Polygonal Approximations

- A digital boundary can be approximated with arbitrary accuracy by a polygon.
- In practice, the goal of polygonal approximation is to capture the “essence” of the boundary shape with the fewest possible polygonal segments.

- For, a closed boundary, the approximation becomes exact when the number of segments of a polygon is equal to number of points in the boundary.

- Here each pair of adjacent points defines a segment of a polygon.

- The goal of a polygon is to capture the essence of the shape in a given boundary using fewest possible number of segments.

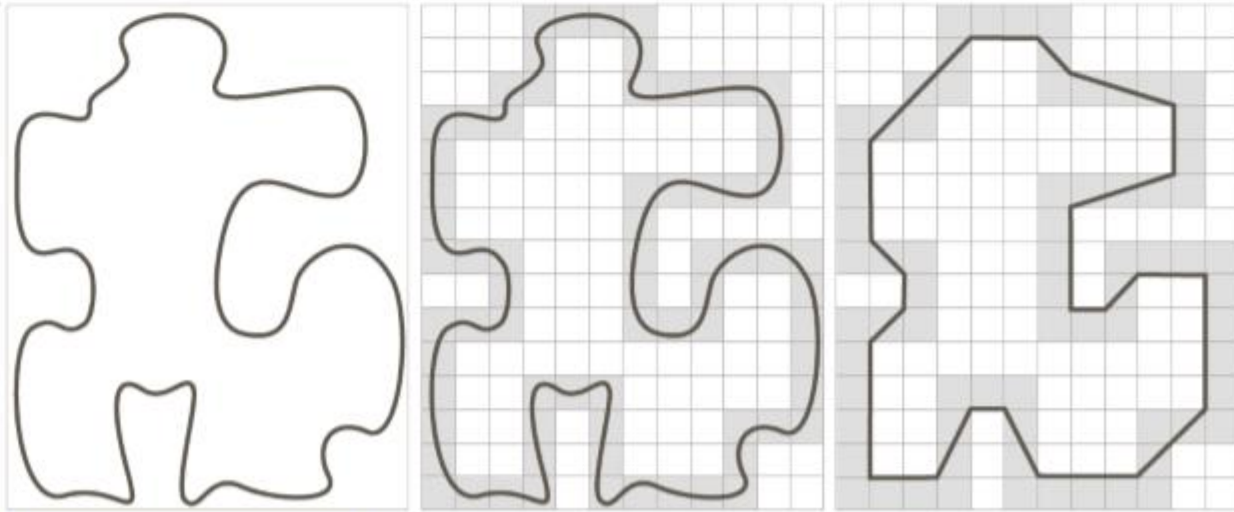
- One of the most powerful method for representing a boundary is by using minimum perimeter polygon(MPP).

Minimum Perimeter Polygons

- Produces a polygon of minimum perimeter that fits the geometry established by the cell strip.
- The size of the cell determines the accuracy of the polygonal approximation.

Minimum-Perimeter Polygon

The goal is to represent the shape in a given boundary using the fewest possible number of sequences.



Object boundary (binary image)

