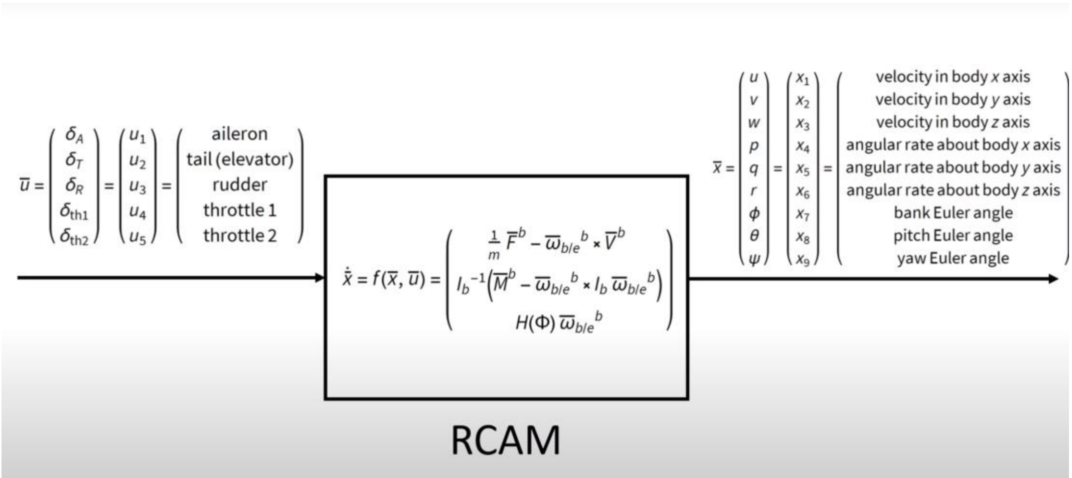
A Nonlinear, 6 DOF Dynamic Model of an Aircraft: The Research Civil Aircraft Model (RCAM)

Introduction:
The aircraft is a twin engine, commercial aircraft (Similar to the specs of a Boeing 757-200 which is also a twin-engine aircraft).

Our focus will be in generating the "Plant Model of the aircraft". The GARTEUR documentation covers the plant model, sensor model, controls design and more.

Plant model: Rigid body model of the aircraft



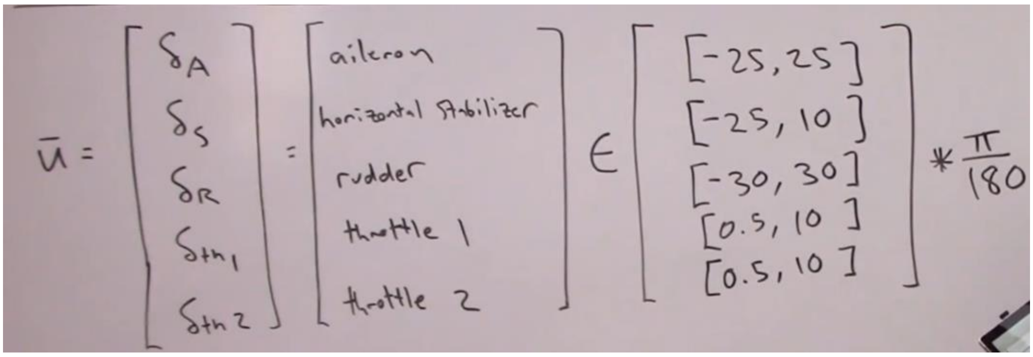
$$\bar{u} = control\ surface\ deflections$$
$$\bar{x} = state\ functions\ of\ the\ aircraft$$

NOTE: u_2 is the entire horizontal tail deflection and not just the elevator deflection.

A set of 10 steps are to be followed to model the Plant Model of the flight and they are:

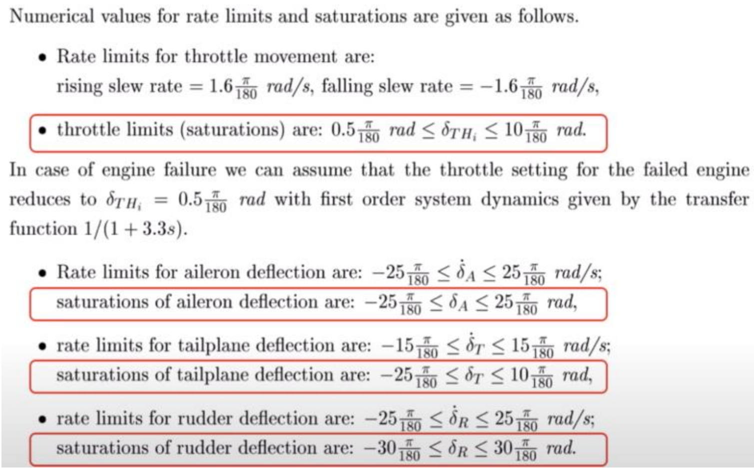
1. Control Limits/Saturation
2. Intermediate Variables
3. Nondimensional Aerodynamic Force Coefficients in F_s
4. Aerodynamic Force in F_b
5. Nondimensional Aerodynamic Moment Coefficient About Aerodynamic Center in F_b
6. Aerodynamic Moment About Aerodynamic Center in F_b
7. Aerodynamic Moment About Center of Gravity in F_b
8. Propulsion Effects
9. Gravity Effects
10. Explicit First Order Form

1. **Control Limits/Saturation:**



This basically deals with the limits on the control surface deflections. (u_1 to u_5)

Assumption:



Numerical values for rate limits and saturations are given as follows.

