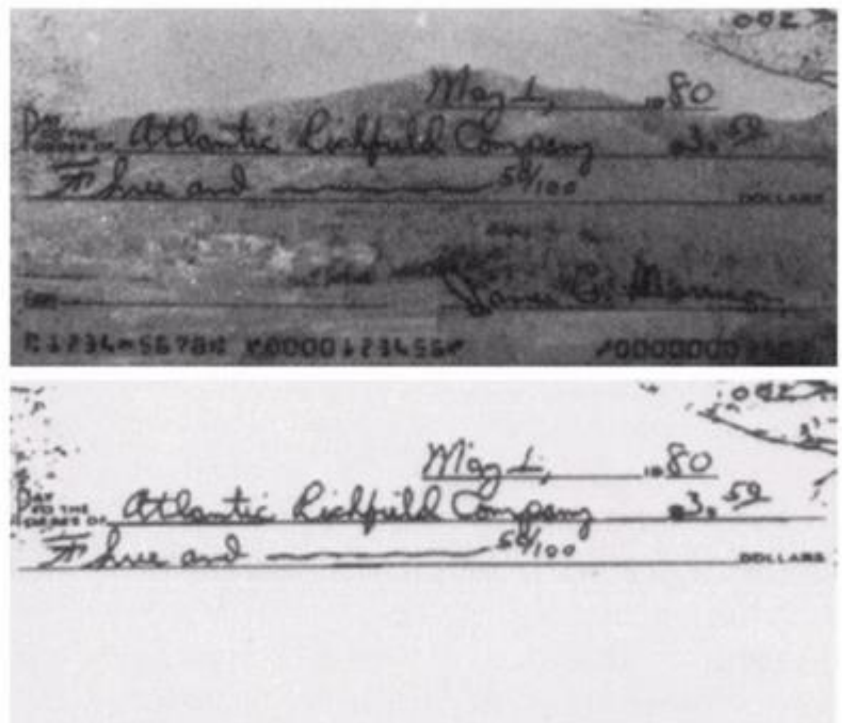
Thresholding
Use of Boundary Characteristics

a
b

FIGURE 10.37
(a) Original image. (b) Image segmented by local thresholding. (Courtesy of IBM Corporation.)



Thresholding
The Role of Illumination

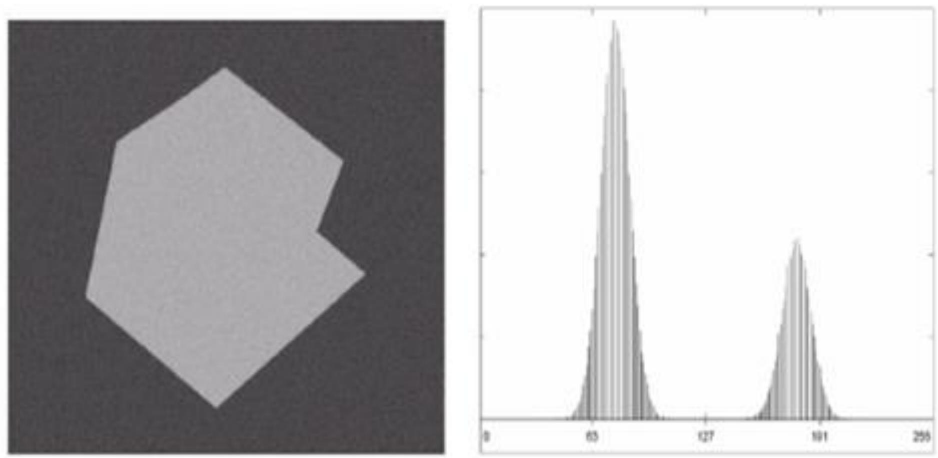
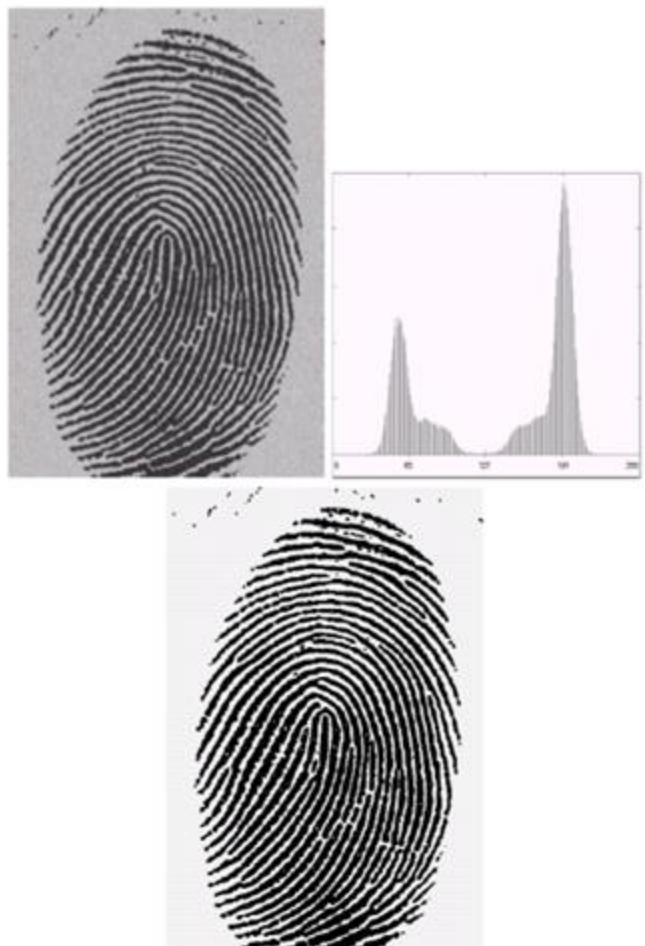


