# Linear and non-linear operations

Example for linear and non linear Function

| | $x$ | $y$ | |
|---|---|---|---|
| | $-7$ | $4$ | |
| $+4$ | $-3$ | $3$ | $-1$ |
| $+4$ | $1$ | $2$ | $-1$ |
| $+6$ | $7$ | $1$ | $-1$ |

Change in x with respect to y

$$\frac{\Delta y}{\Delta x} = -\frac{1}{4}$$

last one $\Rightarrow -1/6$

$\therefore$ Change in x, in this is not linear.

Linear $\longrightarrow$ Constant change in $(x, y)$ point.

Non linear $\longrightarrow$ Difference/Variation in rate of change

# Linear and non-linear operations

- H be an operator whose input and output are images

- H is linear if for any two images f & g and any two scalars a & b

  – $H(a\mathbf{f}+b\mathbf{g}) = aH(\mathbf{f})+bH(\mathbf{g})$
    - then Linear otherwise non-linear

{{ The result of applying linear operator to the sum of two images is identical to applying the operator to the images individually, multiplying the results by the appropriate constants and their adding results. }}

## How to check ?

H be an operator whose I/p & o/p are images.

H is linear if

$$H(af + bg) = aH(f) + bH(g) \quad \text{——(i)}$$

- a & b are arbitary constants.
- f & g are image matrix / Functions.
- H is Function [eg: max, median, min .....]

Eg

$a_1 = 2, \; a_2 = 4$

$f_1 = \begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix} \quad f_2 = \begin{bmatrix} 1 & 5 \\ 6 & 8 \end{bmatrix}$

$$\boxed{H[a_i * f_i(x,y) + a_j * f_j(x,y)] = a_i * H[f_i(x,y)] + a_j * H[f_j(x,y)]}$$

Sum Function is taken as example

$$= \Sigma \left[ 2 * \begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix} + 4 \begin{bmatrix} 1 & 5 \\ 6 & 8 \end{bmatrix} \right]$$

$$= \Sigma \left[ \begin{bmatrix} 6 & 14 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 20 \\ 24 & 32 \end{bmatrix} \right]$$

$$= \begin{pmatrix} 10 & 34 \\ 32 & 36 \end{pmatrix}$$

$$= \underline{\underline{112}}$$

$$2 * \Sigma \begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix} + 4 * \Sigma \begin{bmatrix} 1 & 5 \\ 6 & 8 \end{bmatrix}$$

$$= 2 * 16 + 4 * 20$$

$$= \underline{\underline{112}}$$

$\therefore$ Sum Function is linear Function

Max Function taken as example

$a_1 = 1$, $a_2 = -1$, ~~$x_1 = [0\ 2$~~

$x_1 = [0\ \ 2;\ \ 2,\ 3]$ $\quad x_2 = [6\ \ 5;\ \ 4\ \ 7]$

$$\Rightarrow \max\left[\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix} + (-1)\begin{bmatrix}6 & 5\\ 4 & 7\end{bmatrix}\right] \qquad \Rightarrow \max\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix} + (-1) * \max\begin{bmatrix}6 & 5\\ 4 & 7\end{bmatrix}$$

$$\Rightarrow \max\left[\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix} + \begin{bmatrix}-6 & -5\\ -4 & -7\end{bmatrix}\right] \qquad \Rightarrow 3 + (-1)\,7$$

$$\Rightarrow \max\begin{bmatrix}-6 & -3\\ -2 & -4\end{bmatrix} \qquad \Rightarrow \underline{\underline{-4}}$$

$$\Rightarrow \underline{\underline{-2}}$$

Not equal

Not linear

# Illumination and Reflectance

- The types of images in which we are interested are generated by the combination of an “illumination” source and the reflection or absorption of energy from that source by the elements of the “scene” being imaged.

- **Illumination** is the amount of source light incident on the scene. It is represented as i(x, y).
- **Reflectance** is the amount of light reflected by the object in the scene. It is represented by r(x, y).
- **Illumination** is defined as the energy of light (ε) striking a surface of specific unit area per unit time.

## A Simple image formation model

We denoted image by 2D Function of the form $f(x,y)$. The value or amplitude of $f$ at Spatial Coordinates $(x,y)$ is a +ve Scalar quantity whose Physical meaning is determined by the source of images.

When an image is generated from a Physical Process is its intensity values are Proportional to the energy radiated by a Physical source. As a Consequence $f(x,y)$ must be non-zero & Finite.

ie $0 < f(x,y) < \infty$

The $f(x,y)$ may be characterized by 2 components.

1. The amount of source illimination incident on the scene being viewed.
2. The amount of illumination reflected by the objects in the scene.

These are called illumination & Reflectance $i(x,y)$ and $r(x,y)$

These two Functions are combined to form the Product ie

$$f(x,y) = i(x,y) * r(x,y)$$

where $0 < i(x,y) < \infty$

$0 < r(x,y) < 1$

means total absorption is 0

and total ~~refle~~ reflectance is 1

## Ranges of $i(x,y)$ for visible light

Sun — day time → produces 90000 lm/$m^2$ of illumination on surface of earth.

Clear evening moon → 0.1 lm/$m^2$ illumination

Commercial / official → 1000 lm/$m^2$ — " —

## Average reflectance values $r(x,y)$

0.01 → Black velvet

0.65 → Stainless steel

0.8 → Wall white paint

0.9 → Silver plated metal

0.93 → Snow

lm means lumen — International unit of luminous flux.