Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

0.0.1. Using Nyquist criterion, find out whether the following is stable or not.

$$G(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \tag{0.0.1.1}$$

$$H(s) = 1 \tag{0.0.1.2}$$

Solution: Open loop transfer function (oltf):

$$G(s)H(s) = \frac{100(s+5)}{s(s^2+4)(s+3)} \tag{0.0.1.3}$$

Closed loop transfer function (cltf):

$$\frac{G(s)}{1+G(s)H(s)} \tag{0.0.1.4}$$

Nyquist Stability Criterion can be expressed as:

$$Z = N + P \tag{0.0.1.5}$$

where:

- Z = number of roots of 1+G(s)H(s) in right-hand side (RHS) of s-plane (It is also called zeros of characteristics equation).
- N = number of encirclement of critical point 1+j0 in the clockwise direction.
- P = number of poles of open loop transfer function (OLTF) [i.e. G(s)H(s)] in RHS of s-plane.

The above condition (i.e. Z=N+P) is valid for all the systems whether stable or unstable.

- The system is stable iff Z = 0.

The pole-zero plot of equation 0.0.1.3 is:

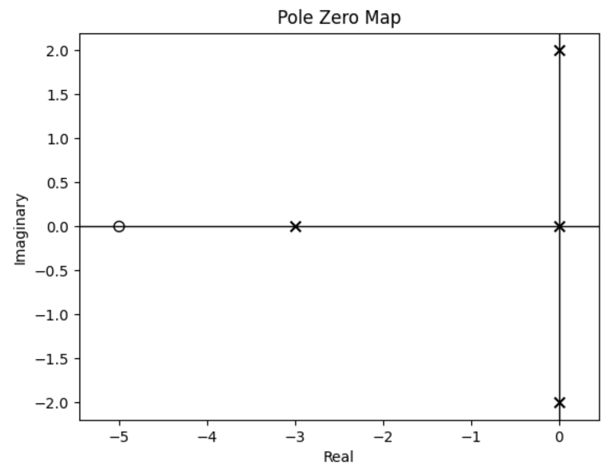

Fig. 0.0.1.1

which gives P = 0.
Since there are poles on the imaginary axis of oltf 0.0.1.3, the Nyquist contour will be (which encloses the right half s-plane) :
The outer semi-circle C is infinitely large and

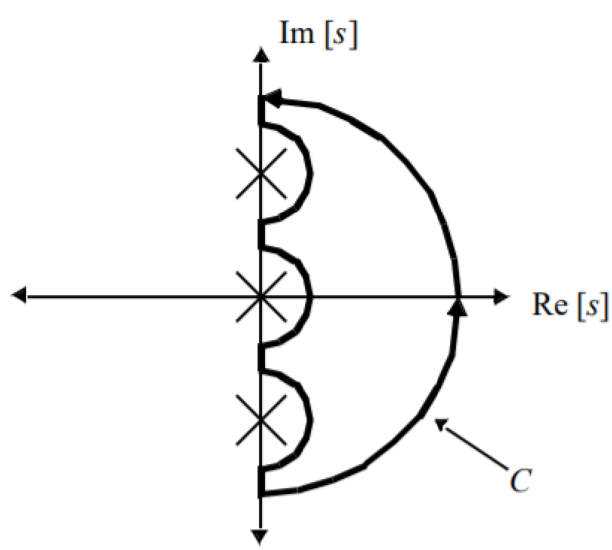

Fig. 0.0.1.2

smaller semicircle's radius goes to almost zero.

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Plot of the Nyquist plot for equation 0.0.1.3 is:

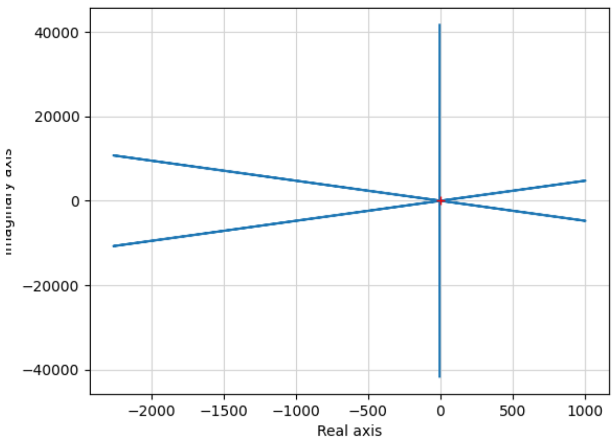

Fig. 0.0.1.3

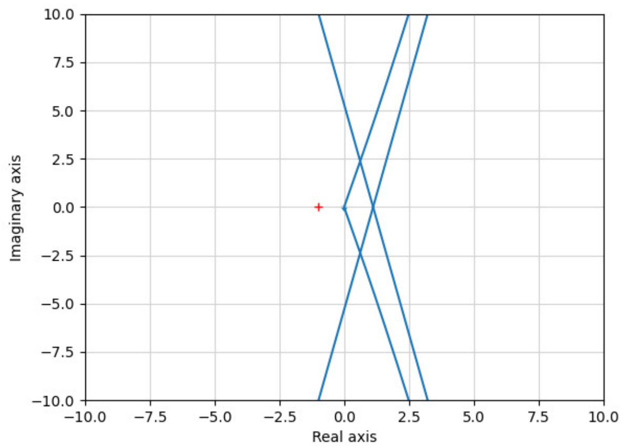

Fig. 0.0.1.4: Zoomed in

- The point -1+0j is encircled twice by the nyquist plot, once due to the semicircle at origin in fig: 0.0.1.2 (which translates to infinitely big semicircle in Nyquist plot).
- Then due to poles on imaginary axis (which also translate to infinitely big semicircle).

which gives us N = 2.
Substituting values of P = 0 and N = 2 in equation 0.0.1.5:

$$\implies Z = 2 \tag{0.0.1.6}$$

This is verified using pole zero plot of 1+G(s)H(s):

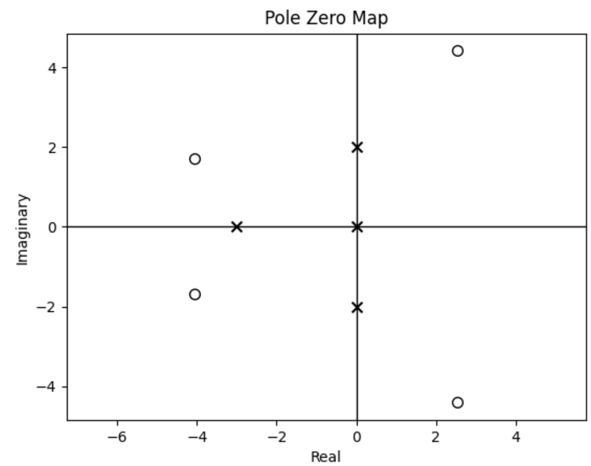

Fig. 0.0.1.5

Two zeroes on RHP of s-plane i.e. Z=2. Since $Z \neq 0$, the system is unstable.