CS 663

ASSIGNMENT 4

es 5) Constraints given:

f is perpendicular to e $(f^t e = 0 = e^t f)$

f is a unit vector $(f^t f = 1)$

$e^t C e = \lambda_1$

We need to maximise $f^t C f$ given $f^t f = 1$ and $f^t e = 0$, using Lagrange multipliers

$$J(f) = f^t C f - \lambda(f^t f - 1) - \delta(f^t e)$$

Take derivative of $J(f)$ wrt f^t and equating it to zero

$$2Cf - 2\lambda f - \delta e = 0$$

(Multiplying both sides by e^t)

$$2e^t C f - 2\lambda e^t f - \delta e^t e = 0$$

Since $Ce = \lambda_1 e \Rightarrow e^t C^t = \lambda_1 e^t$ (transpose)

C is covariance matrix $\Rightarrow$ it is symmetric since covariance of x_i and x_j is same as covariance of x_j and x_i.

Thus, $e^t C = e^t C^t = \lambda_1 e^t$

$\Rightarrow e^t C f = \lambda_1 e^t f = 0$ (as $e^t f = 0$)

$\Rightarrow \delta = 0$ as $e^t C f = 0$, so

$$Cf - \lambda f = 0 \Rightarrow f^t C f = \lambda$$

In order to maximise $f^t C f$, λ must be maximised. Since rank $(C) > 2$ and e is the eigenvector corresponding to λ_1 hence the next largest value of λ can be λ_2 (2nd largest eigenvalue).

Thus f is the eigenvector corresponding to λ_2.