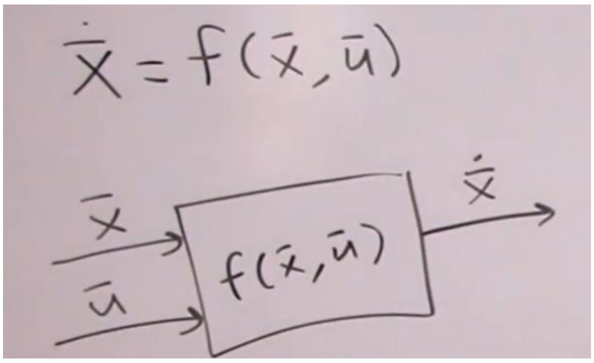
- Rate limits for throttle movement are:
  rising slew rate $= 1.6\frac{\pi}{180}$ rad/s, falling slew rate $= -1.6\frac{\pi}{180}$ rad/s,
- throttle limits (saturations) are: $0.5\frac{\pi}{180}$ $rad \leq \delta_{TH_i} \leq 10\frac{\pi}{180}$ rad.

In case of engine failure we can assume that the throttle setting for the failed engine reduces to $\delta_{TH_i} = 0.5\frac{\pi}{180}$ rad with first order system dynamics given by the transfer function $1/(1 + 3.3s)$.

- Rate limits for aileron deflection are: $-25\frac{\pi}{180} \leq \dot{\delta}_A \leq 25\frac{\pi}{180}$ rad/s;
  saturations of aileron deflection are: $-25\frac{\pi}{180} \leq \delta_A \leq 25\frac{\pi}{180}$ rad,
- rate limits for tailplane deflection are: $-15\frac{\pi}{180} \leq \dot{\delta}_T \leq 15\frac{\pi}{180}$ rad/s;
  saturations of tailplane deflection are: $-25\frac{\pi}{180} \leq \delta_T \leq 10\frac{\pi}{180}$ rad,
- rate limits for rudder deflection are: $-25\frac{\pi}{180} \leq \dot{\delta}_R \leq 25\frac{\pi}{180}$ rad/s;
  saturations of rudder deflection are: $-30\frac{\pi}{180} \leq \delta_R \leq 30\frac{\pi}{180}$ rad.

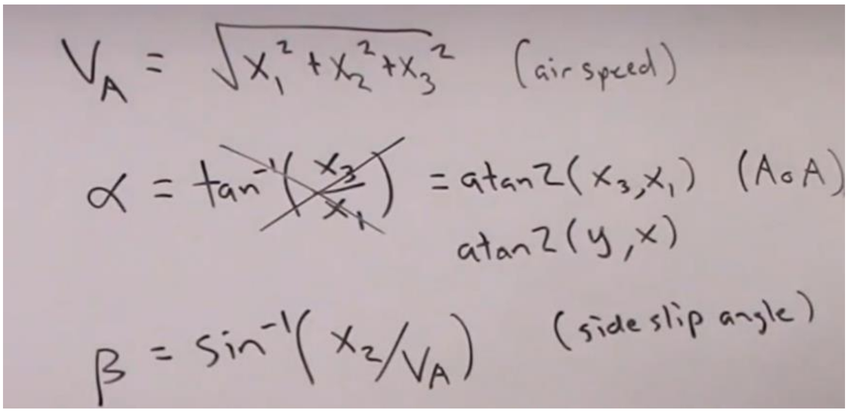
In reality the rates of these control surface deflections are also considered while modelling the aircraft, but to reduce the number of states to ease the calculation we are neglecting the rate of change of these control surface deflections.
The ones marked in red are the control limits ranges.

2. **Intermediate Variables:**

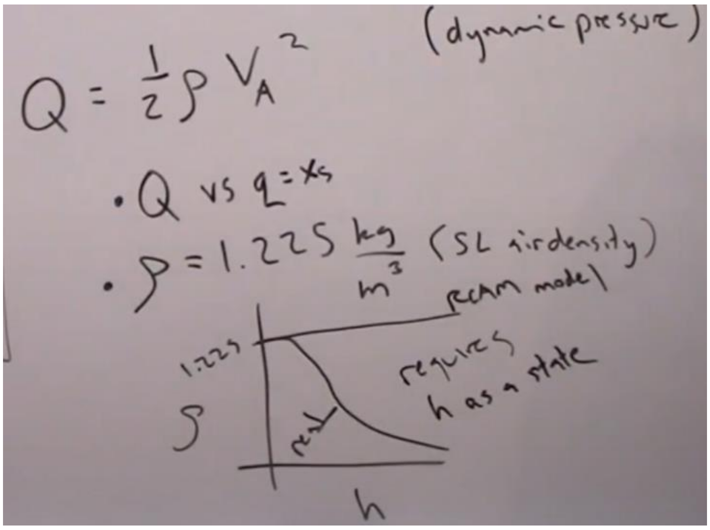


The main purpose of this step is to represent the state rate of change as a function of control surface deflections as well as state variables.

