Tutorial Sheet 1: COL726

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1. **Discretization**

   (a) TODO: Put in proof
   Code for this problem is in the *Codes* folder.
   Result comparison:

   Table 1: Variation of h with result

Value of h	Result
0.1	1.99662925254
0.01	1.99994799207
0.001	1.99999975037

   (b) i. Solution of Differential Equation

   $$\begin{aligned} y(x) &= e^{x^2} + x \\ \implies LHS &= y'(x) = 2xe^{x^2} + 1 \\ \implies RHS &= 2xy(x) - 2x^2 + 1 \\ &= 2xe^{x^2} + 2x^2 - 2x^2 + 1 \\ &= 2xe^{x^2} + 1 \\ \implies LHS &= RHS \end{aligned}$$

   ii. Divided difference
   TODO: Put in proof

   iii. Graph and analysis
   TODO: Write code and plot graphs

   (c) i. Deriving square root

$$
\begin{aligned}
y(x) &= x^2 - c \\
\implies y'(x) &= 2x \\
\implies x_{k+1} &= x_k - \frac{f(x)}{f'(x)} \\
&= x_k - \frac{{x_k}^2 - c}{2x_k} \\
c = 2 \implies x_{k+1} &= \frac{x_k}{2} + \frac{1}{x_k}
\end{aligned}
$$

ii. Proof for digits of accuracy
TODO: Write proof

iii. Table of convergence rate
Code can be found in the *Codes* folder.

Table 2: Variation of digits of accuracy with iterations

Number of Iterations	**Value Guessed**	**Digits of Accuracy**
0	1	0
1	1.5	1
2	1.416666666666666666666666666	2
3	1.414215686274509803921568628	5
4	1.414213562374689910626295579	10
5	1.414213562373095048801689624	23
6	1.414213562373095048801688724	27+

(d) i. Truncation Error
ii. Rounding Error
iii. Truncation Error
iv. Truncation Error

2. **Unstable and Ill-conditioned problems**

(a) i.

$$
\begin{aligned}
y'(x) &= (2/\pi)xy(y-\pi) \\
y(0) &= y_0 \\
\implies (\frac{1}{y-\pi} - \frac{1}{y})dy &= 2xdx \\
\implies \log\frac{y-\pi}{y} &= x^2 + c \\
\implies \frac{y-\pi}{y} &= \frac{y_0-\pi}{y_0} * e^{x^2} \\
\implies y &= \frac{\pi * y_0}{y_0 + (\pi - y_0) * e^{x^2}}
\end{aligned}
$$

ii. TODO: Clear confusion

(b) For the given equations:

$$
\begin{aligned}
x &= 2y + 0.5 \\
cx &= ay - 2
\end{aligned}
$$

The solution is: $x = \frac{8-a}{4c-2a}; y = \frac{4-c}{4c-2a}$.
This problem is not stable.

Table 3: Variation of solution

	x	y
c=2.998 , a=6.001	-199.9	-100.2
c=2.998, a=6	-250	-125.25

(c) The roots of the equation can be seen as follows:
When we vary the value of c it can be seen that the roots first converge to 1

Table 4: Variation of roots

Value of c vs Roots	x_1	x_2	x_3
203	99.9796	1.1527	0.8677
202	99.9898	1.1057	0.9045
201	100	1	1
200	100.01	0.99+0.1i	0.99-0.1i
199	100.02	0.99+0.14i	0.99-0.14i

and then turn complex.

3. **Unstable methods**

(a) The number of digits of accuracy in x will then be 2 as the value of $\sqrt{b^2 - 4ac}$ correctly to 8 digits is . The source for this error is that the value of b and $\sqrt{b^2 - 4ac}$ are very close to each other hence we lose precision while subtracting.

(b) Roots of a quadratic equation are given by

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} * \frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} \\ &= \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \end{aligned}$$

The program for this can be found in the *Codes* folder. The results of this are as follows:

Table 5: Comparison of both methods

	$10^1 < a, c < 10^2; 10^3 < b < 10^6$	$10^1 < a, c < 10^2; 10^6 < b < 10^8$
Mean of difference in roots	0.00049	3573.818
Maximum difference in roots	0.0033	33072.863

(c) The graph and code can be found in the *codes* folder.
There is a large deviation between the real and value computed in this case because the value of error is amplifying with every iteration. The error is increasing by a factor of k while computing the I_k .

(d) i. When the x_i are close to each other, the valye of x_i and $\bar{x}$ is close to each other. On squaring these 2 individually, because of limited digits of accuracy, the squared values will be close to each other. Hence the value of the result will be 0 or something very small.

ii. Refer to *Codes* folder.

(e) TODO: Add reasons

i. No, $sum1$ is not equal to the true value.

ii. No $sum2$ and $sum3$ are not equal to the true values.

iii. No. $sum4$ and $sum5$ are not equal to their true values.