FIGURE 10.27
(a) Computer generated reflectance function.
(b) Histogram of reflectance function.

Thresholding
Basic Global Thresholding



a b
c

FIGURE 10.29 (a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)

Thresholding
Basic Adaptive Thresholding

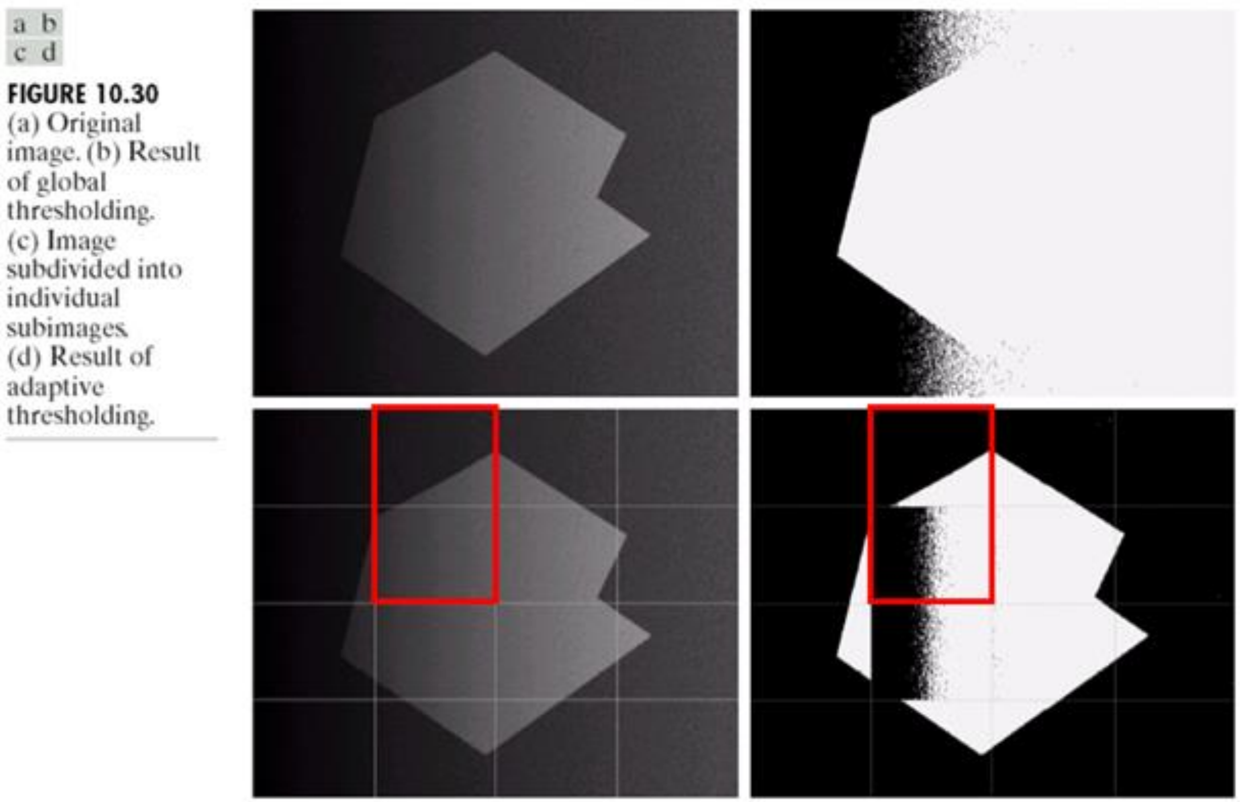


FIGURE 10.30 (a) Original image. (b) Result of global thresholding. (c) Image subdivided into individual subimages. (d) Result of adaptive thresholding.

The Basics of Intensity Thresholding

- The intensity histogram of an image f(x,y), composed of light objects on a dark background, in that object and background pixels have intensity values grouped into two dominant modes.