Let's see how some of the terms can be expressed as f ($\bar{x}$, $\bar{u}$):



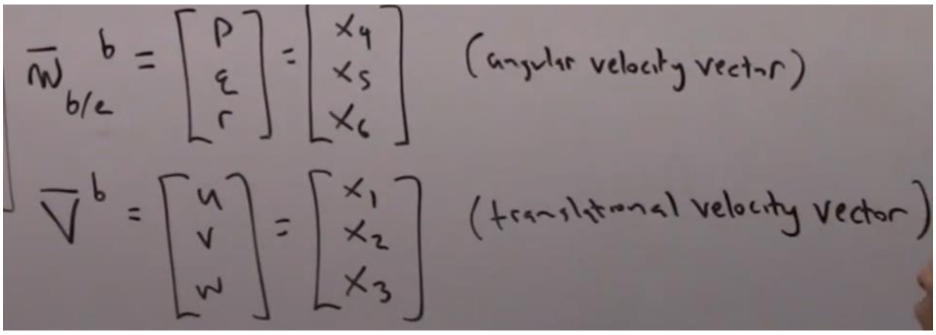
NOTE: For α, arctan2 is taken because arctan will show some problem in the domain and we won’t get the required result according to our expectation.



NOTE: Q: Dynamic Pressure, **q:** Angular Rate about y-axis (x_5)

Assumption:

The density is considered to be constant as if it isn’t we will have to add another state which is the altitude of the aircraft (h).



$\bar{\boldsymbol{\omega}}^{b}_{b/e}$ **:** Angular velocity of the body with respect to the earth when expressed in the body frame.

$\bar{\boldsymbol{v}}^{\boldsymbol{b}}$**:** The velocity of the centre of mass of the aircraft when expressed in the body frame.

3. **Non – Dimensional Aerodynamic Force Coefficients in F^S(Stability frame):**

The reason for using the stability frame can be seen below as stated in section 2.3.4 in RCAM documentation.

2.3.4 Aerodynamic equations

The equations defining aerodynamic forces and moments are determined by means of aerodynamic coefficients. Depending on the method of modelling these coefficients may be defined in different reference frames; e.g. F_W, F_S, or F_B. The reference frame for aerodynamic forces and ~~moments~~ that is used in RCAM is the stability axis frame F_S.

A. Coefficient of Lift (C_L):

a. Wing, Body consideration ($C_{L_{WB}}$)**:**

The coefficient of lift in the wing body frame is given by the piecewise function below:

Wing/body

$$C_{L_{wb}} = \begin{cases} n(\alpha - \alpha_{L=0}) & \text{if } \alpha \leq 14.5\frac{\pi}{180} \\ a_3\alpha^3 + a_2\alpha^2 + a_1\alpha + a_0 & \text{otherwise} \end{cases}$$

where $\alpha_{L=0} = -11.5\frac{\pi}{180}$ (α at $L=0$)

