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Traffic Assignment and Network Design

IOS 2024

Kartikey Sharma

Joint work with Mathieu Besancon, Deborah Hendrych and Sebastian Pokutta

Overview

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Aim: Solve the Network Design problem for the Traffic Assignment Model

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- Introduction

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- Model and Algorithms

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- Model and Algorithms
- Numerical Experiments

Motivation

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- Boscia.jl library leverages Frank-Wolfe to solve discrete optimization problems with convex objectives using MIPs to compute the descent direction

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- Boscia.jl library leverages Frank-Wolfe to solve discrete optimization problems with convex objectives using MIPs to compute the descent direction
- Network Design adds a binary problem on top of a network
- For a fixed network, the underlying model can be solved efficiently
- We leverage this special structure present for efficient computation of these oracles

Example: Network Design

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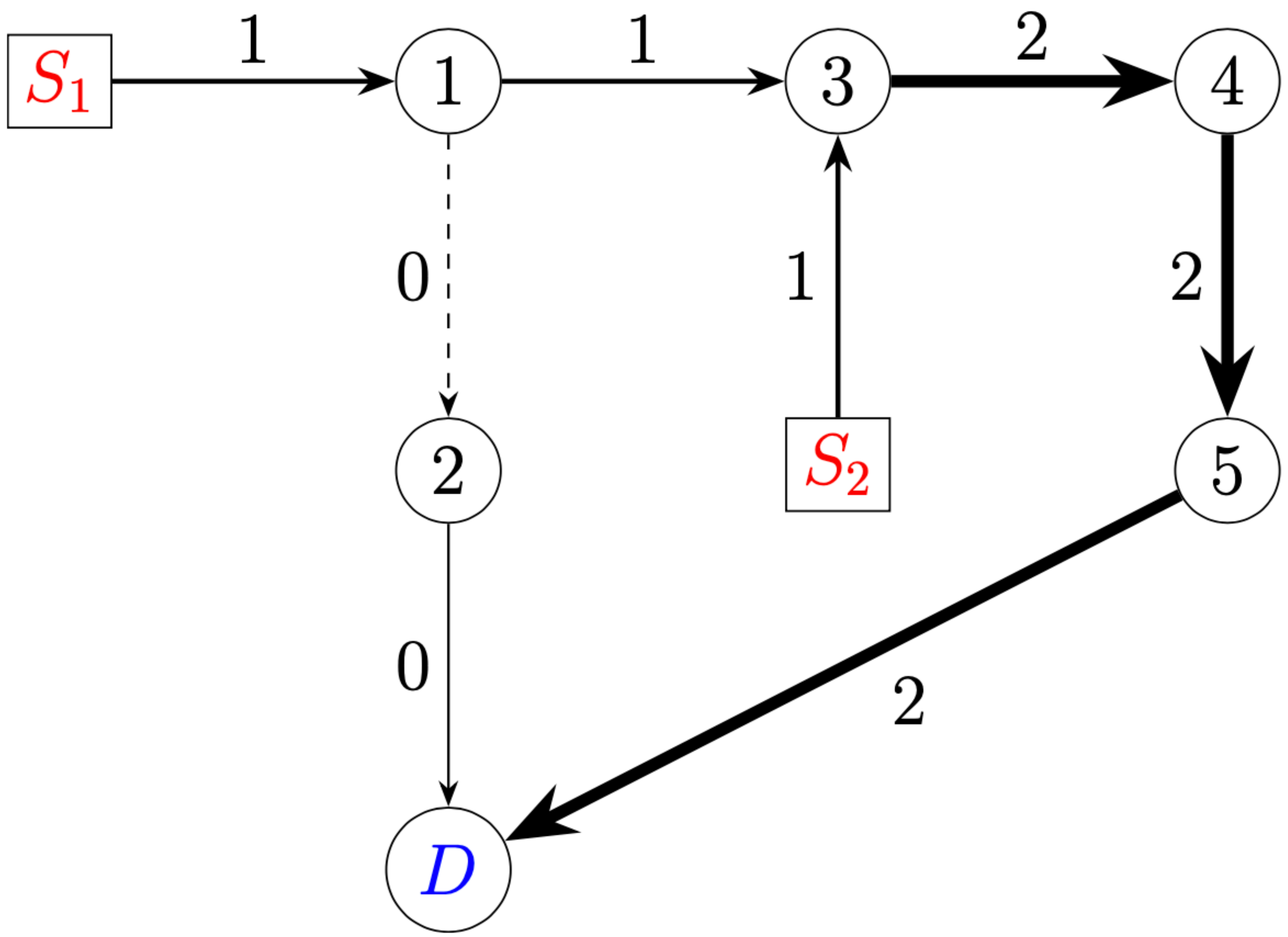

