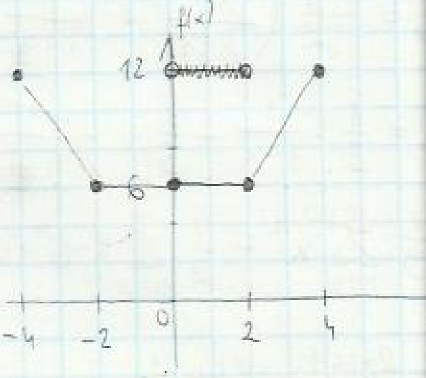
$\boxed{Z6}$

zad. 1.

a) $f(x) = \begin{cases} 6 & ; \ x \in (0,2) \\ 3x & ; \ x \in (2,4) \end{cases}$



rozszerzamy na funkcję parzystą

$f(0) = \frac{1}{2}\left(\lim_{x\to 0^+} f(x) + \lim_{x\to 0^-} f(x)\right) =$

$= \frac{1}{2}\cdot 12 = 6$

$f(2) = \frac{1}{2}\left(\lim_{x\to 2^-} f(x) + \lim_{x\to 2^+} f(x) = \frac{1}{2}\cdot 12 = 6\right.$

$f(-2) = \frac{1}{2}\left(\lim_{x\to -2^-} f(x) + \lim_{x\to -2^+} f(x)\right) = 6\right.$

$f(4) = f(-4) = \frac{1}{2}\left(\lim_{x\to 4^-} f(x) + \lim_{x\to -4^+} f(x)\right) = 12$

f parzysta $\Rightarrow b_n = 0$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\frac{\cos n\pi x}{l}\,dx$$

dla f nieparzystej $\int_{-l}^{l} f(x)\,dx = 0$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx$$

parzystej $\int_{-l}^{l} f(x)\,dx = 2\int_{0}^{l} f(x)\,dx$

$l = 4$

$b_n = 0$

$$a_n = \frac{2}{l}\int_{0}^{l} f(x)\cos\frac{n\pi x}{l}\,dx =$$

$$a_0 = \frac{2}{l}\int_{0}^{4} f(x)\,dx = \frac{1}{2}\left(\int_{0}^{2} 6\,dx + \int_{2}^{4} 3x\,dx\right) =$$

$$= \frac{1}{2}\,6x\Big|_0^2 + \frac{1}{2}\,\frac{3x^2}{2}\Big|_2^4 = 6 + 12 - 3 = 15$$

$$a_n = \frac{2}{4}\cdot\int_{0}^{4} f(x)\cos\frac{n\pi x}{4}\,dx = \frac{1}{2}\left(\int_{0}^{2} 6\cos\frac{n\pi x}{4}\,dx + \int_{2}^{4} 3x\cos\frac{n\pi x}{4}\,dx\right) =$$

$$= 3\cdot\frac{4}{n\pi}\cdot\sin\frac{n\pi x}{4}\Big|_0^2 + \frac{3}{2}\int_{2}^{4} x\cos\frac{n\pi x}{4}\,dx = (*)$$

$$\left(= \frac{12}{n\pi}\sin\frac{n\pi}{2} + \quad\right)$$

$$\int x\cos\frac{n\pi x}{4}\,dx = \left\{\begin{matrix} f = x & g' = \cos\frac{n\pi x}{4} \\ f' = 1 & g = \frac{4}{n\pi}\sin\frac{n\pi x}{4}\end{matrix}\right\} = \frac{4x}{n\pi}\sin\frac{n\pi x}{4} -$$

$$- \frac{4}{n\pi}\int \sin\frac{n\pi x}{4}\,dx = \frac{4x}{n\pi}\cdot\sin\frac{n\pi x}{4} + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4} + C_1 \quad C\in\mathbb{R}$$

$$(*) = \frac{12}{n\pi}\cdot\sin\frac{n\pi}{2} + \frac{3}{2}\left(\frac{4x}{n\pi}\cdot\sin\frac{n\pi x}{4}\Big|_2^4 + \frac{16}{n^2\pi^2}\cos\frac{n\pi x}{4}\Big|_2^4\right) =$$