- One obvious way to extract the objects from background is to select a threshold T, that separates these modes.

- Any point (x,y) in image at which f(x,y) > T is called an object point; otherwise, the point is called background point.

Thresholding of pixel grey level
(Basic Global Thresholding)

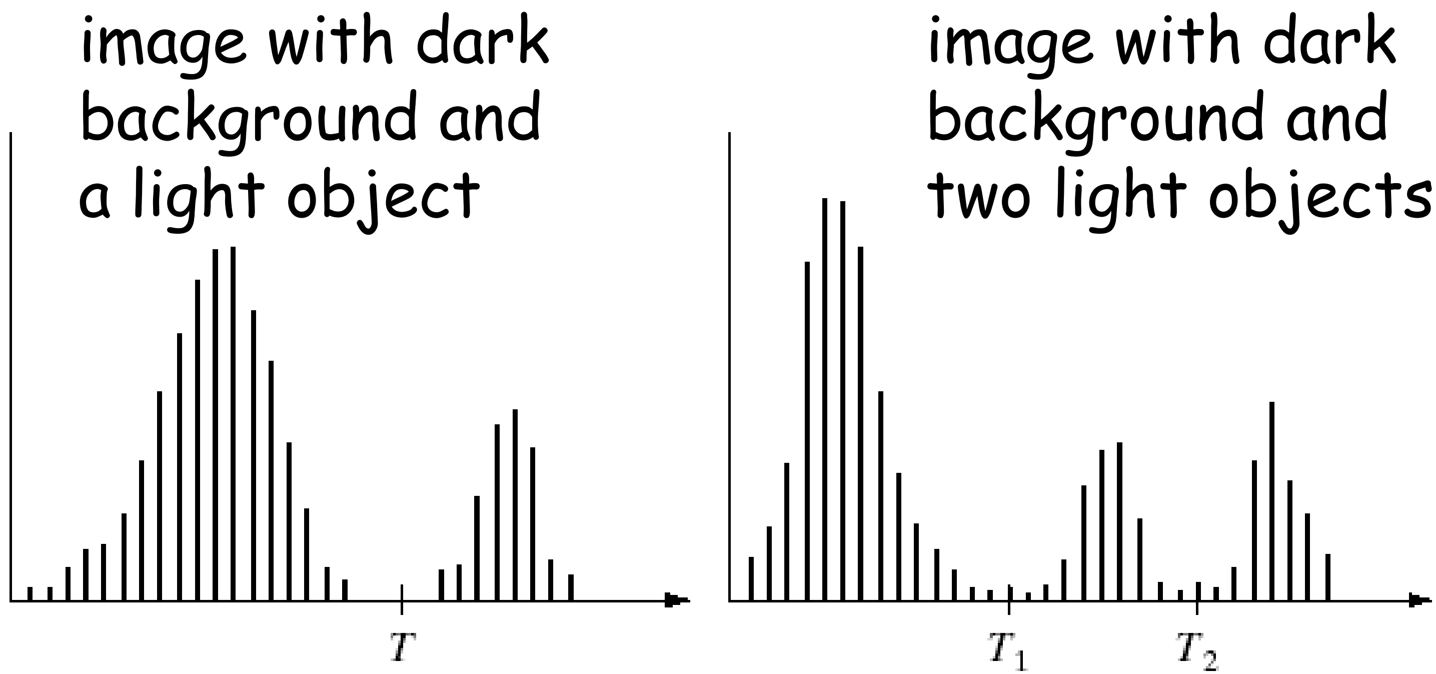
Segmentation is accomplished by scanning the image pixel by pixel and labeling each pixel as object or background, depending on whether the grey level is greater or less than the value of T .