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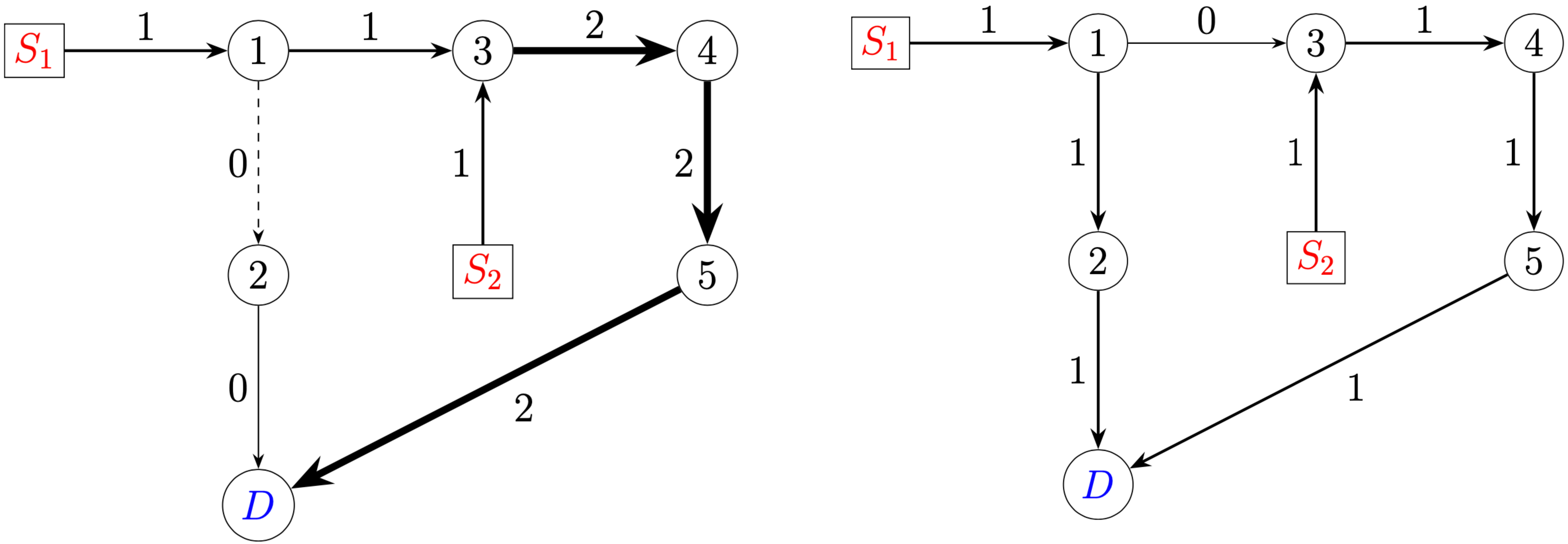

Traffic Assignment Model

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Traffic Assignment Model

- Traffic Assignment models the equilibrium traffic flow between various origins and destinations

$$G = (V, E)$$

$$d_i^t : \text{flow from origin } i \text{ to destination } t$$

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- Equivalent to a network flow problem with convex costs

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$$\begin{aligned} \min_x \; & c(x) \\ \text{s.t. } & x_{ij} = \sum_{d \in \mathcal{D}} x_{ij}^t \\ & \sum_{(i,j) \in \mathcal{E}} x_{ij}^t - \sum_{(j,i) \in \mathcal{E}} x_{ji}^t = D_i^t \\ & x_{ij}^t \geq 0 \end{aligned}$$

$$D_i = \begin{cases} 0 & \text{no net flow at } i \text{ for } t \\ d_i^t & \text{supply from } i \text{ to } t \\ -\sum_{k \in \mathcal{O}} d_k^t & \text{total supply for } t \text{ at } i = t \end{cases}$$

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- Solved using the Frank Wolfe algorithm.
- Linear oracle solved using a sequence of shortest path problems

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Traffic Assignment and Network Design

Traffic Assignment and Network Design

- Implements the Network Design on top of the traffic assignment model

$$
\begin{aligned}
\min_x \; & c(x) + r^\top y \\
\text{s.t. } & x \in \mathcal{X} \\
& x_j \leq M y_j \; \forall j \in J \\
& y_j \in \{0, 1\}
\end{aligned}
$$

Traffic Assignment and Network Design

- Implements the Network Design on top of the traffic assignment model
- Goal: find a new arc which reduces the overall traffic cost at equilibrium

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Traffic Assignment and Network Design

- Implements the Network Design on top of the traffic assignment model
- Goal: find a new arc which reduces the overall traffic cost at equilibrium
- Leads to a discrete optimization problem with convex costs
- Due to graphs and multiple sources and destinations, the problem size is large

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& y_j \in \{0, 1\}
\end{aligned}$$

Stochastic Network Design

$$
\begin{aligned}
\min_{x,y} \; & r^\top y + \sum_{s \in \mathcal{S}} p_s c(x_s) \\
\text{s.t. } & x_s \in \mathcal{X}_s \\
& x_{sj} \leq M y_j \quad \forall j \in J, s \in \mathcal{S} \\
& y_j \in \{0,1\}
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Stochastic Network Design

- Stochastic Network Design further increases size of problem.

