EE1390

Matrix Project

EE18BTECH11016 and EE18BTECH11025

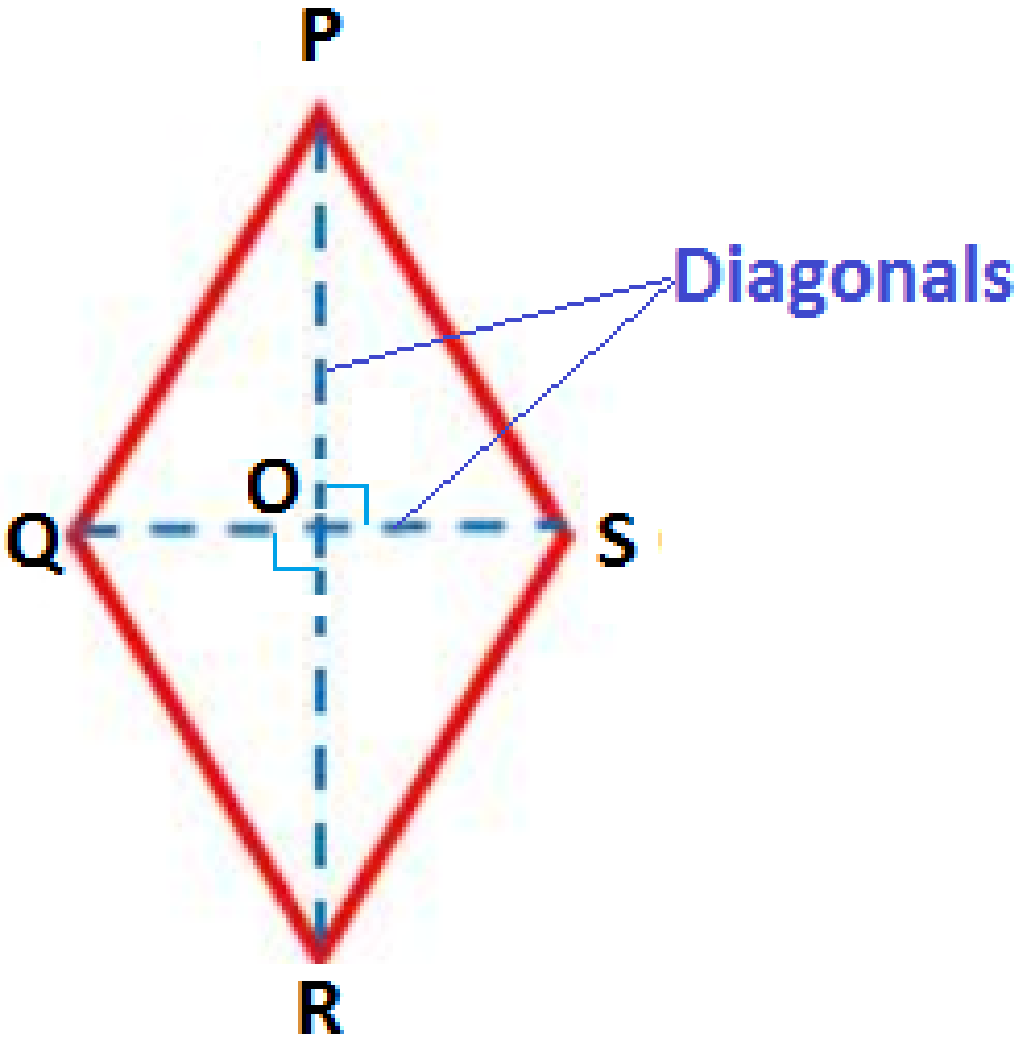
Question

Two sides of a rhombus are along the lines

$$(7\ \text{-}1)\mathbf{x} - 5 = 0$$
$$(1\ \text{-}1)\mathbf{x} + 1 = 0$$

If its diagonals intersect at $\begin{pmatrix}-1\\-2\end{pmatrix}$, find its vertices.

[JEE(main) 2016, Q.no. 13 (code G)]



$PR \perp QS$

Solution

Given the equations of two lines PQ and PS are :

$$(7\ \text{-}1)\mathbf{x} - 5 = 0$$
$$(1\ \text{-}1)\mathbf{x} + 1 = 0$$

Solving these two equations , we get the point of intersection as P.

$$\begin{pmatrix} 7 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1/6 & -1/6 \\ 1/6 & -7/6 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

So coordinates of P in matrix form are $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Now to find coordinates of point R in matrix form, we need to use the mid-point formula.
Hence,

$$M = (P+R)/2$$

where M = point of intersection of diagonals

$$= \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$R = 2M - P$$

$$R = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

The direction vector of PR is $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ So, normal vector of PR is $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

The equation of QS is $(2\ 4)(\mathbf{x} - \begin{pmatrix} -1 \\ -2 \end{pmatrix}) = 0$

$$(2\ 4)\mathbf{x} + 10 = 0$$

We can get the points Q by finding the intersection of PQ and QS;

$$(7\ \text{-}1)\ \mathbf{x} - 5 = 0$$

$$(2\ 4)\ \mathbf{x} + 10 = 0$$

$$\begin{pmatrix} 7 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ -10 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 4/30 & 1/30 \\ -2/30 & 7/30 \end{pmatrix} \begin{pmatrix} 5 \\ -10 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1/3 \\ -8/3 \end{pmatrix}$$

