ASSIGNMENT 5

Camlin Page
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Ques 2) Given $g(x,y) = (h*f)(x,y)$

Take Fourier transform of this equation and use convolution theorem, i.e, Fourier transform of a convolution is the pointwise product of the Fourier transform.

$$G(u,v) = H(u) \quad F(u,v)$$

where $G(u,v) = \mathcal{F}(g(x,y))$

$H(u) = \mathcal{F}(h(x))$

$F(u,v) = \mathcal{F}(f(x,y))$

$\Rightarrow$ $\cancel{\hat{F}(x,y)}$ $\hat{F}(u,v) = \dfrac{G(u,v)}{H(u)} = F(u,v)$

$f(x,y) = \mathcal{F}^{-1}(\hat{F}(u,v))$ (Inverse Fourier Transform)

Since $h(x)$ is a convolution kernel to represent the gradient operation $\Rightarrow$ $H(x,y)$ is a high pass filter. Hence ~~it~~ it will tend to 0 at low frequencies. ~~If~~ $\Rightarrow$ This approach cannot be used if value of $H(u)$ becomes 0.

Now if $H(u)$ doesn't become 0, provided there is no noise $f(x,y)$ can be determined accurately. If we take into consideration noise then ~~and~~ if $H(u)$ tends to 0, $\Rightarrow$ the value of $\hat{F}(u,v)$ will blow up amplifying noise in $f(x,y)$.

This poses a problem as images generally have large magnitude of these components.

For 2D image (2nd Part)

Let g_x, g_y be gradients of 2D image in X and Y directions respectively and h_x and h_y be the convolution kernel for gradient operation in the X and Y direction respectively. f is the original 2D image.

Then,

$$g_x(x,y) = (h_x * f)(x,y)$$

$$g_y(x,y) = (h_y * f)(x,y)$$

Take Fourier Transform of these equations and use convolution theorem, we get

$$G_x(u,v) = H_x(u,v)\, F(u,v)$$
$$G_y(u,v) = H_y(u,v)\, F(u,v)$$

where $G_x(u,v) = \mathcal{F}(g_x(x,y))$, $G_y(u,v) = \mathcal{F}(g_y(x,y))$, $H_x(u,v) = \mathcal{F}(h_x(x,y))$, $H_y(u,v) = \mathcal{F}(h_y(x,y))$, $F(u,v) = \mathcal{F}(f(x,y))$

using these eq^n we get

$$\hat{F}(u,v) = \frac{G_x(u,v)}{H_x(u,v)} = F(u,v) \quad \text{——— (1)}$$

$$\hat{F}(u,v) = \frac{G_y(u,v)}{H_y(u,v)} = F(u,v) \quad \text{——— (2)}$$

As $h_x(x,y)$ is a gradient kernel in x direction thus $H_x(u,v)$ will be a high pass filter in u, hence when u is small $\Rightarrow$ $H_x(u,v)$ will be small and hence using — (1) to calculate $\hat{F}(u,v)$ will be problematic as the denominator tends to zero so its value will blow up. Although — (1) can be used in case u is large. ~~Similarly~~ Similarly ~~analysis~~ ~~true~~ for y direction

Hence if both u and v large $\Rightarrow$ use any of the 2 equations

u is large and v is small $\Rightarrow$ choose ~~eqn~~ — (1)

v is large and u is small $\Rightarrow$ choose — (2)

u and v both are small $\Rightarrow$ both the equations are problematic.

Taking Inverse Fourier Transform

$$f(x,y) = \mathcal{F}^{-1}(\hat{F}(u,v))$$

In case frequency in both u and v directions are low and values of $H_x(u,v)$ and $H_y(u,v)$ becomes 0, then the given approach can't be used to extract low frequency components of $\hat{F}(u,v)$. $f(x,y)$ can be quite accurately determined if any one doesn't become zero and there is no noise.

In case of noise if $H_x(u,v)$ or $H_y(u,v)$ tend to 0 then the corresponding $\hat{F}(u,v)$ will blow up amplifying noise in $f(x,y)$.

This poses a problem as images generally have large magnitude of these components.