$$= \underbrace{\frac{12}{n\pi}\cdot\sin\frac{n\pi}{2} - \frac{12}{n\pi}\sin\frac{n\pi}{2}}_{\text{skraca}} + \frac{16}{n^2\pi^2}\cos n\pi - \frac{16}{n^2\pi^2}\cos\frac{n\pi}{2}$$

$$a_n = \frac{16}{n^2\pi^2}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)$$

$$f^*(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} =$$

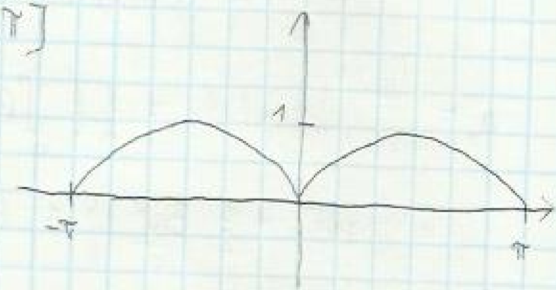
$$= \frac{15}{2} + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)\cdot\cos\frac{n\pi x}{4}$$] rozwinięcie tej funkcji

$$\sin x\cos y = \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)$$

b) $f(x) = |\sin x| \quad x \in [-\pi, \pi]$

warunki Dirichleta są spełnione

funkcja parzysta, $\Rightarrow b_n = 0$



$l = \pi$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx = \frac{2}{\pi}\int_0^{\pi} \sin x \cos(nx)\,dx =$$

$$= \frac{2}{\pi}\cdot\frac{1}{2}\int_0^{\pi}\left[\sin(x-nx) + \sin(x+nx)\right]dx = \frac{1}{\pi}\int_0^{\pi} \sin x(1-n) + \sin x(1+n)\,dx =$$

$$\overset{n\neq 1}{=} \frac{1}{\pi}\left(-\frac{1}{1-n}\cos x(1-n) - \frac{1}{1+n}\cos x(1+n)\right)\Big|_0^{\pi} =$$

$$= -\frac{1}{\pi}\left(\frac{1}{1-n}\cos\pi(1-n) + \frac{1}{1+n}\cos\pi(1+n) - \frac{1}{1-n} - \frac{1}{1+n}\right) =$$

$$= -\frac{1}{\pi}\left(\frac{1}{1-n}(-1)^{n+1} + \frac{1}{1+n}(-1)^{n+1} - \frac{1}{1-n} - \frac{1}{1+n}\right) =$$

$$= -\frac{1}{\pi}\left(\frac{1}{1-n}\left[(-1)^{n+1} - 1\right] + \frac{1}{1+n}\left[(-1)^{n+1} - 1\right]\right)$$

dla $n=2k$: $a_n = -\frac{1}{\pi}\left(\frac{-2}{1-n} + \frac{-2}{1+n}\right) = \frac{2}{\pi}\left(\frac{1}{1-n} + \frac{1}{1+n}\right)$

$n=2k+1$: $a_n = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\pi} = \frac{a_0}{2} + \sum_{k=1}^{\infty} \frac{2}{\pi}\left(\frac{1}{1-2k} + \frac{1}{1+2k}\right)\cdot \cos 2kx$$

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$$a_0 = \frac{2}{\pi}\int_0^{\pi} \sin x \, dx = \frac{2}{\pi}\left[-\cos x\right]_0^{\pi} = \frac{2}{\pi}\left[-(-1)-(1)\right] = \frac{4}{\pi}$$

$$f(x) = \frac{2}{\pi} + \sum_{k=1}^{\infty}\left(\frac{1}{2k+1} - \frac{1}{2k-1}\right)\cos kx \cdot \frac{2}{\pi}$$

$$\frac{2k-1-2k-1}{(2k+1)(2k-1)} = \frac{-2}{(2k+1)(2k-1)}$$

$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k+1)(2k-1)}\cos 2kx$$

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots + \frac{1}{(2k-1)(2k+1)} + \ldots$$