$$g(x,y) = \begin{cases} 0 & f(x,y) < T \\ \hline 1 & f(x,y) => T \end{cases}$$

Thersholding works well when a grey level histogram of the image groups separates the pixels of the object and the background into two dominant modes. Then a threshold T can be easily chosen between the modes.

- T constant is applicable over entire image, the process given in this equation is global thresholding.
- When the value of T changes over an image, then variable thresholding.
- If T depends on the spatial coordinates (x, y) then variable thresholding is referred as dynamic or adaptive thresholding.

Thresholding



a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Multilevel thresholding

- a point (x,y) belongs to
  - to an object class if $T_1 < f(x,y) \leq T_2$
  - to another object class if $f(x,y) > T_2$
  - to background if $f(x,y) \leq T_1$
- T depends on
  - only f(x,y) : only on gray-level values ⇨ Global threshold
  - both f(x,y) and p(x,y) : on gray-level values and its neighbors ⇨ Local threshold

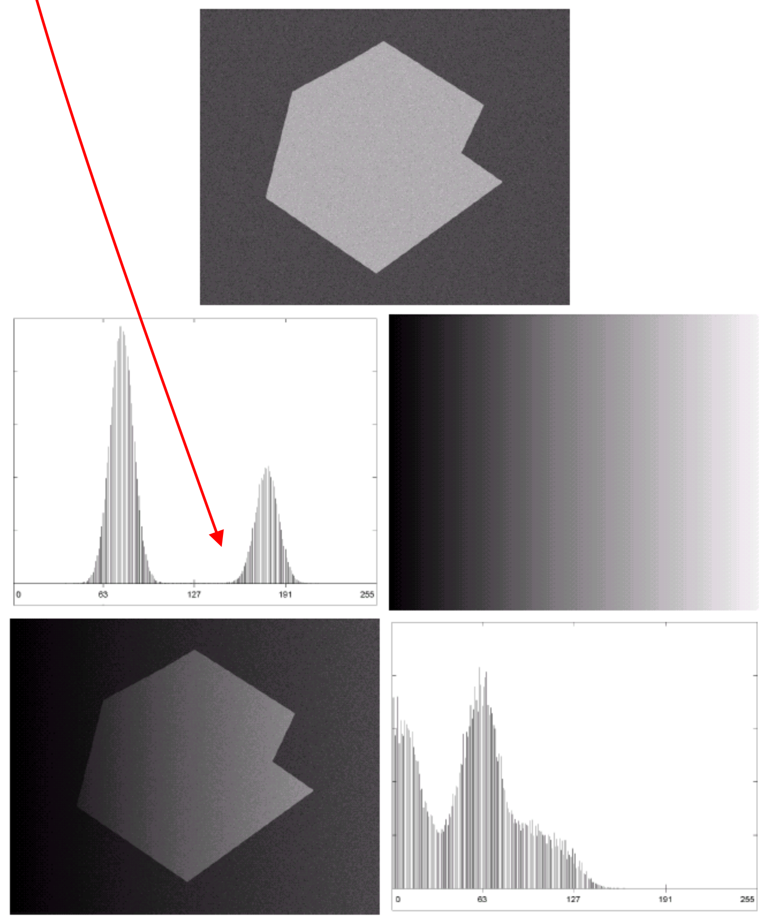
- Thresholding directly related to width and depth of the vallyes ie,
  - Separation between peaks
  - Noise content in the image
  - Relative sizes of the objects and background.
  - The uniformity of the illumination source.
  - The uniformity of the reflectance properties of the image.