Enclose boundary in a grid

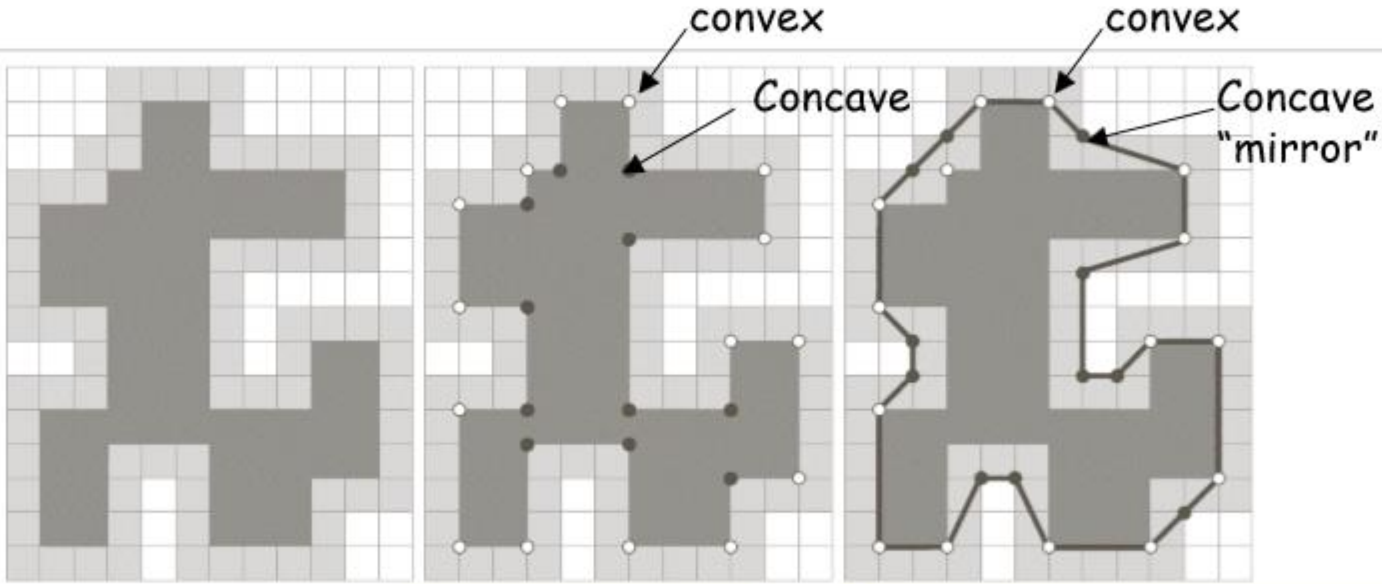
Allow boundary to shrink. The vertices of the polygon are all inner or outer corners of the grid.

a b c

FIGURE 11.6 (a) An object boundary (black curve). (b) Boundary enclosed by cells (in gray). (c) Minimum-perimeter polygon obtained by allowing the boundary to shrink. The vertices of the polygon are created by the corners of the inner and outer walls of the gray region.

Minimum-Perimeter Polygon

The boundary cells from the previous slide enclose the circumscribed shape.



Traverse the 4-connected boundary of the circumscribed shape.

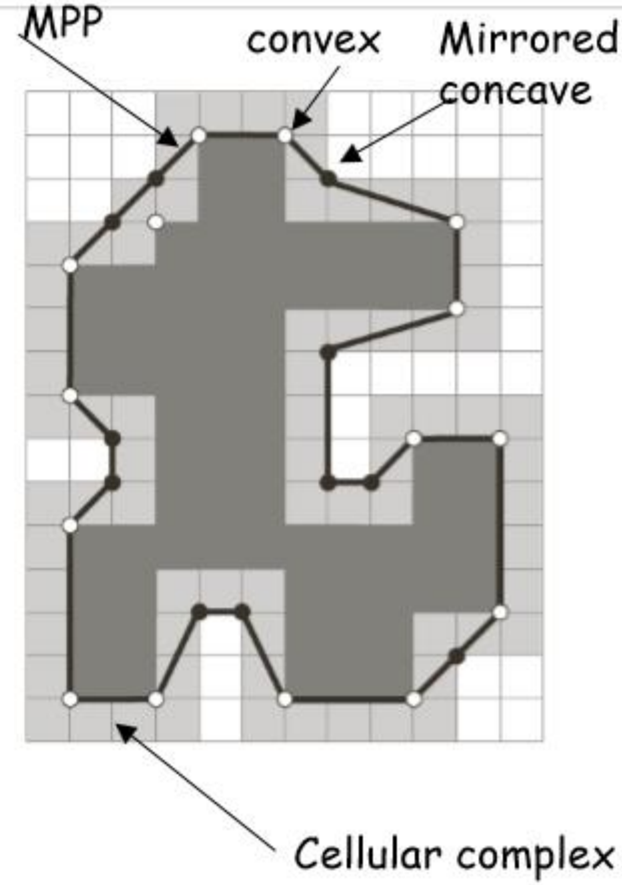
Concave vertices on this boundary have "mirrors" on the outer boundary. The boundary is described by inner convex and outer concave vertices.

a b c

FIGURE 11.7 (a) Region (dark gray) resulting from enclosing the original boundary by cells (see Fig. 11.6). (b) Convex (white dots) and concave (black dots) vertices obtained by following the boundary of the dark gray region in the counterclockwise direction. (c) Concave vertices (black dots) displaced to their diagonal mirror locations in the outer wall of the bounding region; the convex vertices are not changed. The MPP (black boundary) is superimposed for reference.