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$$

Stochastic Network Design

- Stochastic Network Design further increases size of problem.
- Increased benefit of decomposition and parallelisation.

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\text{s.t. } & x_s \in \mathcal{X}_s \\
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Boscia.jl

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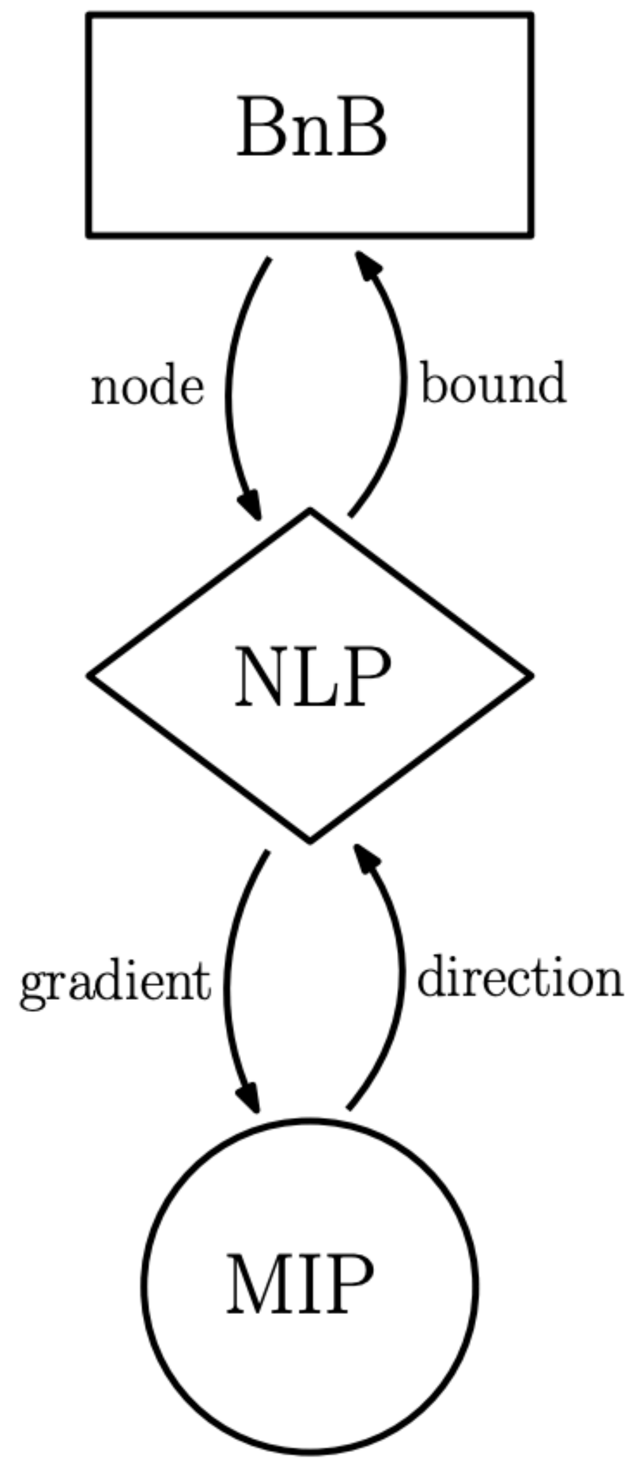
- Solves the MINLP using a B&B approach with an NLP being solved at every node