The Role of Illumination

easily use global thresholding
object and background are separated

$$f(x,y) = i(x,y)\, r(x,y)$$

a) computer generated reflectance function
b) histogram of reflectance function
c) computer generated illumination function (poor)
d) product of a) and c)
e) histogram of product image



a
b c
d e

FIGURE 10.27 (a) Computer generated reflectance function. (b) Histogram of reflectance function. (c) Computer generated illumination function. (d) Product of (a) and (c). (e) Histogram of product image.

difficult to segment

- Illumination
  - Image is a product of reflectance and illuminance
  - Reflection nature of objects and backaground
  - Poor (nonlinear) illumination could impede the segmentation
  - The final histogram is a result of convolution of the histogram of the log reflectance and log illuminance functions
  - Normalization if the illuminance function is known

Three basic approaches to this problem:

1. Correct the shading pattern directly (multiplying image by the inverse pattern)
2. Attempt to correct the global shading pattern via processing(top-hat transformation)
3. Variable thresholding

Basic Global and Local Thresholding

Thresholding may be viewed as an operation that involves tests against a function T of the form:

$T = T[x,y,p(x,y),f(x,y)]$

Where $f(x,y)$ is the gray level, and $p(x,y)$ is some local property.

Simple thresholding schemes compare each pixels gray level with a single global threshold. This is referred to as **Global Tresholding.**

If T depends on both f(x,y) and p(x,y) then this is referred to a **Local Thresholding**.

Basic Global and Local Thresholding

- When intensity distributions of objects and background pixels are sufficiently distinct, it is possible to use a single (global) threshold applicable over the entire image.
- An algorithm capable of estimating automatically the threshold value for each image is required.
- The following iterative algorithm can be used for this purpose.

So how can we determine the threshold value automatically?

Example automatic thresholding methods

1. Otsu’s method

2. K-means clustering

Otsu's method

- Automatic thresholding method
  - automatically picks "best" threshold **t** given an image histogram

- Assumes 2 groups are present in the image:

1. Those that are <= t.
2. Those that are > t.

Otsu’s method

Best choices for t.

