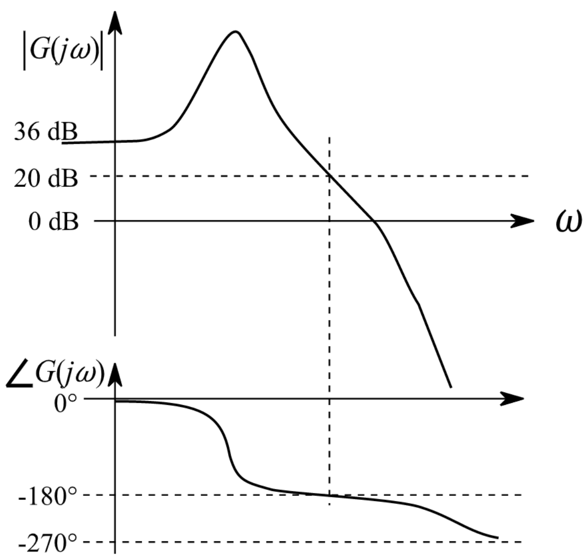
Control Systems

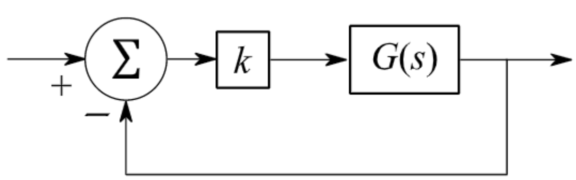
CONTENTS

I. GAIN MARGIN

1. The figure below shows the Bode magnitude and phase plots of a stable transfer function:

$$G(s) = \frac{n_0}{s^3 + d_2 s^2 + d_1 s + d} \quad (1)$$





Consider the negative unity feedback configuration with gain k in the feed forward path. The closed loop is stable for . The maximum value of k_o is:

Solution:

- For a stable system, Gain margin at the phase cross-over frequency $>$ 0dB.
- The phase crossover frequency is the frequency at which the phase angle first reaches -180°.
- The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable.
- Gain margin is the factor by which the gain must be multiplied at the phase crossover to have the value 1.
- The phase crossover frequency is the frequency at which the phase angle first reaches -180°and thus is the point where the Nyquist plot crosses the real axis.

The gain margin is defined as

$$K_g = \frac{1}{|G(j\omega)|} \quad (2)$$

at the frequency at which phase angle is -180°.

In terms of decibels:

$$K_g dB = -20log(|G(j\omega)|)dB \quad (3)$$

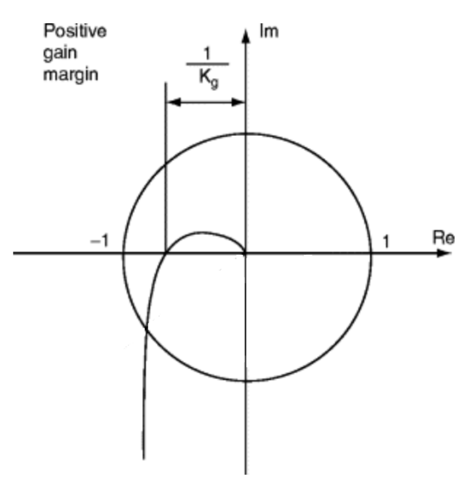


Fig: nyquist plot of stable transfer function

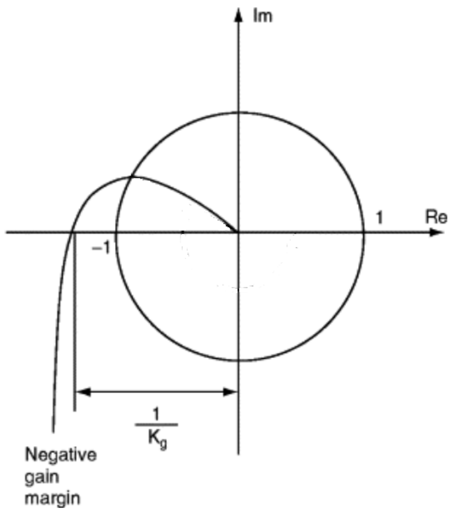


Fig: nyquist plot of unstable transfer function

For a stable system, Gain margin at the phase cross-over frequency >1.

G(s) is cascaded with k, so,

$$G_1(s) = kG(s) \tag{4}$$

$$K_g = \frac{1}{|G_1(j\omega_{pc})|} > 1 \tag{5}$$

$$K_{g(dB)} = -20log(|G_1(j\omega_{pc})|) > 0dB \tag{6}$$

$$-20log(|G(j\omega_{pc})k|) > 0dB \tag{7}$$

$$-20 - 20log(|k|) > 0dB \tag{8}$$

$$20log(k) < -20 \tag{9}$$

$$k < 10^{-1} \tag{10}$$

$$k_{max} = 0.1 \tag{11}$$

$\therefore$ k_o = 0.1