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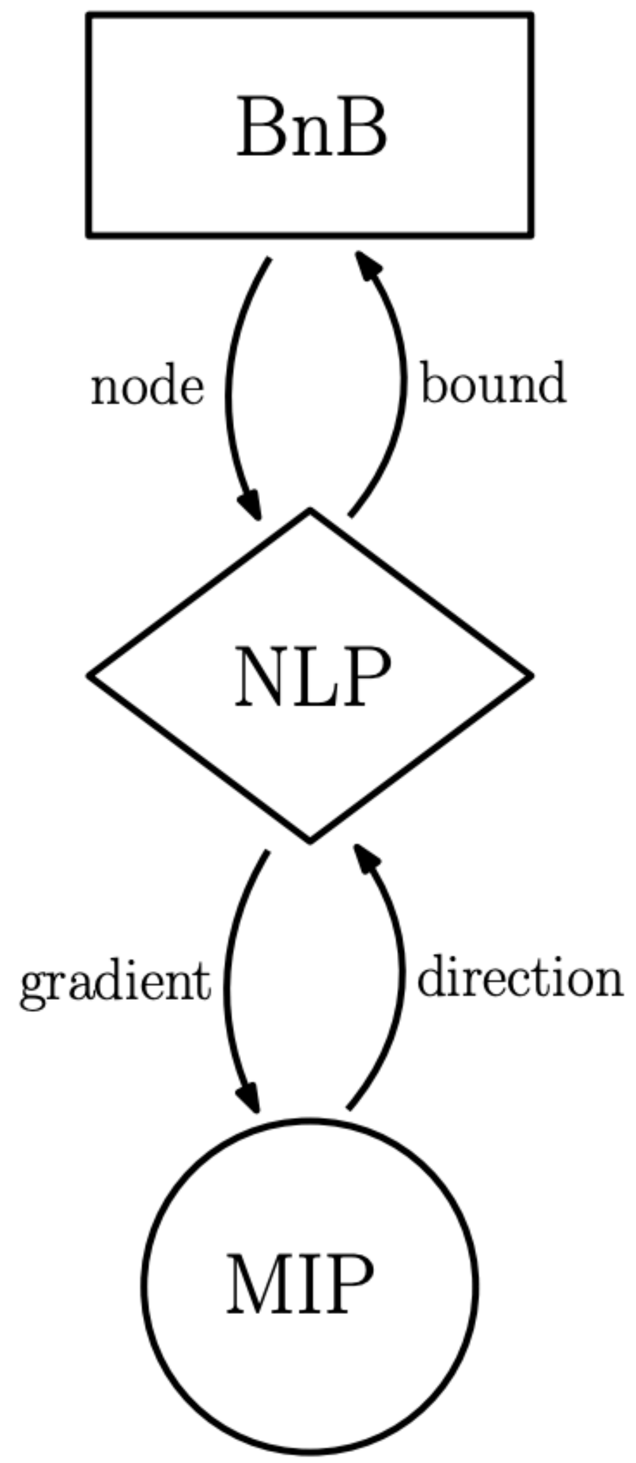
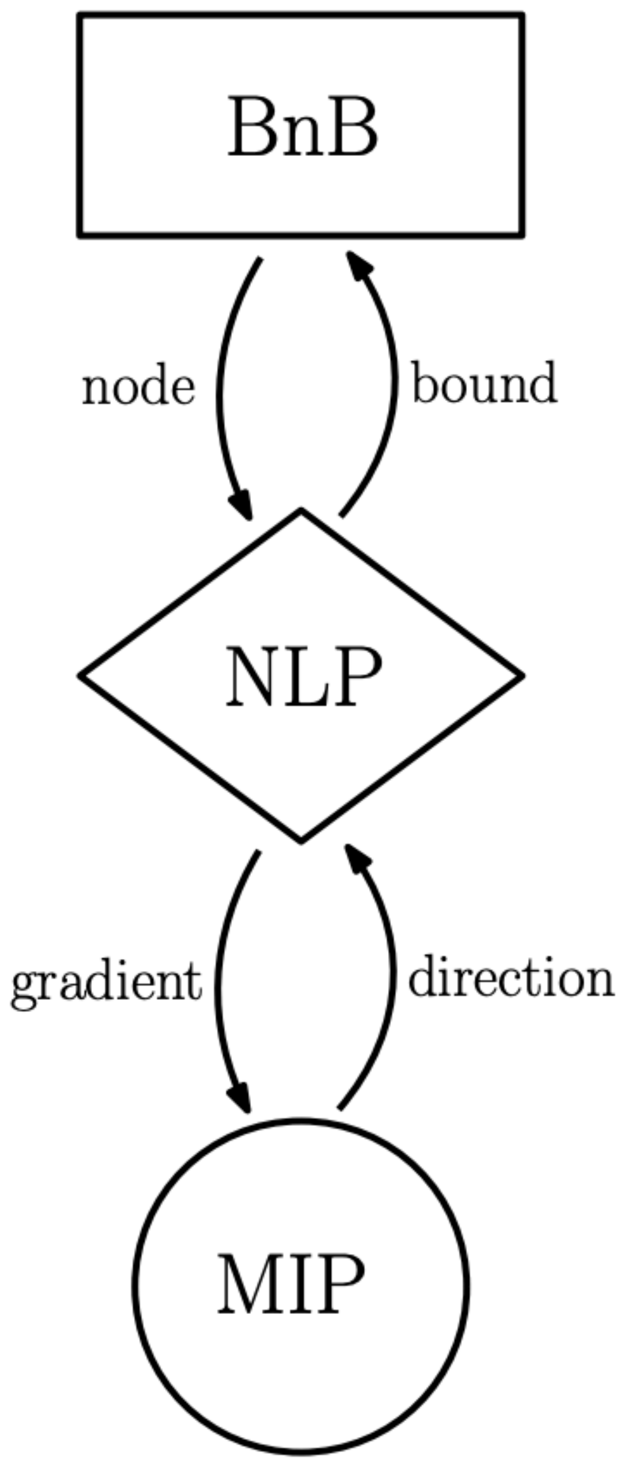


Boscia.jl

- Solves the MINLP using a B&B approach with an NLP being solved at every node
- The NLP is solved using the FW algorithm
- The FW algorithm requires a MIP oracle



Integer LMO : IFW

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- Solves an integer optimization problem to find the descent direction.