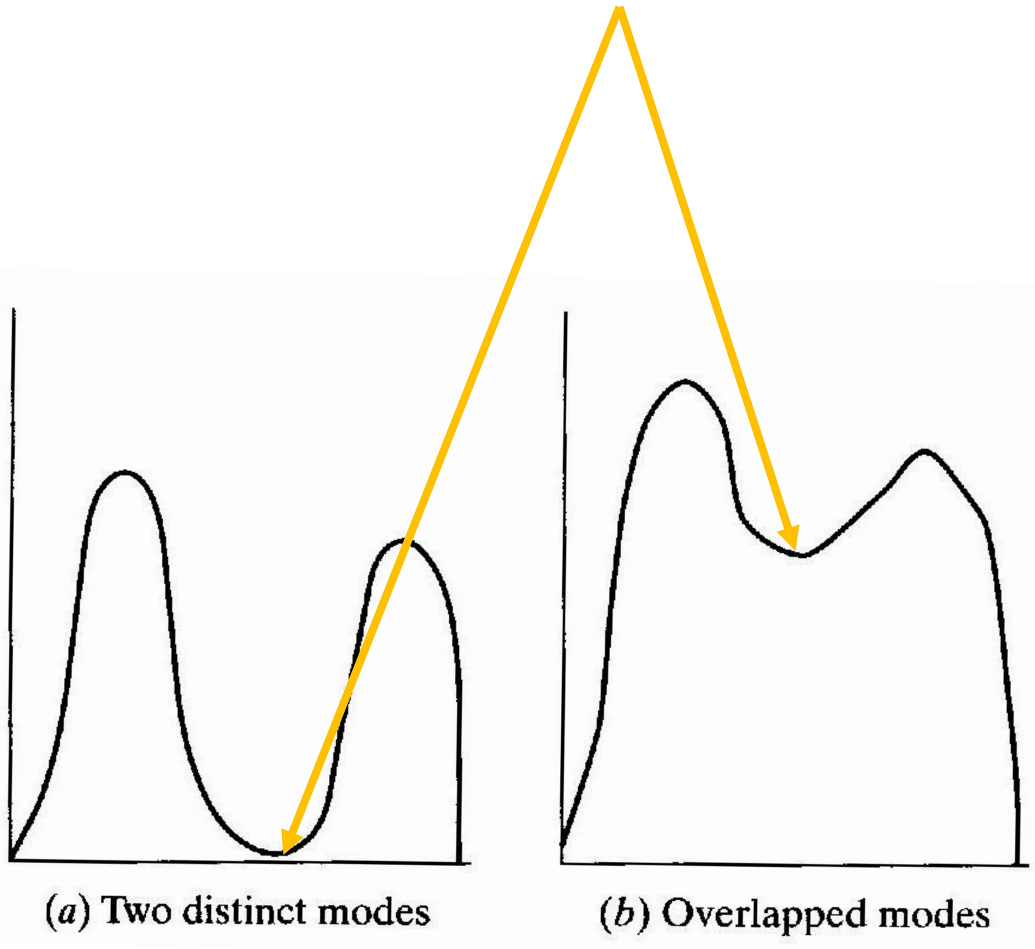


Figure 3.23 Two image histograms. (a) The histogram has two easily separable modes; (b) the histogram has overlapped modes that make it more difficult to find a suitable threshold.

Otsu’s method

For *every* possible t:

A. Calculate within group variances:

1. Probability of being in group 1; Probability of being in group 2

2. Determine mean of group 1; Determine mean of group 2

3. Calculate variance for group 1; Calculate variance for group 2

4. Calculate weighted sum of group variances

B. Remember which t gave rise to minimum.

Optimum Global Thresholding Using Otsu's Method

- Thresholding may be viewed as a statistical-decision theory problem whose objective is to minimize the average error incurred in assigning pixels to two or more groups.

- **Otsu's method** is used to automatically perform clustering-based image thresholding, or, ***the reduction of a gray level image to a binary image.***

- The algorithm assumes that the image contains two classes of pixels following bi-modal histogram (foreground pixels and background pixels).

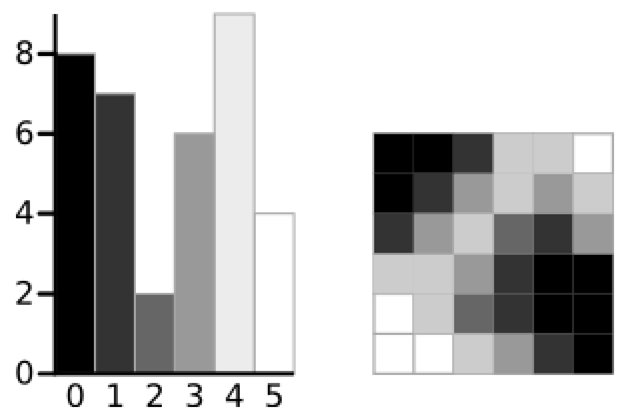
- It calculates the optimum threshold separating the two classes so that their combined spread (intra-class variance) is minimal.

- **Otsu's method is named after Nobuyuki Otsu.**

- Otsu's thresholding method involves iterating through all the possible threshold values and calculating a measure of spread for the pixel levels each side of the threshold.i.e. the pixels that either fall in foreground or background.

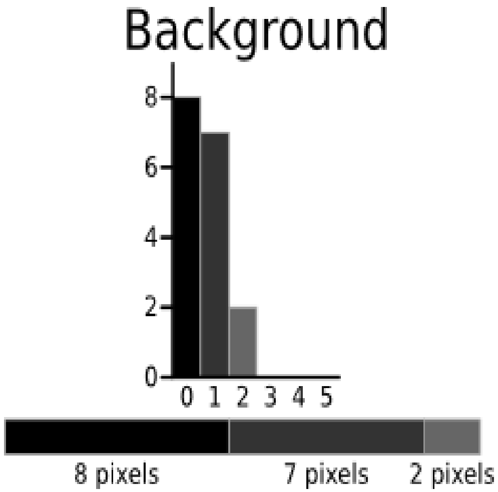
- The aim is to find the threshold value where the sum of foreground and background spreads is at its minimum.

