Short Assignment 3

This is an individual assignment.

Due: Friday, November 4 @ 11:59PM

Problem 1

Consider the two classes represented by Gaussians distributions P1 and P2 in Figures 1 and 2. Calculate Fisher's univariate separation indices to answer the following questions.

In [2]:

```python
from IPython.display import Image
Image('figures/two-gaussian-distributions.png',width=800)
```

Out[2]:

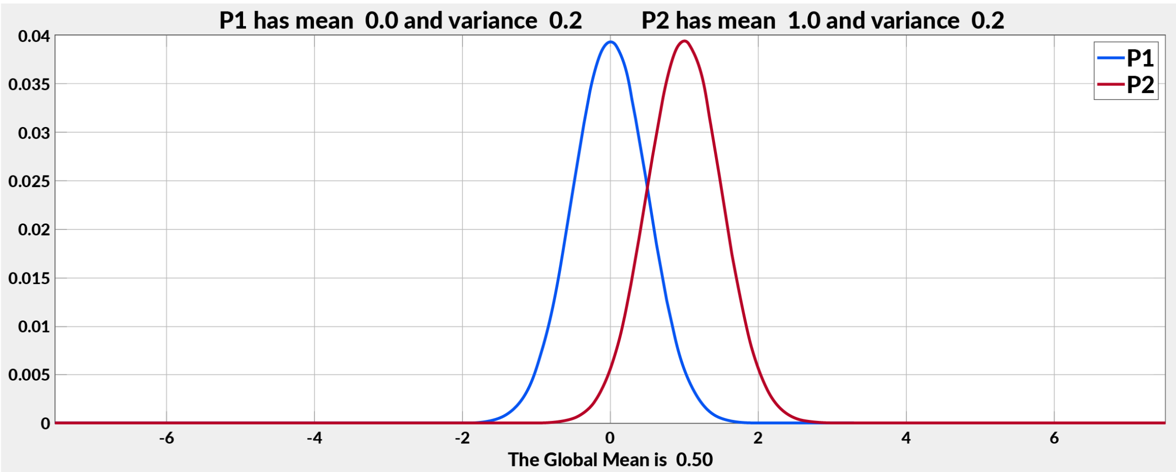


Figure 1: Two Gaussian Classes

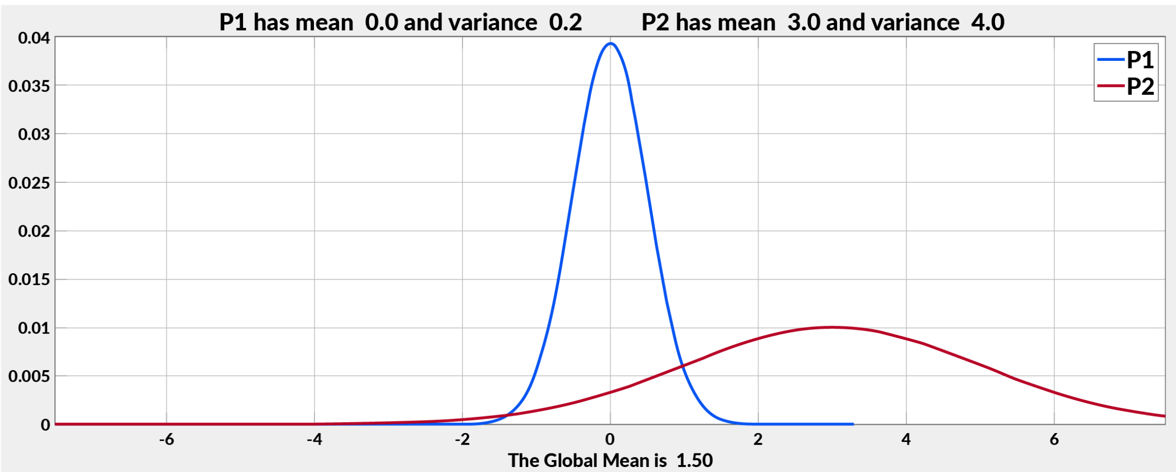


Figure 2: Two Gaussian Classes

1. What is the Within-Class Separation in Figure 1?
2. What is the Within-Class Separation in Figure 2?
3. What is the Between-Class Separation in Figure 1?
4. What is the Between-Class Separation in Figure 2?