Minimum-Perimeter Polygon

<u>MPP Observations:</u>
1. The MPP bounded by a simply connected cellular complex is not self-intersecting.
2. Every *convex* vertex of the MPP is a W vertex, but not every W vertex of a boundary is a vertex of the MPP.
3. Every *mirrored concave* vertex of the MPP is a B vertex, but not every B vertex of a boundary is a vertex of the MPP.
4. All B vertices are on or outside the MPP, and all W vertices are on or inside the MPP.
5. The uppermost, leftmost vertex in a sequence of vertices contained in a cellular complex is always a W vertex of the MPP.



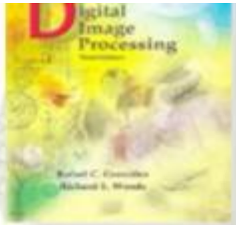
Not all vertices in the MPP become vertices of the MPP

Minimum-Perimeter Polygon

Let a=(x_1,y_1), b=(x_2,y_2), and c=(x_3,y_3)

$$A = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$\operatorname{sgn}(a,b,c) = \det(\boldsymbol{A}) = \begin{cases} > 0 & \textit{if } (a,b,c) \textit{ is a counterclockwise sequence} \\ = 0 & \textit{if } (a,b,c) \textit{ are collinear} \\ < 0 & \textit{if } (a,b,c) \textit{ is a clockwise sequence} \end{cases}$$



Minimum-Perimeter Polygon

<u>Definitions:</u>

Form a list whose rows are the coordinates of each vertex and whether that vertex is W or B. The concave verttices must be mirrored, the vertices must be in sequential order, and the first uppermost, leftmost vertex V_O is a W vertex. There is a white crawler (W_C) and a black crawler (B_C). The Wc crawls along the convex W vertices, and the B_C crawls along the mirrored concave B vertices.

Minimum Perimeter Polygon

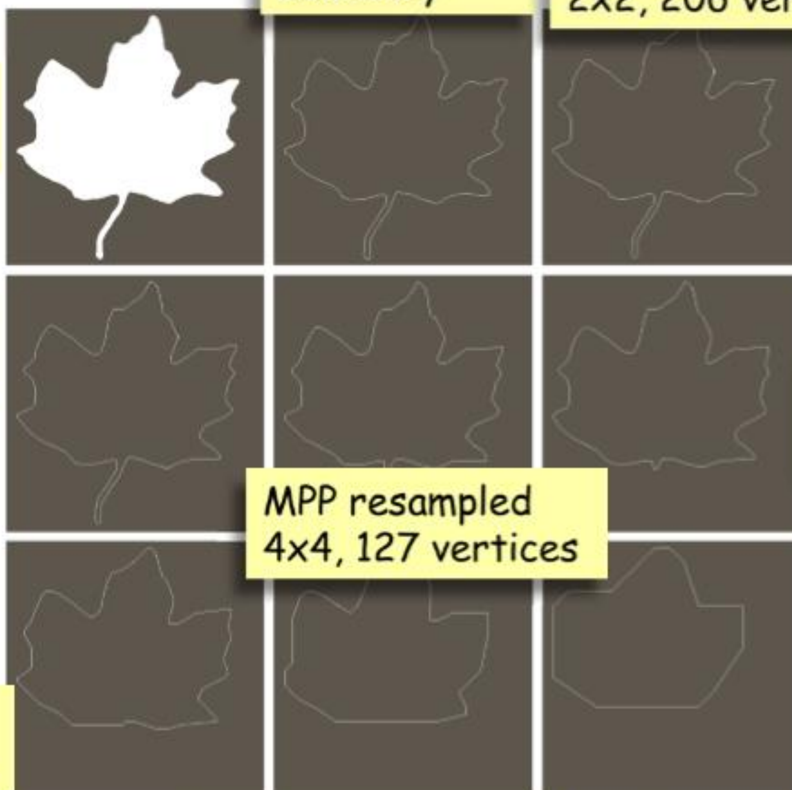


FIGURE 11.8 (a) 566 × 566 binary image. (b) 8-connected boundary. (c) through (i), MMPs obtained using square cells of sizes 2, 3, 4, 6, 8, 16, and 32, respectively (the … in (b) is 1900. The numbers of vertices in (c) through (i) are 206, 160, 127, 92, 66, 32, and 13, respectively.