- The algorithm will be demonstrated using the simple 6x6 image shown below. The histogram for the image is shown next to it. To simplify the explanation, only 6 grayscale levels are used.



A 6-level greyscale image and its histogram

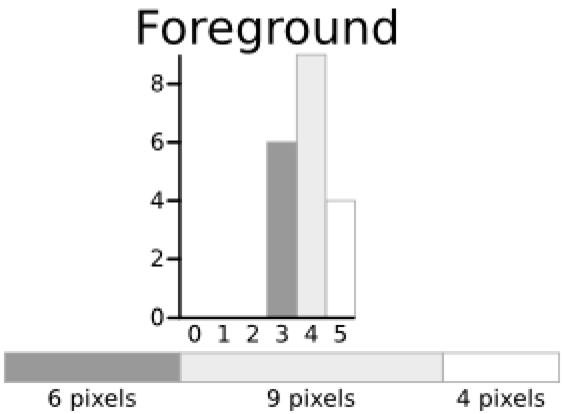
The calculations for finding the foreground and background variances (the measure of spread) for a single threshold are now shown. In this case the threshold value is 3.



$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

$$\text{Mean} \quad \mu_b = \frac{(0\times 8)+(1\times 7)+(2\times 2)}{17} = 0.6471$$

$$\begin{aligned}\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2\times 8)+((1-0.6471)^2\times 7)+((2-0.6471)^2\times 2)}{17} \\ &= \frac{(0.4187\times 8)+(0.1246\times 7)+(1.8304\times 2)}{17} \\ &= 0.4637\end{aligned}$$



$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

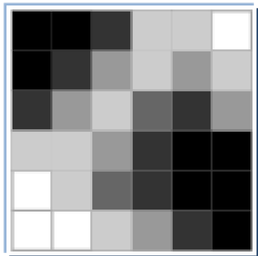
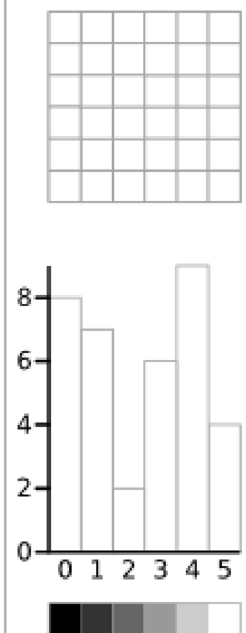
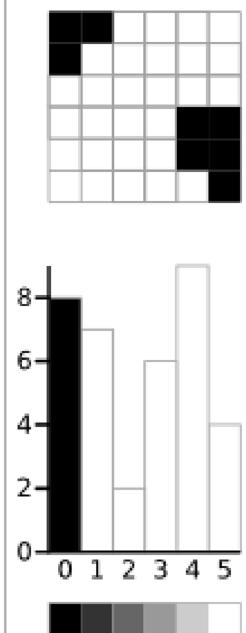
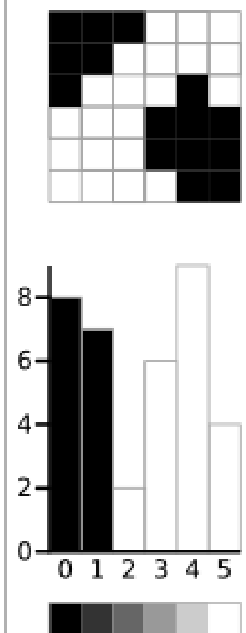
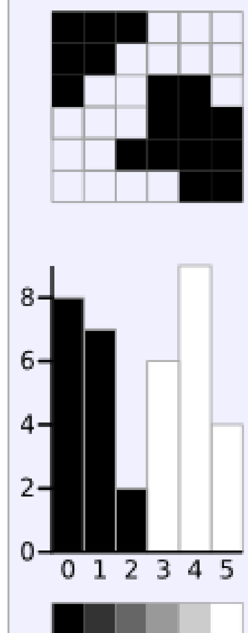
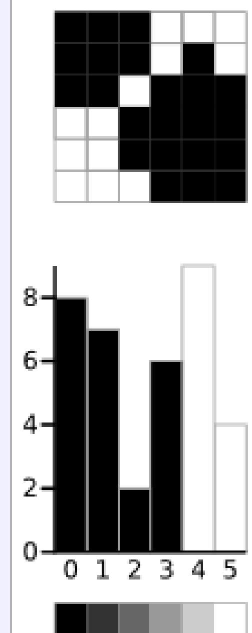
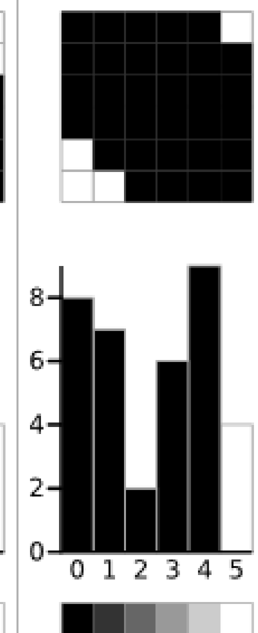
$$\text{Mean} \quad \mu_f = \frac{(3\times 6)+(4\times 9)+(5\times 4)}{19} = 3.8947$$

