Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

0.0.1. Using Nyquist criterion, find out whether the following is stable or not.

$$G(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \quad (0.0.1.1)$$

$$H(s) = 1 \quad (0.0.1.2)$$

Solution: Open loop transfer function (oltf):

$$G(s)H(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \quad (0.0.1.3)$$

Closed loop transfer function (cltf):

$$\frac{G(s)}{1+G(s)H(s)} \quad (0.0.1.4)$$

Nyquist Stability Criterion can be expressed as:

$$Z = N + P \quad (0.0.1.5)$$

where:

- Z = zeros of $1 + G(s)H(s)$ in RHS of s-plane
- N = number of encirclement of critical point 1+0j in the clockwise direction.
- P = poles of G(s)H(s) in RHS of s-plane.

The pole-zero plot of equation (0.0.1.3) is fig. (0.0.1.1) which gives $\mathbf{P = 0}$.

- Since the multiplicity of zero pole is 1 (fig.0.0.1.1), it should be assumed that the phasor travels one time clockwise along a semicircle of infinite radius.
- Same applies for poles at 2j and -2j.

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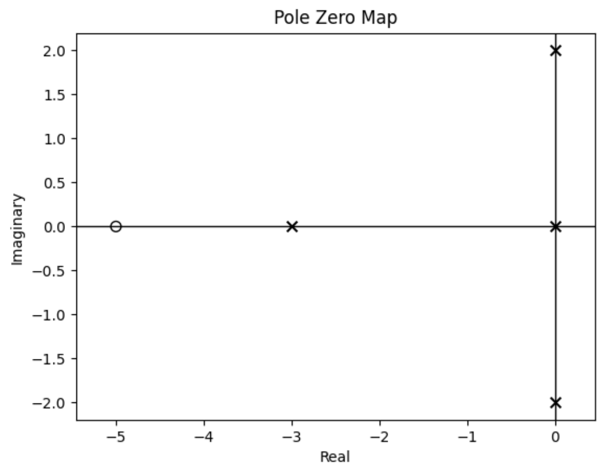

Fig. 0.0.1.1

- Fig. (0.0.1.2) shows a schematic, the dotted lines are infinite radii semi-circles.

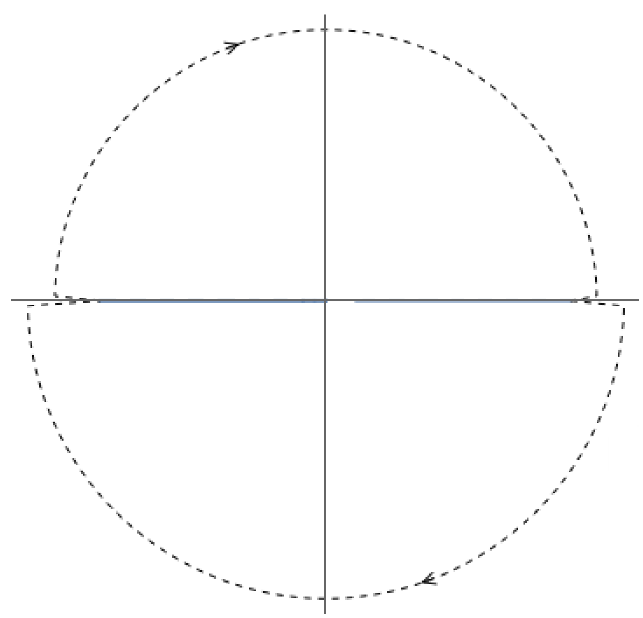

Fig. 0.0.1.2

- The point -1+0j is not encircled by the nyquist plot (fig. 0.0.1.4).

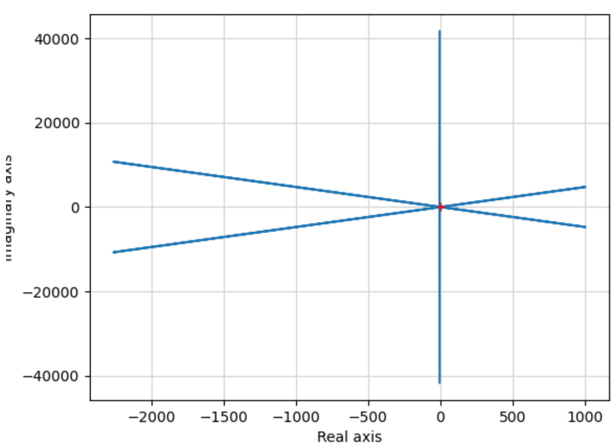

Fig. 0.0.1.3: Nyquist plot of G(s)H(s)

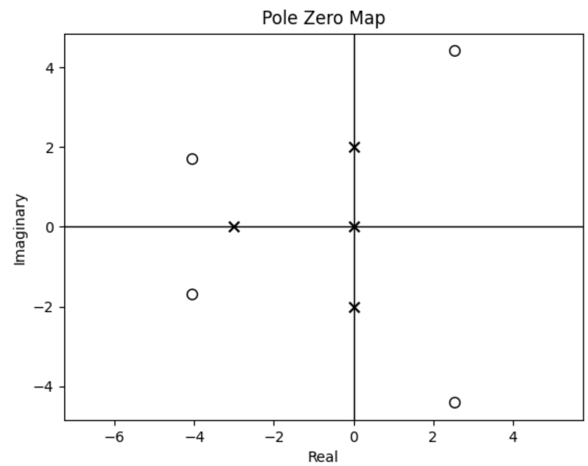

Fig. 0.0.1.5

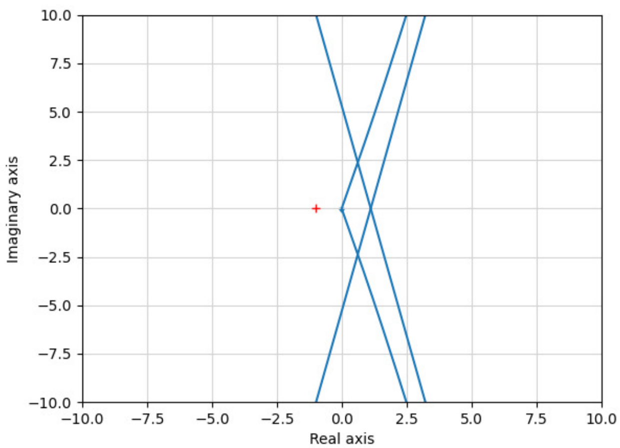

Fig. 0.0.1.4: Zoomed in

From the nyquist plot (fig. 0.0.1.4), -1+0j is not encircled by the plot. So from above points, the only clockwise encirclement is considered due to the mentioned poles (zero, 2j and -2j) with multiplicity of 1.
Therefore, **N=2**
Substituting values of P = 0 and N = 2 in equation (0.0.1.5):

$$\implies Z = 2 \tag{0.0.1.6}$$

This is verified using pole zero plot of 1+G(s)H(s) (fig. 0.0.1.5). Two zeroes on RHS of s-plane i.e. Z=2.
Since $Z \neq 0$, **the closed loop system is unstable**.
The **open loop system is stable** as there are **no poles on RHS** of s-plane.
The following code plots the pole zero plot and the nyquist plot.

```
codes/ee18btech11025.py
```