5. Which Distributions are more separated: (A) P1,P2 in Figure 1 or (B) P1,P2 in Figure 2? Justify your answer.

Short Assignment 3

Answer

Diagram 1

For P_1

mean $\mu_1 = 0$

variance $\varepsilon^2 = 0.2$

P_2

$\mu_2 = 1.0$

$\varepsilon_2^2 = 0.2$

For Diagram 1

objective function

$$J(w) = \frac{(\mu_2 - \mu_1)}{\varepsilon_1^2 + \varepsilon_2^2}$$

$$= \frac{1-0}{0.2+0.2}$$

$$= 2.5$$

within class separation

$= (\mu_2 - \mu_1)^2$

$= 1 - 0$

$= 1$

Between class seperation

$= \varepsilon_1^2 + \varepsilon_2^2$

$= 0.2 + 0.2$

$\boxed{= 0.4}$

Diagram 2

P_1

$\mu_1 = 0$

$\varepsilon^2 = 0.2$

P_2

$\mu_2 = 3.0$

$\varepsilon_2^2 = 4.0$

For Diagram 2

objective function

$$J(w) = \frac{(\mu_2 - \mu_1)}{\varepsilon_1^2 + \varepsilon_2^2}$$

$$= \frac{(3-0)^2}{0.2+4}$$

$= 2.14$

Within class seperation

$= (\mu_2 - \mu_1)^2$

$= 3 - 0$

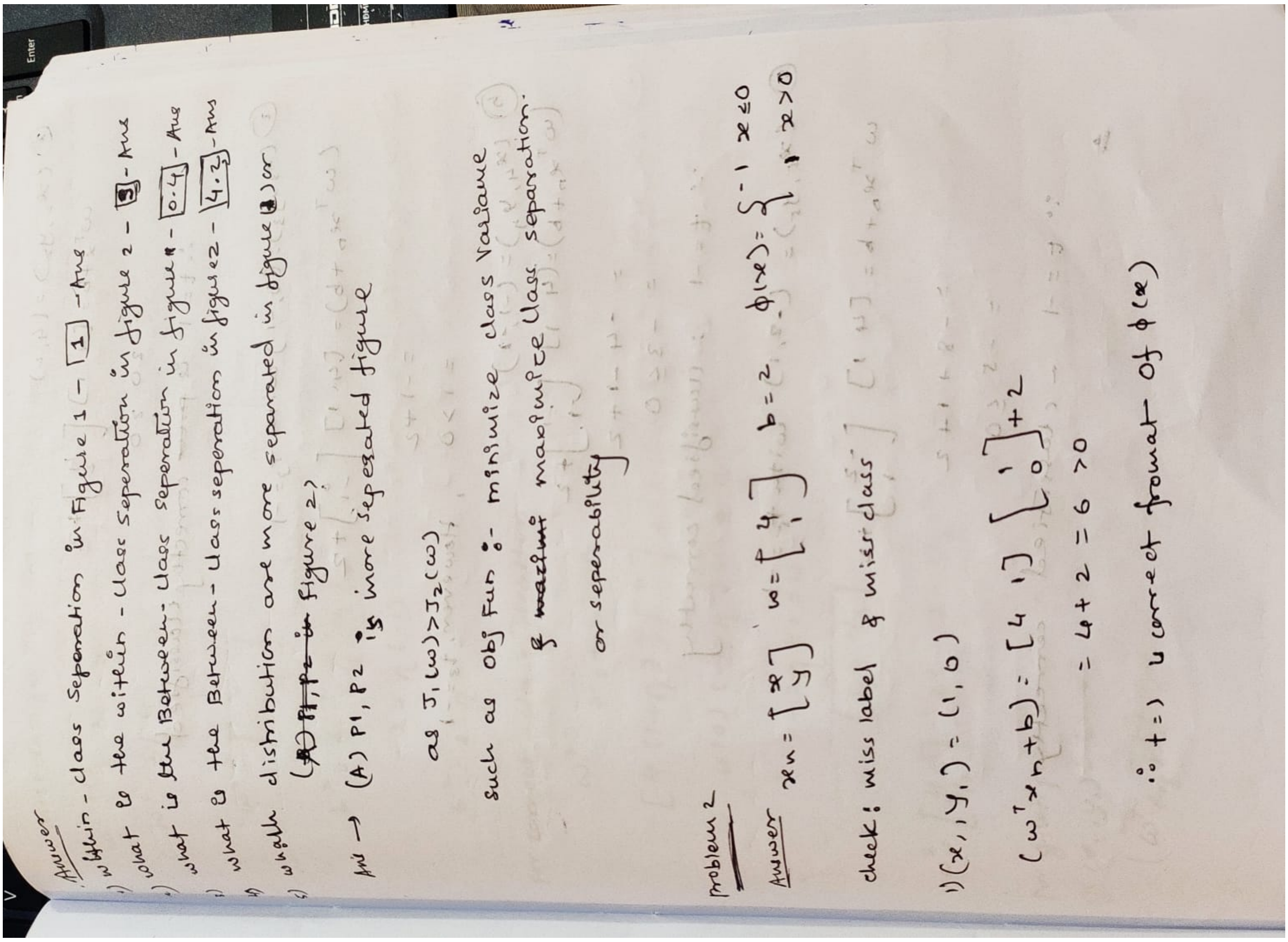
$= (3)^2 = 9$

Between class seperation

$= \varepsilon_1^2 + \varepsilon_2^2$

$= 0.2 + 4$

$\boxed{= 4.2}$



Answer

1) Within - class Separation in Figure 1 - $\boxed{1}$ - Ans

2) what is the within - class Seperation in figure 2 - $\boxed{3}$ - Ans

3) what is the Between - class Seperation in figure 1 - $\boxed{0.4}$ - Ans

4) what is the Between - class seperation in figure 2 - $\boxed{4.2}$ - Ans

5) which distribution are more separated in figure 1 or figure 2?

Ans → (A) P1, P2 is more seperated figure

as $J_1(w) > J_2(w)$

such as obj Fun :- minimize class variance & maximize class separation. or seperability

Problem 2