$n = 5.5$

$a_3 = -768.5$

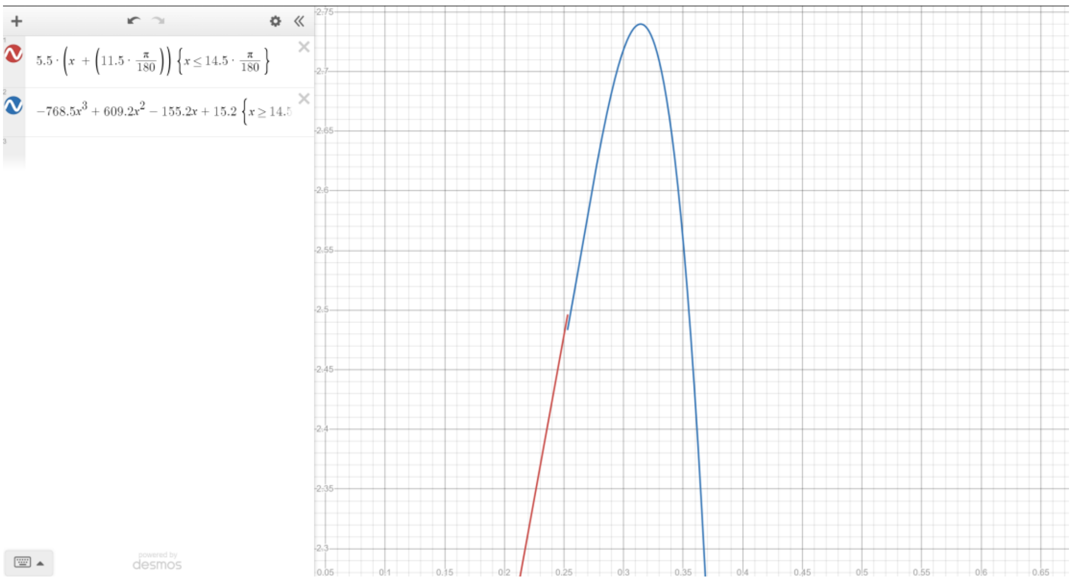
$a_2 = 609.2$

$a_1 = -155.2$

$a_0 = 15.2$ *

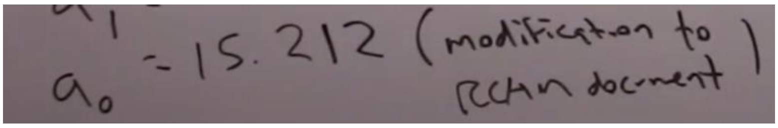
(We will discuss in regard to the value of a_0 in more detail later and that's why it has a "" symbol around it)*

Let's try to understand whether the function mentioned above is reasonable for consideration and how is the function plotted below.

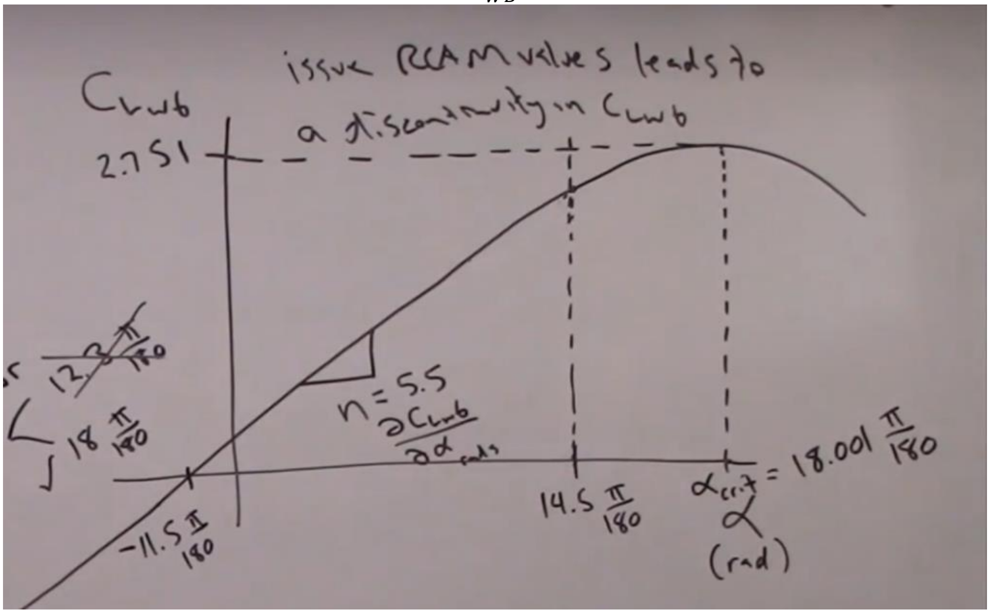


When the 2 curves are plotted, we see a discontinuity at the point α = 14.5*(π/180) rad.

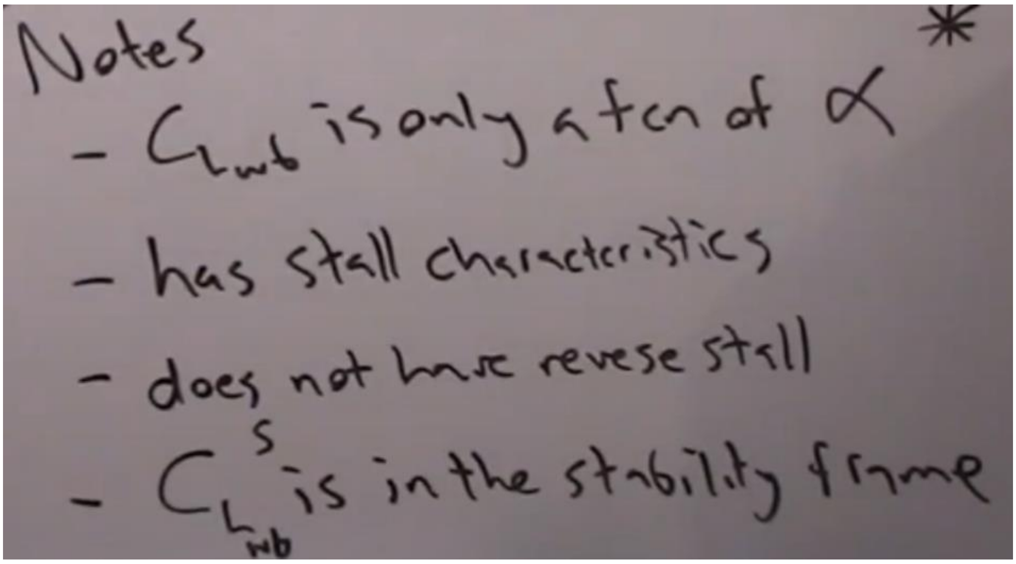
So as to make the function continuous, the term a_0 has to be made more accurate and hence we will be adding more numbers after the decimal place. The new value of a_0 is:



Which will in return give me the curve of $C_{L_{WB}}$ vs α as follows:

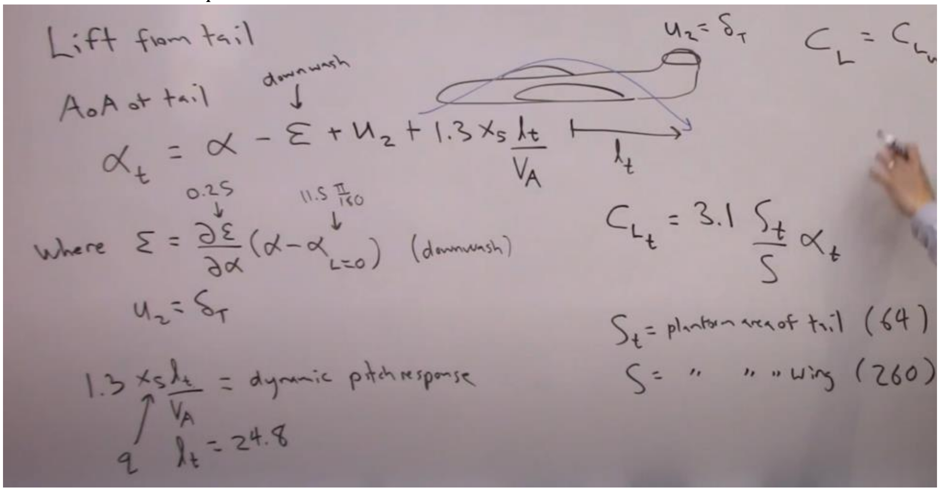


The critical value of the angle of attack can be found from making the slope of the cubic equation 0 and finding the points. We get 2 points and we must take the point that is after α = 14.5*(π/180) rad. The point obtained is α = 18.001*(π/180) rad and it produces a lift of 2.751.



Here the dependency of sideslip angle isn't considered to find the coefficient of lift effects. This will get deal

b. Tail consideration (C_{L_T}):



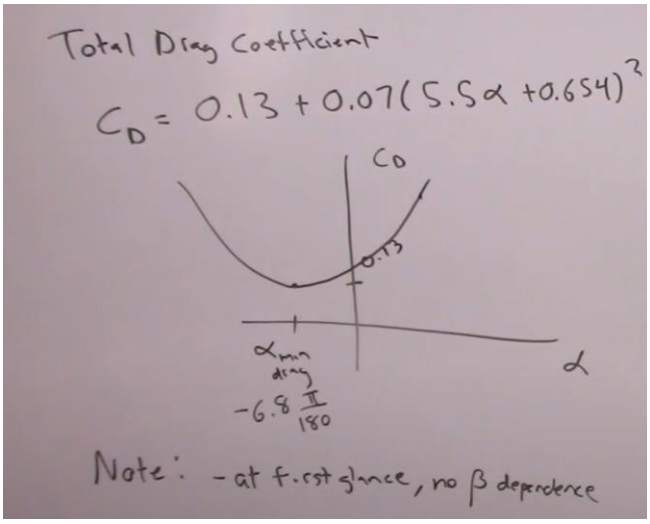
(**ϵ:** Change in angle of attack on the tail due to the lift generated by the wing.)

($\boldsymbol{\alpha_{L=0}}$: 11.5*(π/180) rad (As obtained from the curve of $C_{L_{WB}}$ vs α))

The net lift acting on the aircraft can be given as:

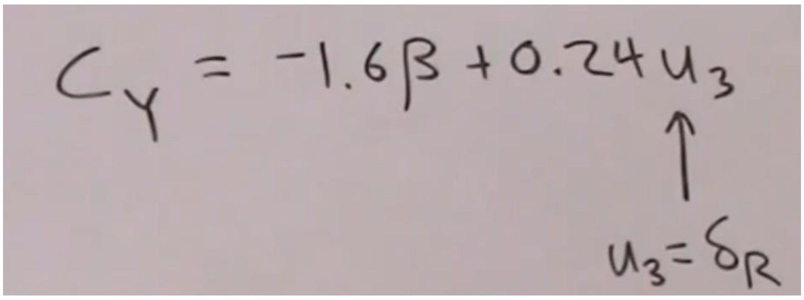
NOTE: $\boldsymbol{C_L = C_{L_T} + C_{L_{WB}}}$