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\begin{aligned}
\min_{v} \ & \nabla_x c(x_t)^\top v^x + r^\top v^y \\
\text{s.t. } & v^x \in \mathcal{X} \\
& v^x_j \leq M v^y_j \ \forall j \in J \\
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Integer LMO : IFW

- Solves an integer optimization problem to find the descent direction.

- Solves the math programming model without leveraging the special structure

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Network LMO with Penalty : NLMO-P

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- x and y only connected by network design constraints.

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- Move the constraint into the objective and penalise its violation

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\begin{aligned}
\min_{x,y}\ & c(x) + r^\top y + \lambda \sum_{j \in J} \max(x_j - M y_j, 0)^p \\
\text{s.t. } & x \in \mathcal{X} \\
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Network LMO with Penalty : NLMO-P

- x and y only connected by network design constraints.
- Move the constraint into the objective and penalise its violation
- Allows the descent direction problem to be separated into network flow and a selection problems.

$$\min_{x,y}\ c(x) + r^\top y + \lambda \sum_{j \in J} \max(x_j - My_j, 0)^p$$

$$\text{s.t. } x \in \mathcal{X}$$

$$y_j \in \{0,1\}$$

$$\min_{x}\ (\nabla c(x_t) + \lambda g_t)^\top v^x \qquad \min_{y}\ (r + \lambda h_t)^\top v^y$$

$$\text{s.t. } v^x \in \mathcal{X} \qquad \text{s.t. } v^y_j \in \{0,1\}$$

$$g_{ti} = \begin{cases} 0 & i \notin J \\ 0 & i \in J \ \&\ x_{ti} \leq My_{ti} \\ p(x_{ti} - My_{ti})^{p-1} & i \in J \ \&\ x_{ti} > My_{ti} \end{cases}$$

$$h_{ki} = \begin{cases} 0 & i \in J \ \&\ x_{ti} \leq My_{ti} \\ -Mp(x_{ti} - My_{ti})^{p-1} & i \in J \ \&\ x_{ti} > My_{ti} \end{cases}$$

Network LMO with Penalty : NLMO-P

- x and y only connected by network design constraints.
- Move the constraint into the objective and penalise its violation
- Allows the descent direction problem to be separated into network flow and a selection problems.
- 2 parameters: λ (penalty) and p (power)

$$\min_{x,y} c(x) + r^\top y + \lambda \sum_{j \in J} \max(x_j - M y_j, 0)^p$$
$$\text{s.t. } x \in \mathcal{X}$$
$$y_j \in \{0,1\}$$

$$\min_x (\nabla c(x_t) + \lambda g_t)^\top v^x \qquad \min_y (r + \lambda h_t)^\top v^y$$
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- The subproblem at each step then simplifies to network flow problem

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\begin{aligned}
\min_{x,y} \ & r^\top v^y + \eta \\
\text{s.t. } & \eta \ge p_k^\top b - s_k^\top M v^y \quad \forall k \in OPT \\
& 0 \ge p_k^\top b - s_k^\top M v^y \quad \forall k \in FEAS \\
& v^y_j \in \{0,1\} \quad \forall j \in J
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Network LMO with Benders : NLMO-B

- We solve the descent problem using Benders's decomposition
- The subproblem at each step then simplifies to network flow problem
- Disadvantage: Requires a iterative algorithm for the descent problem which can be slow