EXTRA

Minimum-Perimeter Polygon

<u>MPP Algorithm:</u>

1. Set $W_C=B_C=V_O$

2.
(a) V_K is on the positive side of the line (V_L,W_C) $[sgn(V_L,W_C,V_K)>0$
(b) V_K is on the negative side of the line (V_L,W_C) or is collinear with it $[sgn(V_L,W_C,V_K)\leq 0$; V_K is on the positive side of the line (V_L,B_C) or is collinear with it $[sgn(V_L,B_C,V_K)\geq 0$
(c) V_K is on the negative side of the line (V_L,B_C) $[sgn(V_L,B_C,V_K)<0$

If condition (a) holds the next MPP vertex is W_C and $V_L=W_C$; set $W_C=B_C=V_L$ and continue with the next vertex.
If condition (b) holds V_K becomes a candidate MPP vertex. Set $W_C=V_K$ if V_K is convex otherwise set $B_C=V_K$. Continue with next vertex.
If condition (c) holds the next vertex is B_C and $V_L=B_C$.
Re-initialize the algorithm by setting $W_C=B_C=V_L$ and continue with the next vertex after V_L.

3. Continue until the first vertex is reached again.

Minimum-Perimeter Polygon

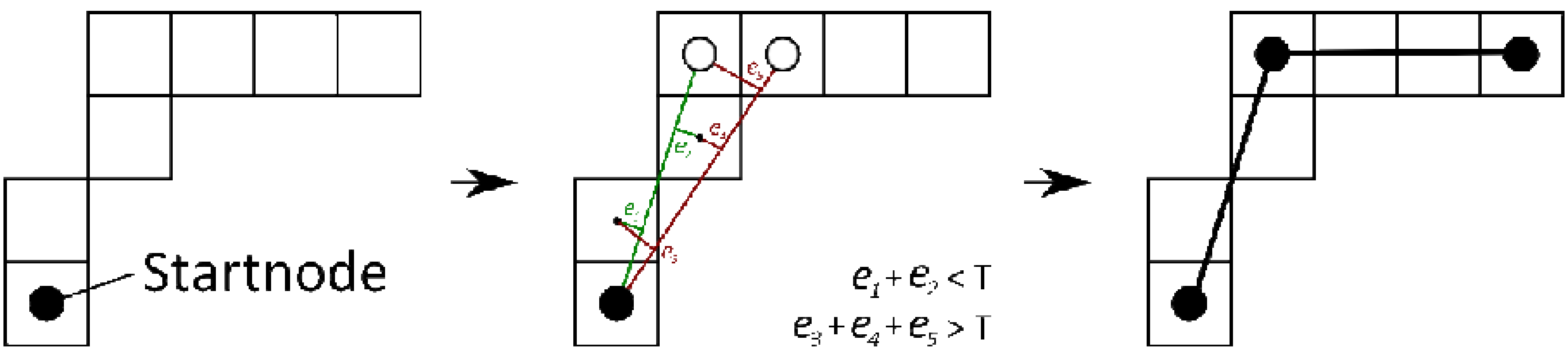
V_0	(1,4)	W
V_1	(2,3)	B
V_2	(3,3)	W
V_3	(3,2)	B
V_4	(4,1)	W
V_5	(7,1)	W
V_6	(8,2)	B
V_7	(9,2)	B

The fundamental concept is to move the crawlers along the perimeter, calculate the curvatures, and determine if the vertex is a vertex of the MPP.