$$\begin{aligned}\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2\times 6)+((4-3.8947)^2\times 9)+((5-3.8947)^2\times 4)}{19}\\ &= \frac{(4.8033\times 6)+(0.0997\times 9)+(4.8864\times 4)}{19}\\ &= 0.5152\end{aligned}$$

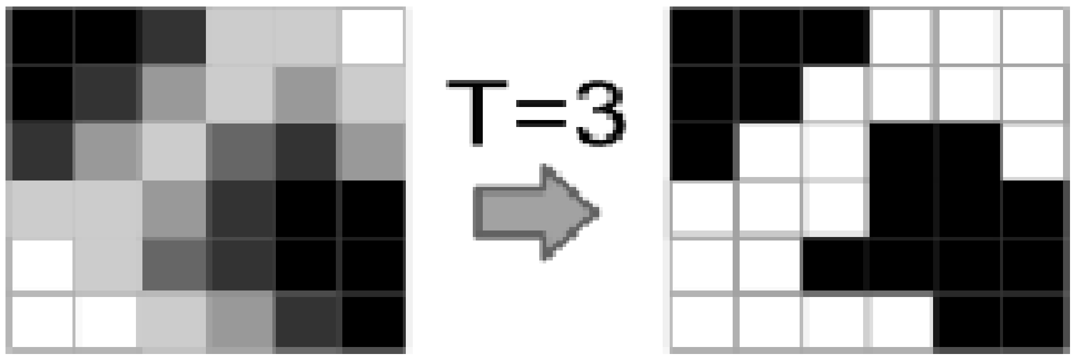
The next step is to calculate the 'Within-Class Variance'. This is simply the sum of the two variances multiplied by their associated weights.

$$\begin{aligned}\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 &= 0.4722 * 0.4637 + 0.5278 * 0.5152\\ &= 0.4909\end{aligned}$$

This final value is the 'sum of weighted variances' for the threshold value 3. This same calculation needs to be performed for all the possible threshold values 0 to 5. The table below shows the results for these calculations. The highlighted column shows the values for the threshold calculated above

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
						
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\sigma^2_W = \mathbf{0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

It can be seen that for the threshold equal to 3, as well as being used for the example, also has the lowest sum of weighted variances. Therefore, this is the final selected threshold. All pixels with a level less than 3 are background, all those with a level equal to or greater than 3 are foreground. As the images in the table show, this threshold works well.



Result of Otsu's Method

This approach for calculating Otsu's threshold is useful for explaining the theory, but it is computationally intensive, especially if you have a full 8-bit greyscale.

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Within Class Variance	σ^2_W = 3.1196	σ^2_W = 1.5268	σ^2_W = 0.5561	**σ^2_W = 0.4909**	σ^2_W = 0.9779	σ^2_W = 2.2491
Between Class Variance	σ^2_B = 0	σ^2_B = 1.5928	σ^2_B = 2.5635	**σ^2_B = 2.6287**	σ^2_B = 2.1417	σ^2_B = 0.8705