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Methods Compared

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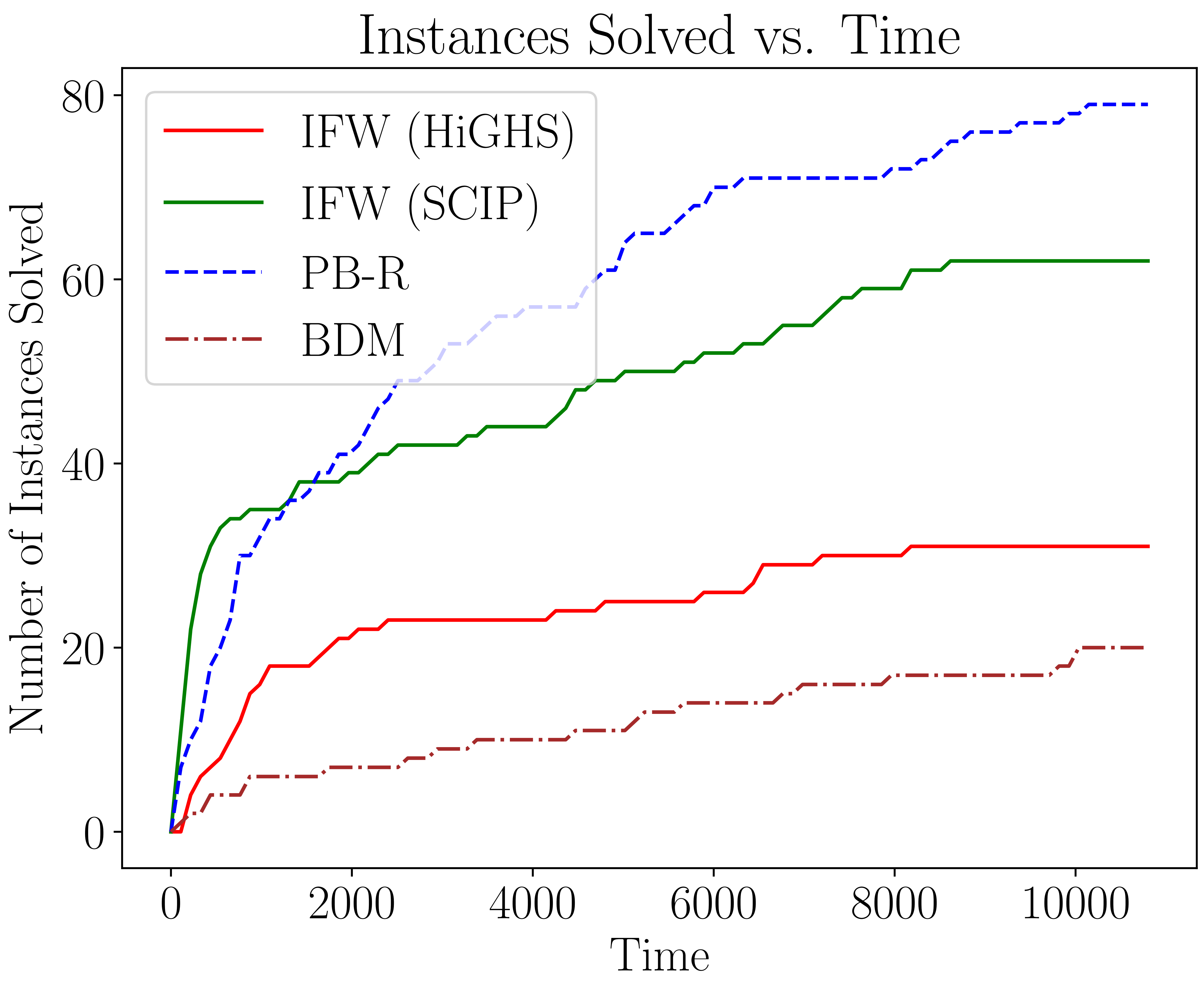
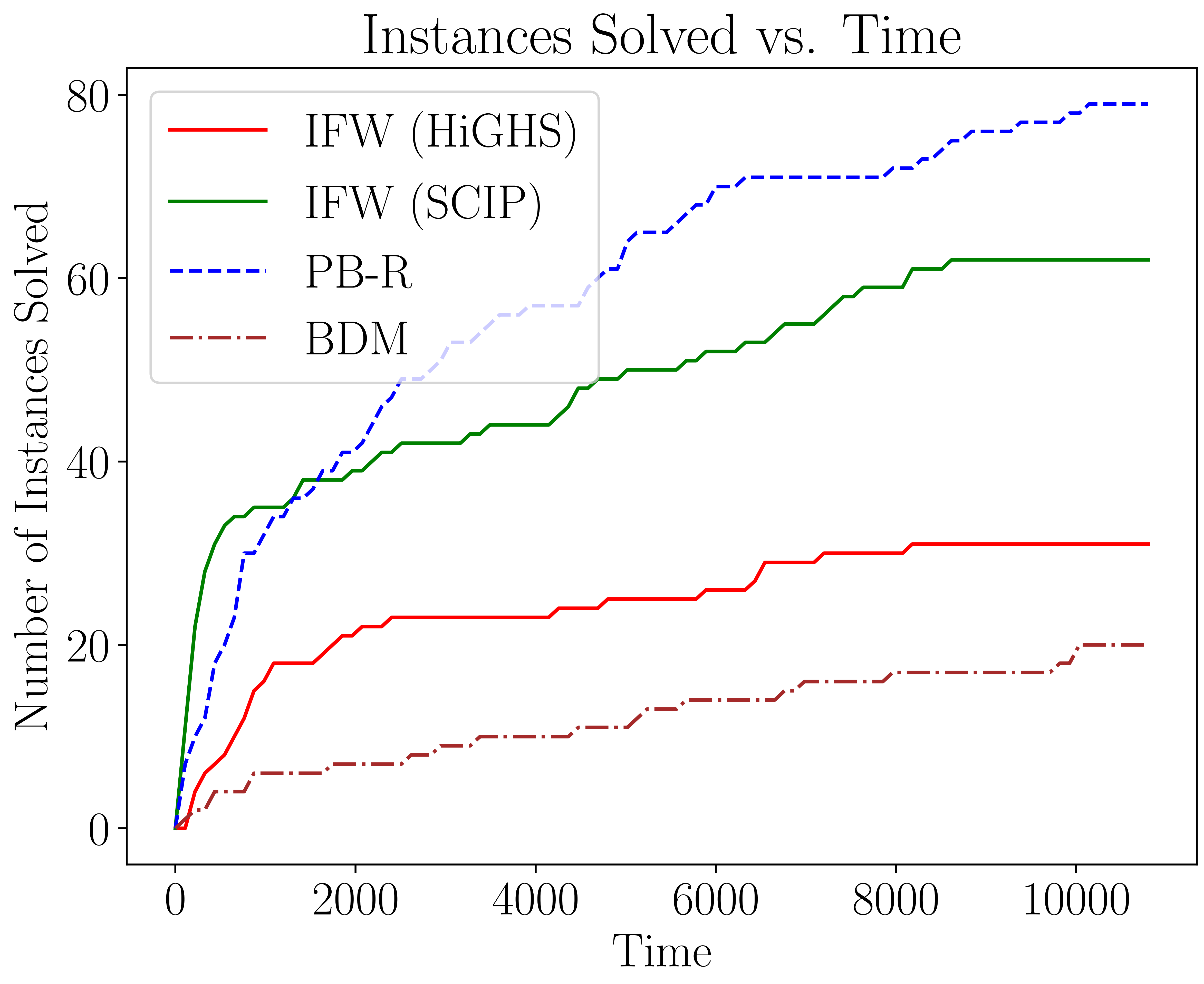
- IFW
- NLMO-P
- NLMO-B

