ASSIGNMENT 5

Question 1) Given :

$$g_1(x,y) = f_1(x,y) + (h_2 * f_2)(x,y)$$

$$g_2(x,y) = (h_1 * f_1)(x,y) + f_2(x,y)$$

Now take the Fourier transform of these equations and use convolution theorem, ie, fourier transform of a convolution is the pointwise product of the Fourier transform.

$\Rightarrow$

$$G_1(u,v) = F_1(u,v) + H_2(u,v)\,F_2(u,v) \quad \text{——— (1)}$$
$$G_2(u,v) = H_1(u,v)\,F_1(u,v) + F_2(u,v) \quad \text{——— (2)}$$

From (1) after multiplying $H_1(u,v)$ both sides

$$H_1(u,v)\,G_1(u,v) = H_1(u,v)\,H_2(u,v)\,F_2(u,v) + H_1(u,v)\,F_1(u,v)$$

Now subtract (2) from the above equation

$$\Rightarrow H_1(u,v)\,G_1(u,v) - G_2(u,v) = H_1(u,v)\,H_2(u,v)\,F_2(u,v) - F_2(u,v)$$

$$\hat{F}_2(u,v) = \frac{G_2(u,v) - H_1(u,v)\,G_1(u,v)}{1 - H_1(u,v)\,H_2(u,v)} = F_2(u,v)$$

$f_2(x,y) = \mathcal{F}^{-1}(\hat{F}_2(u,v))$ (Inverse Fourier Transform)

Similarly we get f_1 as

$$f_1(x,y) = \mathcal{F}^{-1}(\hat{F}_1(x,y)) \quad \text{where}$$

$$\hat{F}_1(u,v) = \frac{G_1(u,v) - H_2(u,v)\,G_2(u,v)}{1 - H_2(u,v)\,H_1(u,v)} = F_1(u,v)$$

Even after the assumptions, the formula derived is problematic. As $h_1(x,y)$ and $h_2(x,y)$ are blur kernels $\Rightarrow$ $H_1(u,v)$ and $H_2(u,v)$ are low pass filters and hence they will both tend to 1 at low frequencies.

$\Rightarrow 1 - H_1(u,v) H_2(u,v)$ will tend to 0 and thus pose a problem and hence this approach cannot be used to get the values of low frequency components of $\hat{F}_1(u,v)$ and $\hat{F}_2(u,v)$.

Now if $1 - H_1(u,v) H_2(u,v)$ doesn't become 0, then $f_1(x,y)$ and $f_2(x,y)$ can be calculated ~~quite~~ quite accurately assuming there is no noise.

If we consider noise then if $1 - H_1(u,v) H_2(u,v)$ tends to zero, ~~then~~ values of $\hat{F}_1(u,v)$ and $\hat{F}_2(u,v)$ will blow up amplifying noise in $f_1(x,y)$ and $f_2(x,y)$