B. Coefficient of Drag (C_D):



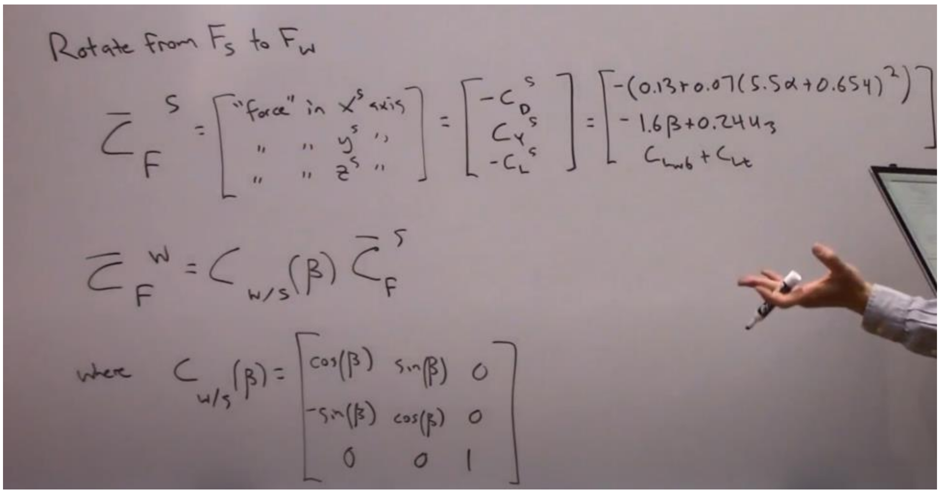
NOTE: We don’t see any terms with the sideslip angle because here the considerations are made with the consideration of the stability axis. When the discussion is moved to the wind axis, a more well-defined effect of sideslip angle will be seen.

C. Coefficient of Side – force (C_Y)**:**



We will be able to see more effect of the rudder when we are modelling the equations for the yawing moment coefficient for the RCAM model.

Rotation from the Stability frame to the wind frame:



Let's use the help of Mathematica to understand this matrix multiplication so as to get the final coefficients in the wind frame.

Define $\overline{C}_F^{\,s}$

In[133]:=

$$\text{CFs} = \begin{pmatrix} -\text{CD} \\ \text{CY} \\ -\text{CL} \end{pmatrix};$$

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CFs // MatrixForm
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Out[134]//MatrixForm=

$$\begin{pmatrix} -0.13 - 0.07\,(0.654 + 5.5\,\alpha)^2 \\ 0.24\,\text{u3} - 1.6\,\beta \\ -5.5\,(0.200713 + \alpha) - 0.763077\left(\text{u2} + \frac{32.24\,\text{x5}}{\text{VA}} + \alpha - 0.25\,(0.200713 + \alpha)\right) \end{pmatrix}$$

Rotating this to F_w

Rotating this to F_w

In[135]:=

$$\text{Cws}[\beta\_] = \begin{pmatrix} \text{Cos}[\beta] & \text{Sin}[\beta] & 0 \\ -\text{Sin}[\beta] & \text{Cos}[\beta] & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

In[136]:=

```
CFw = Cws[β].CFs;
CFw // MatrixForm
```

Out[137]//MatrixForm=

$$\begin{pmatrix} \left(-0.13 - 0.07\,(0.654 + 5.5\,\alpha)^2\right)\text{Cos}[\beta] + (0.24\,\text{u3} - 1.6\,\beta)\,\text{Sin}[\beta] \\ (0.24\,\text{u3} - 1.6\,\beta)\,\text{Cos}[\beta] - \left(-0.13 - 0.07\,(0.654 + 5.5\,\alpha)^2\right)\text{Sin}[\beta] \\ -5.5\,(0.200713 + \alpha) - 0.763077\left(\text{u2} + \frac{32.24\,\text{x5}}{\text{VA}} + \alpha - 0.25\,(0.200713 + \alpha)\right) \end{pmatrix}$$

NOTE: To get to know the drag coefficient, it will be – (1,1) of CFw matrix.

To get to know the side – force coefficient, it will be (2,1) of CFw matrix.

To get to know the lift coefficient, it will be –(3,1) of CFw matrix.

$C_D^{\,w}$

Out[139]=

$$-\left(-0.13 - 0.07\,(0.654 + 5.5\,\alpha)^2\right)\text{Cos}[\beta] - (0.24\,\text{u3} - 1.6\,\beta)\,\text{Sin}[\beta]$$

$C_Y^{\,w}$

Out[142]=

$$(0.24\,\text{u3} - 1.6\,\beta)\,\text{Cos}[\beta] - \left(-0.13 - 0.07\,(0.654 + 5.5\,\alpha)^2\right)\text{Sin}[\beta]$$