Results



Instances Solved vs. Time
Number of Instances Solved
Time
IFW (HiGHS)
IFW (SCIP)
PB-R
BDM
0
20
40
60
80
0
2000
4000
6000
8000
10000

Results



The penalty-based method outperforms all other methods.

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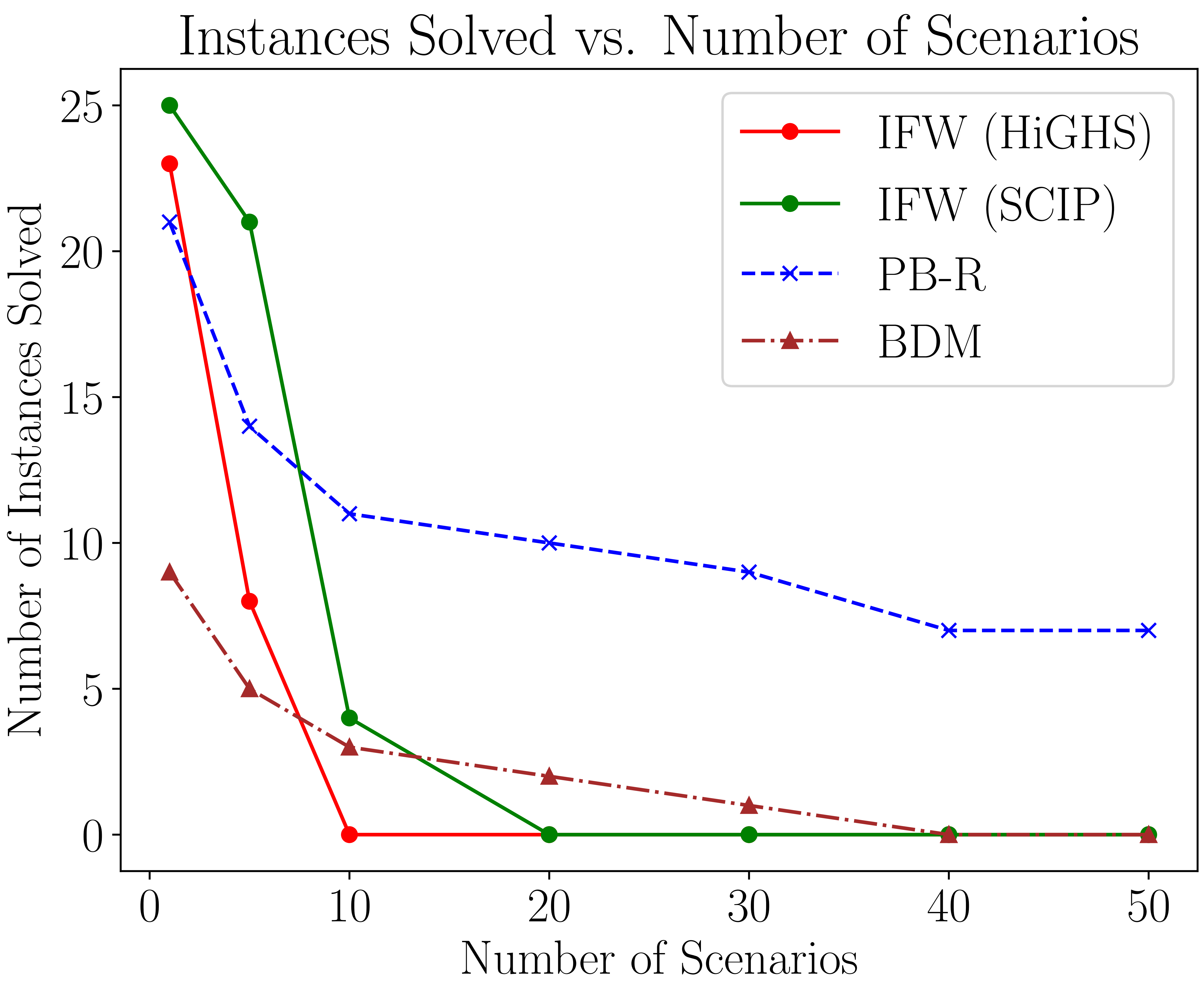