$x = 0$

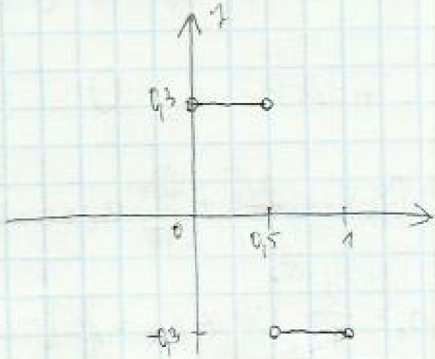
$$0 = f(0) = \frac{2}{\pi} - \frac{4}{\pi}\cdot\sum_{k=1}^{\infty}\frac{1}{(2k+1)(2k-1)}$$

$$\frac{4}{\pi}\cdot\sum_{k=1}^{\infty}\frac{1}{(2k+1)(2k-1)} = \frac{2}{\pi}$$

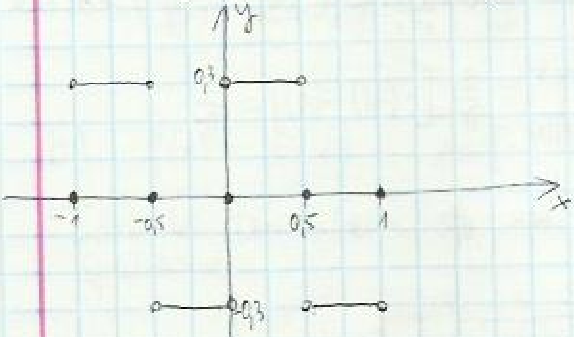
$$\sum_{k=1}^{\infty}\frac{1}{(2k+1)(2k-1)} = \frac{1}{2}$$

zad. 4.

$$f(x) = \begin{cases} 0{,}3 & \text{dla} \quad 0 < x < 0{,}5 \\ -0{,}3 & \text{dla} \quad 0{,}5 < x < 1 \end{cases}$$



a) sinusy $\Rightarrow$ $f(x)$ - nieparzysta



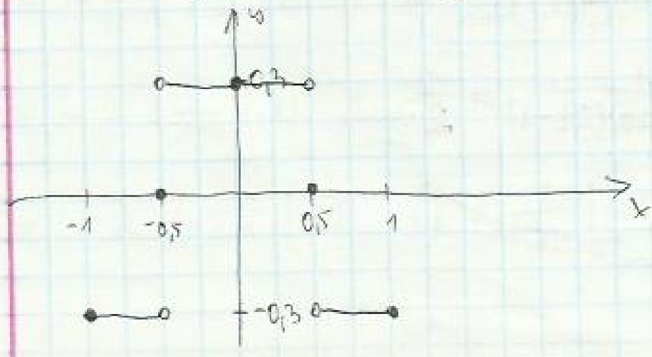
$$f(-1) = f(1) = \frac{\lim\limits_{x \to -1^+} f(x) + \lim\limits_{x \to 1^-} f(x)}{2} = 0$$

$$f(0{,}5) = f(0) = f(-0{,}5)$$

będą tylko b_n!, $a_n = 0$

$$b_n = \frac{2}{l}\left[\int_0^{1/2} 0{,}3 \cdot \sin n\pi x \, dx - \int_{1/2}^{1} 0{,}3 \sin n\pi x \, dx\right]$$

b) cosinusy $\Rightarrow$ $f(x)$ - parzysta



$$f(1)=f(-1)=0{,}3$$

$$f(0{,}5)=f(-0{,}5)=0$$

$$f(0)=0{,}3$$

$$a_0=\frac{2}{1}\left[\int_0^{1/2} 0{,}3\,dx - \int_{1/2}^{1} 0{,}3\,dx\right]$$

$$a_n=\frac{2}{1}\left[\int_0^{1/2} 0{,}3\cdot\cos n\pi x\,dx - \int_{1/2}^{1} 0{,}3\cos n\pi x\,dx\right]$$