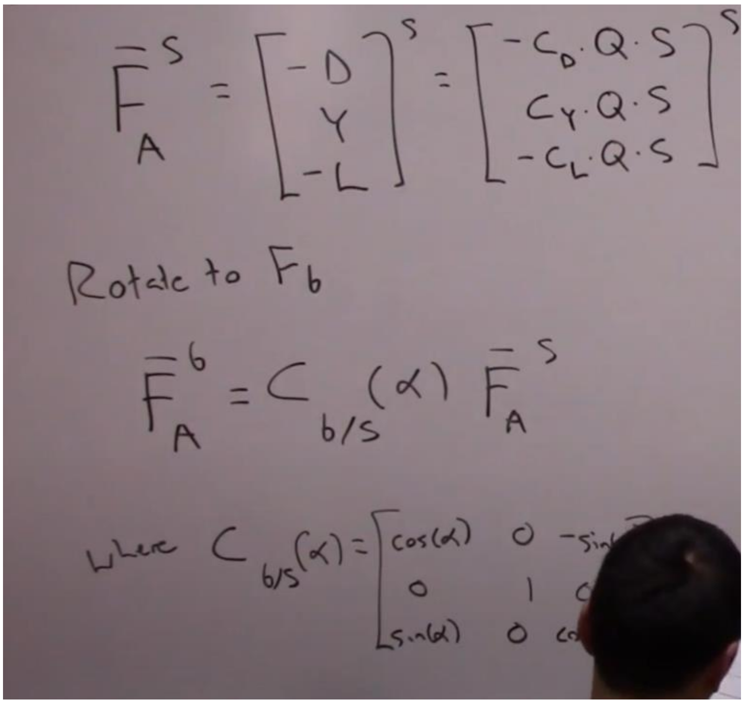
$C_L^{\,w}$

Out[145]=

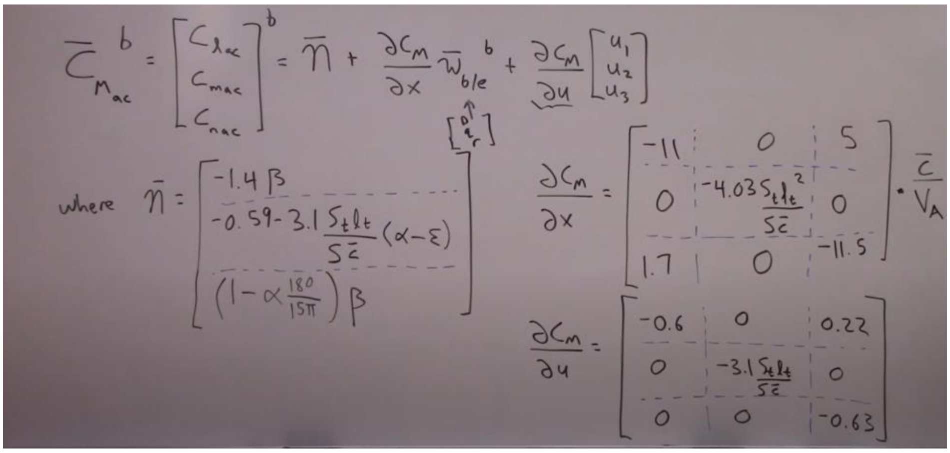
$$5.5\,(0.200713 + \alpha) + 0.763077\left(\text{u2} + \frac{32.24\,\text{x5}}{\text{VA}} + \alpha - 0.25\,(0.200713 + \alpha)\right)$$

NOTE: There is no dependence of the sideslip angle in the C_L modelling in RCAM.

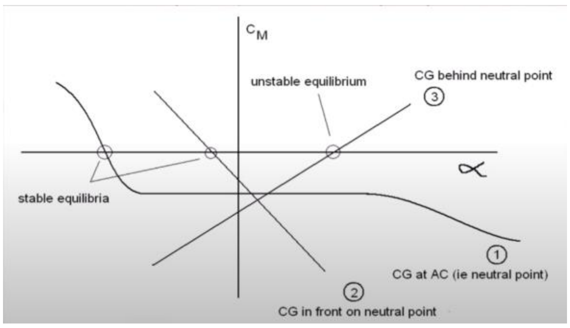
4. **Aerodynamic Force in F_b:**



5. **Non-Dimensional Aerodynamic Moment Coefficient about Aerodynamic Centre in the F_b:**



The 1st term in the RHS of the equation will give me the static stability component of the aircraft. The (2,1) element in the first terms is kept in such a way that there is static stability provided as the slope wrt the angle of attack is negative in nature which in turn gives stability. This can be seen from the graph shown below.



The 2nd term lets me know the dynamic angular rates behaviour which basically tells that if you have some non – zero angular velocities, you will induce some moments to the aircraft. These terms may dampen or exaggerate the angular velocities/angular rates.

The 3rd term measures the effectiveness of the control surface deflection in producing aerodynamic moments and lets me know if they do impart any kind of rolling, pitching or yawing moment.

