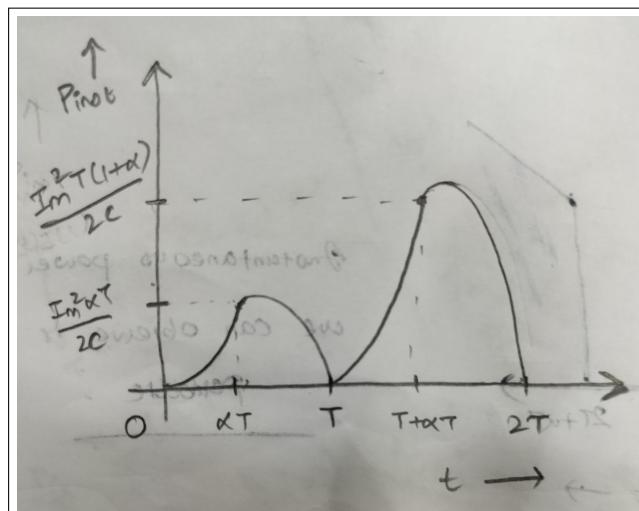


Figure 4: P_{inst} of the capacitor

As shown in Fig. 4, the instantaneous power of the capacitor is **not periodic**.

8. The expression for energy $e(t)$ stored in the capacitor for the first cycle:

$$e(t) = \frac{I_m^2 t^4}{8c\alpha^2 T^2} \quad 0 < t < \alpha T$$

$$e(t) = \frac{I_m^2}{2c(1-\alpha)} \left[\frac{(t-T)^4}{4(1-\alpha)T^2} - \frac{t^2}{2} + tT + \frac{T^2}{4}(\alpha^3 - \alpha^2 - \alpha - 1) \right] + \frac{I_m^2 \alpha^2 T^2}{8c} \quad \alpha T < t < T$$

9. The sum of Q_1 and Q_2 is as given below:

$$Q_1 + Q_2 = \frac{125}{\pi} - \frac{25\pi \times 10^{-4}}{2} = +39.785 \text{ var}$$

If the frequency of the source is **doubled**, then the reactive power of inductor(Q_1) is **halved**, and the reactive power of capacitor(Q_2) is **doubled**. Hence, the updated value of the sum of Q_1 and Q_2 will be:

$$Q_1 + Q_2 = \frac{125}{2\pi} - 25\pi \times 10^{-4} = +19.886 \text{ var}$$

Hence, effectively the total reactive power is **halved** since Q_2 is negligible when compared with Q_1 .