Deterministic Optimization

Illustration of the optimization process

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A portfolio optimization problem

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Learning objectives:

- Identify basic portfolio optimization and associated issues
- Examine the Markowitz Portfolio Optimization approach

The problem

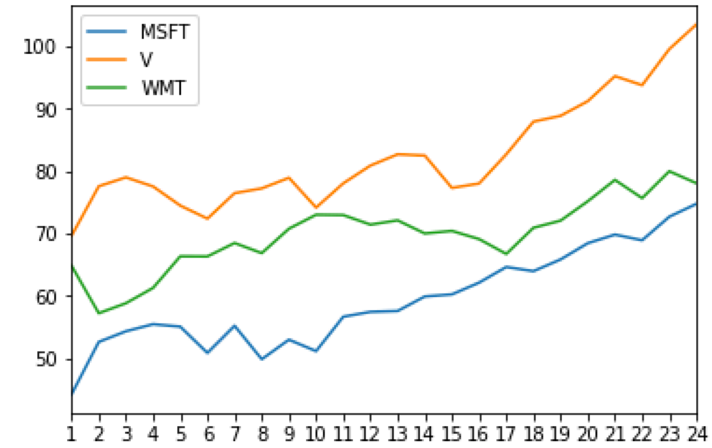
- Want to invest $1000 dollars in 3 stocks (MSFT= Microsoft, V = Visa, WMT = Walmart) for a one month period
- How should we distribute the budget among the 3 stocks?

- Available data:
  - File: monthly_prices.csv
  - Closing price of each stock on the first day of each month for the last 24 months
  - The last row indicates the current price (i.e. buy price)

	MSFT	V	WMT
1	44.259998	69.660004	64.839996
2	52.639999	77.580002	57.240002
3	54.349998	79.010002	58.84
4	55.48	77.550003	61.299999
5	55.09	74.489998	66.360001
6	50.880001	72.389999	66.339996
7	55.23	76.480003	68.489998
8	49.869999	77.239998	66.870003
9	53	78.940002	70.779999
10	51.169998	74.169998	73.019997
11	56.68	78.050003	72.970001
12	57.459999	80.900002	71.440002
13	57.599998	82.699997	72.120003
14	59.919998	82.510002	70.019997
15	60.259998	77.32	70.43
16	62.139999	78.019997	69.120003
17	64.650002	82.709999	66.739998
18	63.98	87.940002	70.93
19	65.860001	88.870003	72.080002
20	68.459999	91.220001	75.18
21	69.839996	95.230003	78.599998
22	68.93	93.779999	75.68
23	72.699997	99.559998	79.989998
24	74.769997	103.519997	78.07

Price and Return

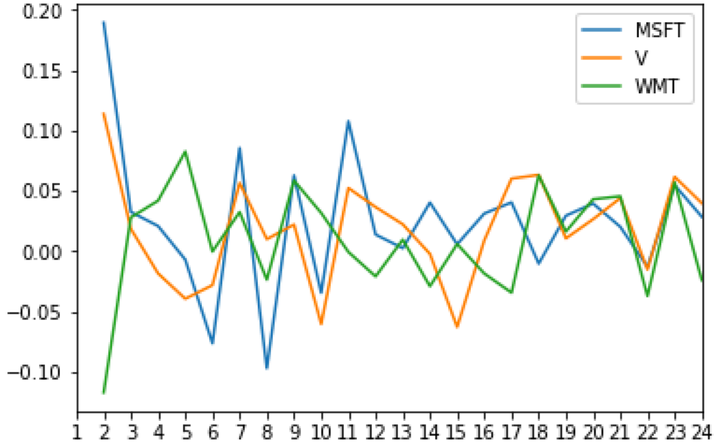
- Stock prices are uncertain and so is their return



- Monthly return:

$$r_{jt} = \frac{p_{jt} - p_{jt-1}}{p_{jt-1}}$$

where
r_{jt} = return of stock j in month t
p_{jt} = price of stock j in month t



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Portfolio

- Model the uncertain stock returns as a random vector, i.e. a vector whose each component is a random variable

$$\tilde{\mathbf{r}} = \begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2 \\ \tilde{r}_3 \end{bmatrix} \text{ where } \tilde{r}_j \text{ is the random return of stock } j.$$

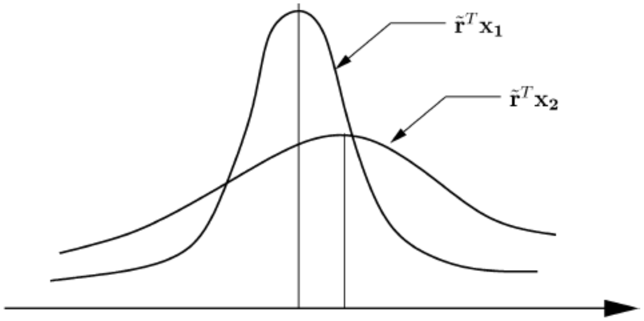
$$\text{Here } j = 1 : \text{ MSFT}, \; j = 2\text{: V, and } j = 3 : \text{ WMT}$$

- The portfolio is given by the vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ where } x_j \text{ is the \$ invested in stock } j$$

Random returns

- The return of the portfolio $\tilde{\mathbf{r}}^\top \mathbf{x}$ is a random variable
- How can we compare random variables (corresponding to different portfolios) to select a "best" portfolio?



- Markowitz (1952): Select a portfolio that to **maximizes expected return** and **minimizes the variance (or standard deviation) of returns** (a measure of"risk")

Markowitz Portfolio Optimization

- The Markowitz portfolio optimization criteria is a multi-objective problem
- Approaches: $\lambda \cdot$
  - Maximize Expected Return - Risk $\leq s^{\max}$
  - Maximize Expected Return s.t. Risk $\geq r^{\min}$
  - Minimize Risk s.t. Expected Return
- By changing the parameters we can find a set of optimal portfolios that tradeoff Expected return and Risk
- From the remainder of this example, we will consider the third approach

Optimization Problem Statement

- Given $1000 dollars, how much should we invest in each of the three stocks MSFT, V and WMT so as to
  - have a one month expected return of at least a given threshold
  - Minimize the risk (variance) of the portfolio return

- Decision: Investment in each stock
- Alternatives: Any investment that meets the budget and the minimum expected return requirement
- Best: Minimize variance

Summary

- Portfolio: Investments in a set of instruments (stocks)
- Return of a portfolio is uncertain
- Markowitz approach: Select a portfolio that trades off expected return and variance of returns
- We will consider

Minimize Risk
s.t. Expected Return $\geq r^{\min}$