6. **Aerodynamic Moment about the Aerodynamic Centre in F_b:**

$$\bar{M}_{A_{ac}}^{\,b} = \bar{C}_{M_{ac}}^{\,b} \cdot q S \bar{c}$$

NOTE: Here q is "Q"

7. **Aerodynamic Moment about the C.G. in F_b:**

$$\bar{M}_{A_{ac}}^{\,b} = \bar{C}_{M_{ac}}^{\,b} \cdot q S \bar{c}$$

Moment transfer

$$\bar{M}_{A_{CG}}^{\,b} = \underbrace{\bar{M}_{A_{ac}}^{\,b}}_{\text{Step 6}} + \underbrace{\bar{F}_{A}^{\,b}}_{\text{Step 4}} \times \left(\bar{r}_{CG}^{\,b} - \bar{r}_{AC}^{\,b}\right)$$

$$\bar{r}_{CG} = \begin{bmatrix} X_{CG} \\ Y_{CG} \\ Z_{CG} \end{bmatrix} = \begin{bmatrix} 0.23\bar{c} \\ 0 \\ 0.1\bar{c} \end{bmatrix} \qquad \bar{r}_{ac} = \begin{bmatrix} X_{ac} \\ Y_{ac} \\ Z_{ac} \end{bmatrix} = \begin{bmatrix} 0.12\bar{c} \\ 0 \\ 0 \end{bmatrix}$$

8. Propulsion Effects:

$$F_i = \delta_{th_i} \cdot m \cdot g$$

$$F_1 = u_4 \cdot mg \qquad \text{recall: } u_4, u_5 \leq 10\frac{\pi}{180}$$

$$F_2 = u_5\, mg$$

at max thrust $\left(u_4 = u_5 = 10\frac{\pi}{180}\right)$

$$\frac{F_{1_{max}} + F_{2_{max}}}{mg} = 0.35$$

$$\bar{F}_{E_i}^{\,b} = \begin{bmatrix} F_i \\ 0 \\ 0 \end{bmatrix}^b$$

$$\boxed{\bar{F}_E^{\,b} = \bar{F}_{E_1}^{\,b} + \bar{F}_{E_2}^{\,b}}$$

propulsive engine force in Fb

$$\bar{M}_{E_{cg_i}}^{\,b} = \bar{M}_i^{\,b} \times \bar{F}_{E_i}^{\,b}$$

$$\bar{M}_i^{\,b} = \begin{bmatrix} X_{CG} - X_{APT_i} \\ Y_{APT_i} - Y_{CG} \\ Z_{cg} - Z_{APT_i} \end{bmatrix}$$

Engine Parameters					
X_{APT1}	XAPT1	=	x position of application point of thrust of engine 1 in F_M	0.0	m
Y_{APT1}	YAPT1	=	y position of application point of thrust of engine 1 in F_M	−7.94	m
Z_{APT1}	ZAPT1	=	z position of application point of thrust of engine 1 in F_M	−1.9	m
X_{APT2}	XAPT2	=	x position of application point of thrust of engine 2 in F_M	0.0	m
Y_{APT2}	YAPT2	=	y position of application point of thrust of engine 2 in F_M	7.94	m
Z_{APT2}	ZAPT2	=	z position of application point of thrust of engine 2 in F_M	−1.9	m

Table 2.4 Parameters definitions

$$\boxed{\bar{M}_{E_{CG}}^{\,b} = \bar{M}_{E_{CG1}}^{\,b} + \bar{M}_{E_{CG2}}^{\,b}}$$

$\Rightarrow$ propulsive/engine moment about CG in F_b

9. Gravity Effects:

$$\bar{F}_g = m \cdot \bar{g}$$

Easy frame F_e

$$\bar{F}_g^{\,e} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} m$$

Need to rotate to F_b

$$\bar{F}_g^{\,b} = C_{b/e}(\phi, \theta, \psi)\,\bar{F}_g^{\,e}$$

$$\boxed{\bar{F}_g^{\,b} = \begin{bmatrix} -g\sin(X_8) \\ g\cos(X_8)\sin(X_7) \\ g\cos(X_8)\cos(X_7) \end{bmatrix} \cdot m}$$

assume m = constant

10. Explicit First Order form:

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m}\bar{F}^b - \bar{\omega}_{b/e}^{\,b} \times \bar{V}^b$$

note: constant mass

where $\bar{F}^b = \bar{F}_g^{\,b} + \bar{F}_E^{\,b} + \bar{F}_A^{\,b}$

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = I_b^{-1}\left(\bar{M}_{cg}^{\,b} - \bar{\omega}_{b/e}^{\,b} \times I_b \bar{\omega}_{b/e}^{\,b}\right)$$

note: I_b is constant

where $\bar{M}_{cg}^{\,b} = \bar{M}_{E_{cg}}^{\,b} + \bar{M}_{A_{cg}}^{\,b}$

$$\begin{bmatrix} \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\phi\, t\theta & c\phi\, t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}^b$$

$$\dot{\bar{X}} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$\bar{x}$, $\bar{u}$ $\rightarrow$ $f(\bar{x}, \bar{u})$ — RCAM aircraft $\rightarrow$ $\dot{\bar{x}}$