	W_C	B_C	V_L	W_C curvature	B_C curvature
	V_0	V_0	V_0		
V_1	V_0	V_0	V_0	$V_L, W_C, V_1 = 0$	$V_L, W_B, V_1 = 0$
V_2	V_0	V_1	V_0	$V_L, W_C, V_2 = 0$	$V_L, W_B, V_2 > 1$
V_3	V_2	V_1	V_0	$V_L, W_C, V_3 < 0$	$V_L, W_B, V_3 = 0$
V_4	V_2	V_3	V_0	$V_L, W_C, V_4 < 0$	$V_L, W_B, V_4 = 0$
V_5	V_4	V_3	V_4	$V_L, W_C, V_5 > 0$	
V_5	V_4	V_4	V_4	$V_L, W_C, V_5 = 0$	$V_L, W_B, V_5 = 0$
V_6	V_5	V_4	V_4	$V_L, W_C, V_6 > 0$	
V_7	V_5	V_6	V_5	$V_L, W_C, V_7 = 0$	$V_L, W_B, V_7 = 0$

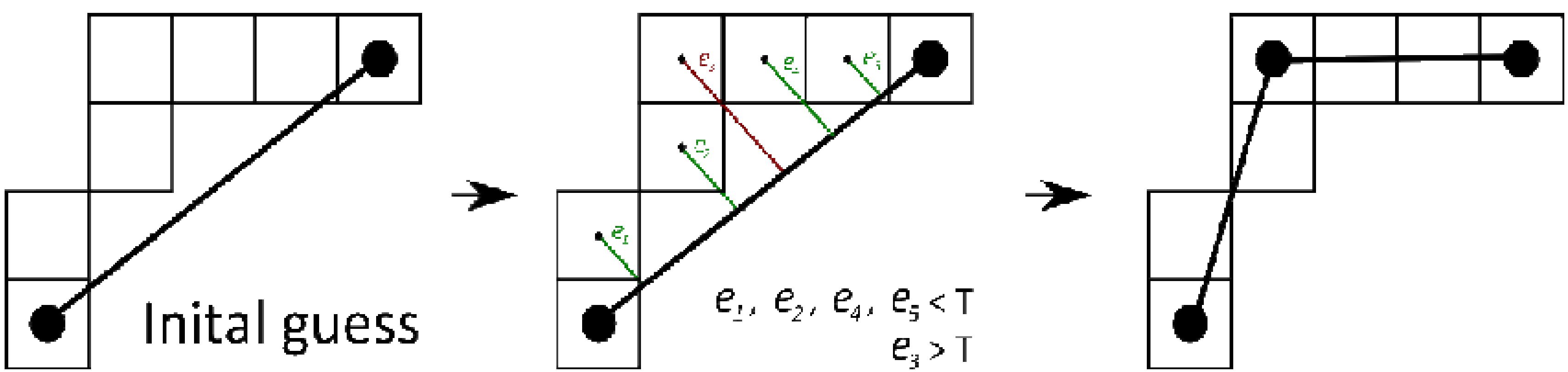
Polygonal Approximations

- **Merging techniques**
  1. Walk around the boundary and fit a least-square-error line to the points until an error threshold is exceeded
  2. Start a new line, go to 1
  3. When start point is reached the intersections of adjacent lines are the vertices of the polygon



Polygonal Approximations

- **Splitting techniques**
  1. Start with an initial guess, e.g., based on majority axes
  2. Calculate the orthogonal distance from lines to all points
  3. If maximum distance > threshold, create new vertex there
  4. Repeat until no points exceed criterion



- A digital boundary can be approximated with arbitrary accuracy by a polygon.
- Minimum perimeter polygons.