Instances Solved vs. Number of Scenarios
Number of Instances Solved
Number of Scenarios
0
5
10
15
20
25
0
10
20
30
40
50
IFW (HiGHS)
IFW (SCIP)
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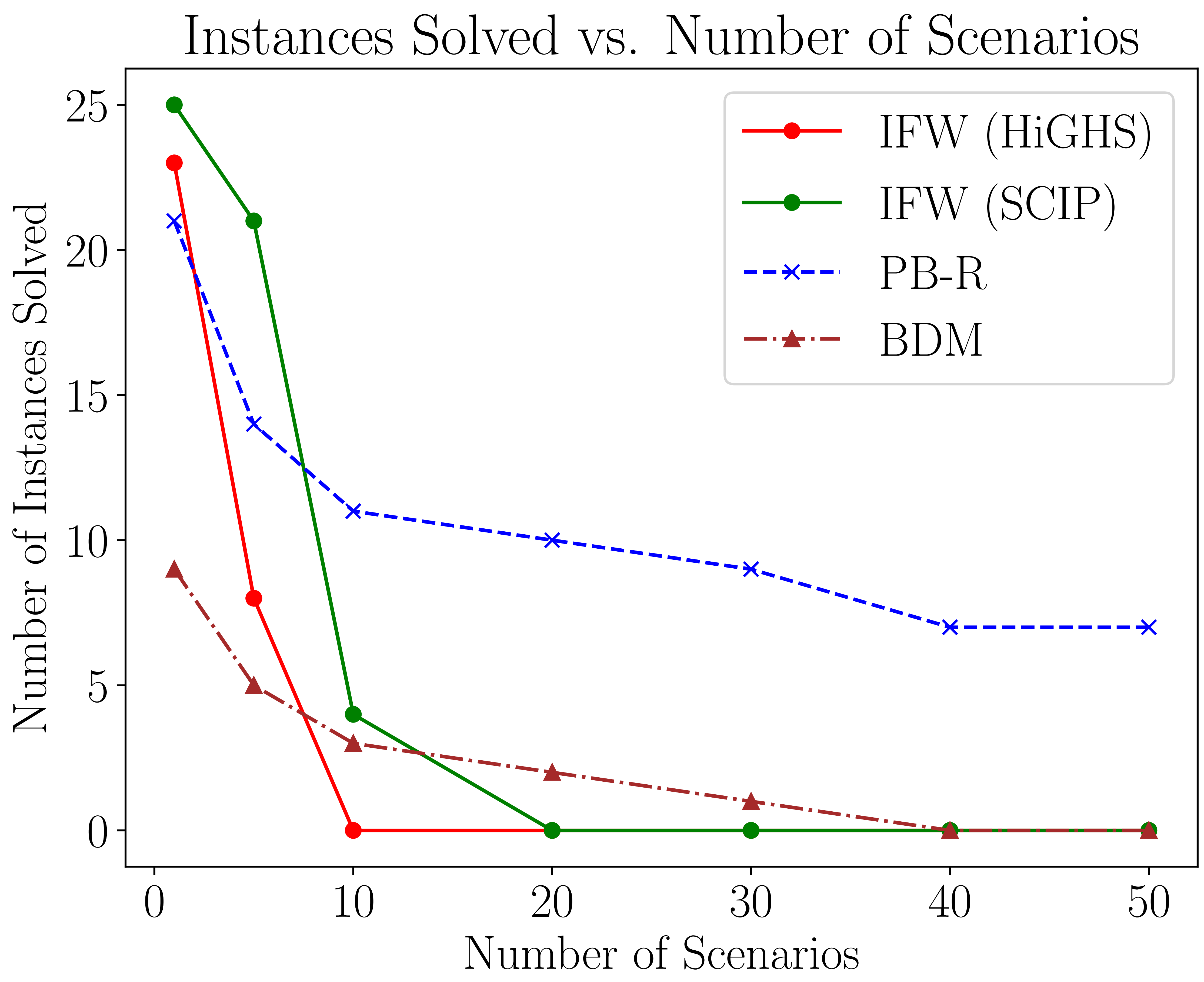
Results



For single scenario, the IFW approach is better.

For large number of scenarios, the penalty-based approach outperforms the others

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- Perspective formulations to achieve a tighter relaxation

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Future Directions

- Perspective formulations to achieve a tighter relaxation
- Recapture the information lost during relaxation.

thank you!