Answer $x_n = \begin{bmatrix} x \\ y \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $b = 2$, $\phi(x) = \begin{cases} -1 & x \le 0 \\ 1 & x > 0 \end{cases}$

check: miss label & miss class

1) $(x_1, y_1) = (1, 0)$

$(w^T x_n + b) = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2$

$= 4 + 2 = 6 > 0$

$\therefore t = 1$ correct format of $\phi(x)$

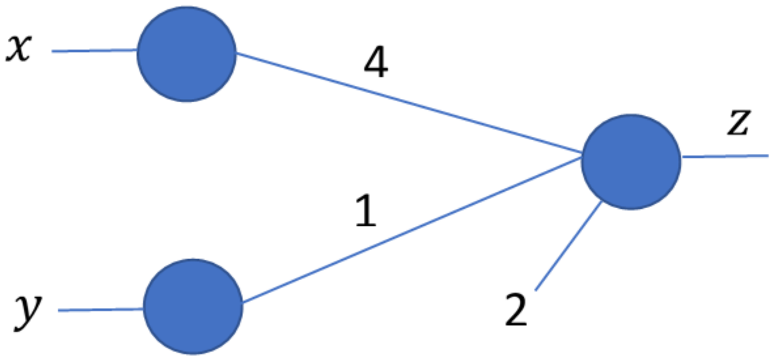
Problem 2

Consider the following perceptron:

In [3]:

```
Image('figures/Perceptron.png', width=400)
```

Out[3]:



Recall that the perceptron uses the activation function:

$$\phi(x) = \begin{cases} -1 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

And the cost function is:

$$E_p(\mathbf{w}, b) = - \sum_{m \in \mathcal{M}} (\mathbf{w}^T \mathbf{x}_n + b)^T t_n$$

where is the set of all misclassified points. The update equations for the weights and bias term are:

$$\begin{aligned} \mathbf{w}^{(t+1)} \leftarrow & \quad \mathbf{w}^{(t)} - \eta \frac{\partial E_p(\mathbf{w}, b)}{\partial \mathbf{w}} = \mathbf{w}^{(t)} + \eta \mathbf{x}_n t_n \\ b^{(t+1)} \leftarrow & \quad b^{(t)} - \eta \frac{\partial E_p(\mathbf{w}, b)}{\partial b} = b^{(t)} + \eta t_n \end{aligned}$$

Suppose you have the following 5 data samples (x, y) and their corresponding labels t:

$$\begin{aligned} (x_1, y_1) &= (1, 0) \text{ with } t_1 = 1 \\ (x_2, y_2) &= (4, 2) \text{ with } t_2 = 1 \\ (x_3, y_3) &= (0, -1) \text{ with } t_3 = -1 \\ (x_4, y_4) &= (-1, -1) \text{ with } t_4 = -1 \\ (x_5, y_5) &= (-2, 1) \text{ with } t_5 = -1 \end{aligned}$$

What is the smallest value for the learning rate η such that the updated network will result in zero misclassified points using only one iteration?