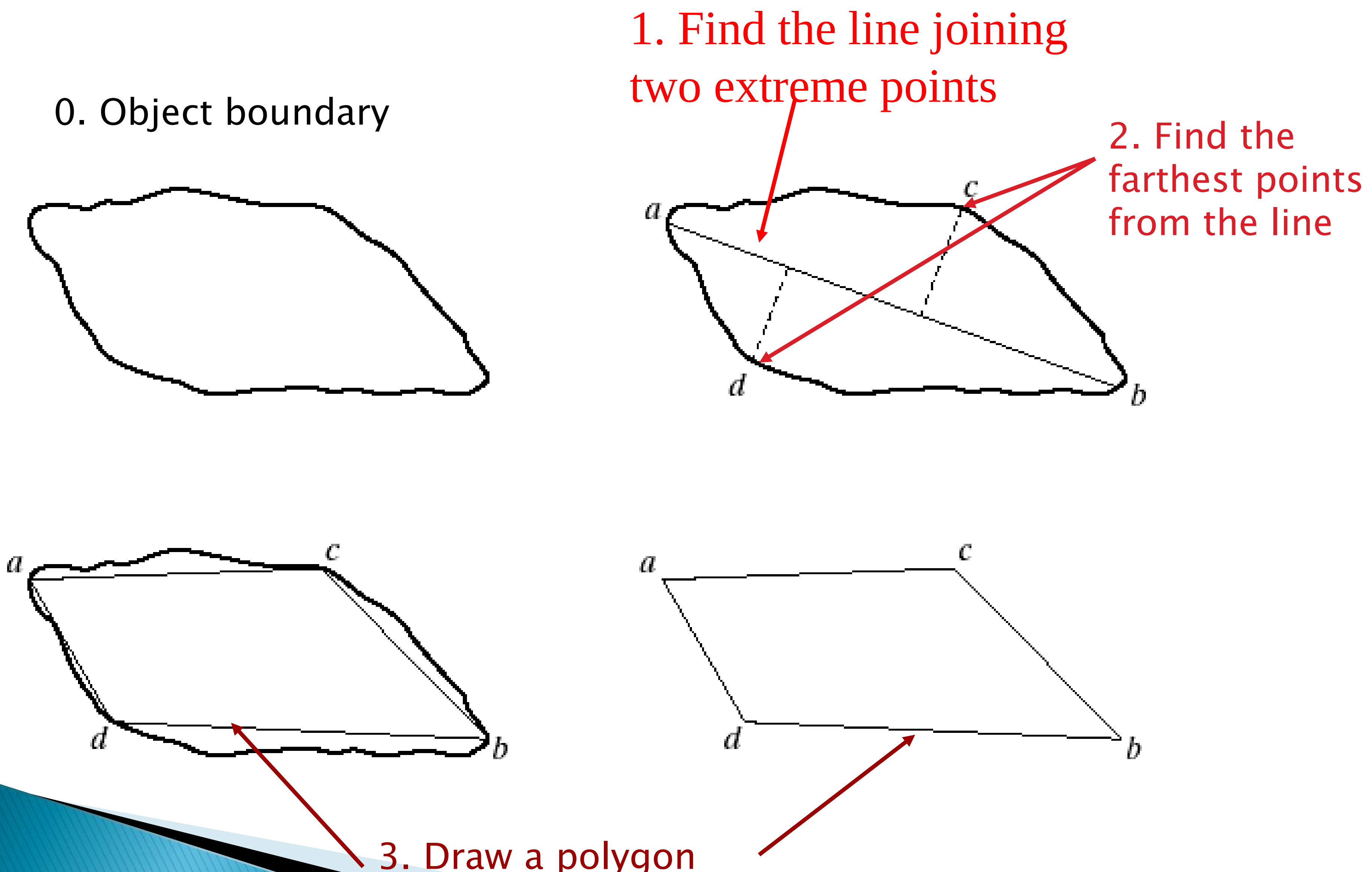
  A polygon of minimum perimeter fitted to the object boundary enclosed by cells.
- Splitting techniques.

  Subdivision of a segment successively into two parts until a given criterion is satisfied.

Splitting techniques

- One approach to boundary segment splitting is to subdivide a segment successively into two part until a specified criterion is satisfied.
- For a closed boundary, the best starting points usually are two farthest points in the boundary.
- Fig 11.4(c) shows the result of using the splitting procedure with a threshold equal to 0.25 times the length of line *ab*.

Polygon Approximation:Splitting Techniques



(Images from Rafael C. Gonzalez and Richa
Wood, Digital Image Processing, 2nd Edition

Representation
polynomial approximations

- Splitting Techniques