Chair for Scientific Computing
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Algorithmic Differentiation
Exercise 1

Consider the scalar function $f : x \in \mathbb{R}^3 \to y \in \mathbb{R}$:

$$y = f(x) = \sin(x_1 * x_2) + x_1^2 * x_3 - x_1^3 * e^{(x_2 - x_3)}$$

Single-Assignment-Code (SAC)

a) Implement the above function as a single-assignment-code in C. Name the intermediate variables in your code as `v1,v2,v3,...`

Tangent expressions and forward differentiated code

b) Derive tangent expressions for the all intermediate variables and finally for the output variable y in the single-assignment-code.

c) Augment the tangent expressions with the single-assignment code to obtain a forward-differentiated code, which computes the directional derivative $\nabla f(x) \cdot \mathbf{v}, \mathbf{v} \in \mathbb{R}^3$ for an arbitrary vector $\mathbf{v}$.

d) Using the forward-differentiated code, evaluate the derivatives of the output y with respect to the inputs x_1, x_2 and x_3 at the point $(x_1, x_2, x_3) = (1.0, -1.0, 2.5)$. Validate your results using analytical results.

Adjoint expressions and reverse differentiated code

e) Derive adjoint expressions for the all intermediate variables `v1,v2,v3,...` and finally for the input variables x_1, x_2 and x_3.

f) Augment the adjoint expressions with the single-assignment code to obtain a reverse-differentiated (adjoint) code, which computes the gradient vector $\nabla f(x)$.

g) Evaluate the derivatives of y with respect to x_1, x_2 and x_3 at $(x_1, x_2, x_3) = (1.0, -1.0, 2.5)$. Validate your results using the results from (d).