problem 2

Answer $x_n = \begin{bmatrix} x \\ y \end{bmatrix}$ $w = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $b = 2$ $\phi(x) = \begin{cases} -1 & x \leq 0 \\ 1 & x > 0 \end{cases}$

check: miss label & miss class

1) $(x_1, y_1) = (1, 0)$

$(w^T x_n + b) = [4 \ \ 1] \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2$

$= 4 + 2 = 6 > 0$

$\therefore t = 1$ ⇒ correct fromat of $\phi(x)$

(2) $(x_2, y_2) = (4, 2)$

$$w^T x + b = [4 \ \ 1] \begin{bmatrix} 4 \\ 2 \end{bmatrix} + 2$$

$$= 20 \geq 0$$

$\therefore t = 1$ is ~~pred~~ correctly classified

(3) $(x_3, y_3) = (0, -1)$ with $t_3 = -1$

$$(w^T x_n + b) = [4 \ \ 1] \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 2$$

$$= -1 + 2$$

$$= 1 > 0$$, However, $t_3 = -1$

(4) $(x_4, y_4) = (-1, -1)$

$$(w^T x_n + b) = [4 \ \ 1] \begin{bmatrix} -1 \\ -1 \end{bmatrix} + 2$$

$$= -4 - 1 + 2$$

$$= -3 \leq 0$$

$\because t = -1$ $\therefore$ classified correctly

(5) $(x_5, y_5) = (-2, 1)$ with $t_5 = -1$

$$w^T x_n + b = [4 \ \ 1] \begin{bmatrix} -2 \\ 1 \end{bmatrix} + 2$$

$$= -8 + 1 + 2$$

$$= -5 \leq 0$$,

$\because t = -1$ — classified correctly

for miss classified point ie (x_3, y_3)

$$w^{(1+1)} = w' + \eta x_n t_n$$

$$w^2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \eta \begin{bmatrix} 0 \\ -1 \end{bmatrix} (-1)$$

$$w^2 = \begin{bmatrix} 4 \\ 1+\eta \end{bmatrix}$$

$$b^2 = b^{(1)} + \eta (t_n)$$

$$= 2 + \eta(-1)$$

$$b^2 = 2 - \eta$$

for correct classification

$$w^{2t} x_3 + b^2 = 0$$

$$[4 \ (1+\eta)] \begin{bmatrix} 0 \\ -1 \end{bmatrix} + (2-\eta) = 0$$

$$4(0)\ (-1-\eta) + 2 - \eta = 0$$

$$-2\eta + 1 = 0$$

$$\boxed{\eta = 1/2}$$

$$w^2 = \begin{bmatrix} 4 \\ 3/2 \end{bmatrix} \quad b^2 = 2 - 1/2 = 3/2$$

for given points

① $(x_1, y_1) = (1, 0)$

$$(w^T x_n + b) = [4 \ \ 3/2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3/2$$

$$= 6 + 3/2$$

$$= \frac{15}{2} > 0 \quad \therefore t = 1 \rightarrow \text{correctly classified}$$

(2) $(x_2, y_2) = [4, 2]$

$w^T x + b = [4 \ \ 3/2] \begin{bmatrix} 4 \\ 2 \end{bmatrix} +$

$= 16 + 3 + 3/2$

$= 19 + 3/2$

$= \frac{41}{2} > 0$

$\boxed{\therefore t = 1}$ correctly classified

(3) $(x_3, y_3) = (0, -1)$ with $t_3 = -1$

$(w^T x_n + b) = [4 \ \ 3/2] \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 3/2$

$= -3/2 + 3/2$

$= 0 \quad \therefore t_3 = -1$ correctly classified

(4) $(x_4, y_4) = (-1, -1)$

$(w^T x_n + b) = [4 \ \ 3/2] \begin{bmatrix} -1 \\ -1 \end{bmatrix} + 3/2$

$= -4 - 3/2 + 3/2$

$= -4 \leq 0$

$\therefore t_4 = -1$ correctly classified

(5) $(x_5, y_5) = (-2, 1)$

$= [4 \ \ 3/2] \begin{bmatrix} -2 \\ 1 \end{bmatrix} + 3/2$

$= -8 + 3/2 + 3/2$

$= -8 + 3$

$\boxed{= -5}$

$t_5 = -1$ — classified.

Hence, learning rate

$\boxed{\eta = 1/2}$

update in one iteration (w, b)

Submit Your Solution

Confirm that you've successfully completed the assignment.

Along with the Notebook, include a PDF of the notebook with your solutions.

`add` and `commit` the final version of your work, and `push` your code to your GitHub repository.

Submit the URL of your GitHub Repository as your assignment submission on Canvas.