Similarly for getting S, we find intersection of PS and QS

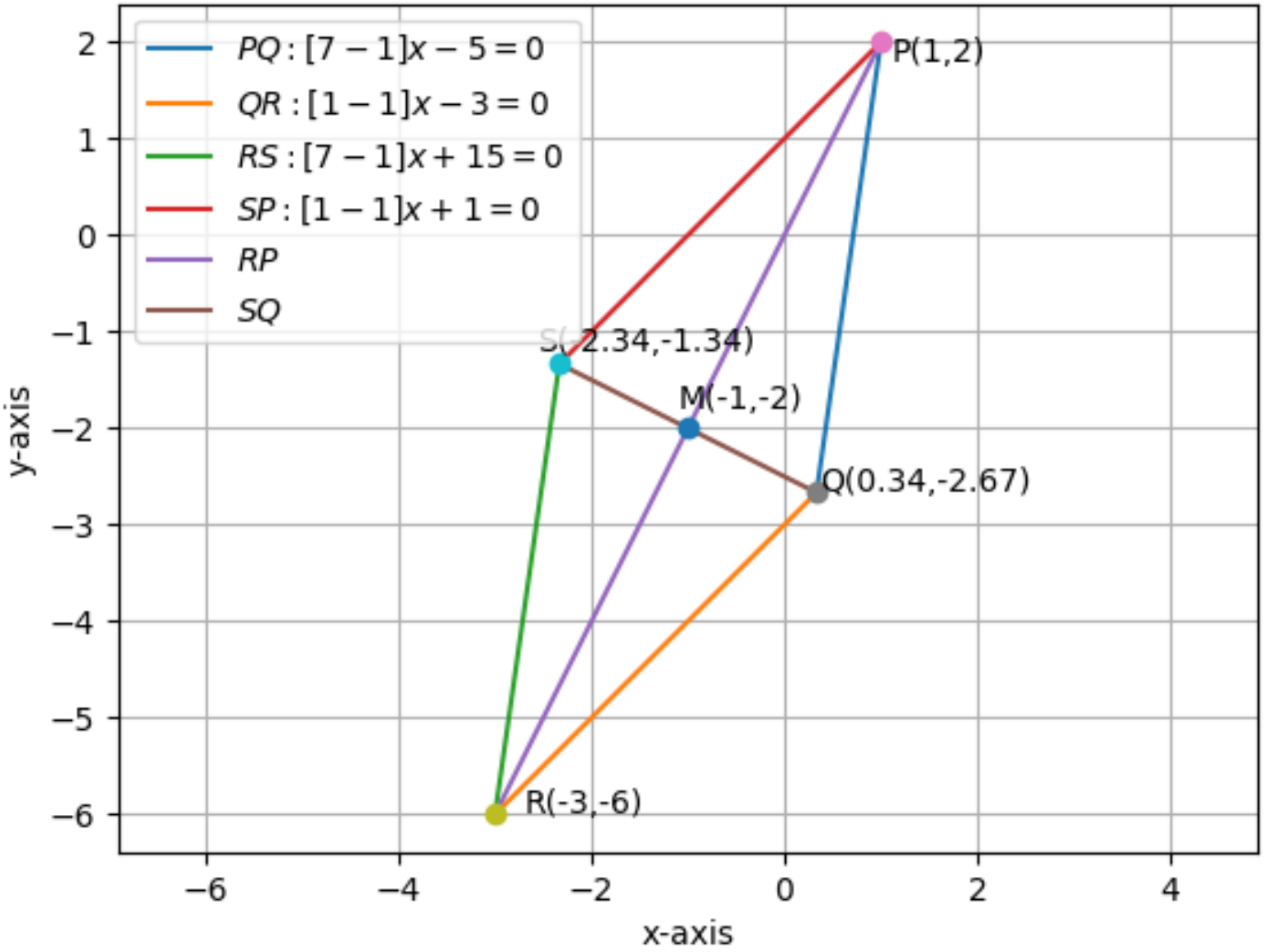
$$(1\ \text{-}1)\ \mathbf{x} + 1 = 0$$

$$(2\ 4)\ \mathbf{x} + 10 = 0$$

$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ -10 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 4/6 & 1/6 \\ -2/6 & 1/6 \end{pmatrix} \begin{pmatrix} -1 \\ -10 \end{pmatrix}$$

$$S = \begin{pmatrix} -7/3 \\ -4/3 \end{pmatrix}$$


$PQ : [7-1]x-5=0$
$QR : [1-1]x-3=0$
$RS : [7-1]x+15=0$
$SP : [1-1]x+1=0$
RP
SQ
P(1,2)
S(-2.34,-1.34)
M(-1,-2)
Q(0.34,-2.67)
R(-3,-6)
y-axis
x-axis
2
1
0
−1
−2
−3
−4
−5
−6
−6
−4
−2
0
2
4