$\boxed{27}$

zad. 5.

a) $f(z) = \dfrac{Re(z)}{1+|z|}$; $z = x+iy$ $\quad Re z = x$

$|z| = \sqrt{x^2+y^2}$

$$\lim_{z\to 0} \frac{Re z}{1+|z|} = \lim_{\substack{x\to 0 \\ y\to 0}} \frac{x}{1+\sqrt{x^2+y^2}} = 0$$

b) $f(z) = \dfrac{Re(z^2)}{z^2}$ $\quad z^2 = (x+iy)^2 = x^2+2xiy-y^2$

$$f(z) = \frac{Re(x+iy)^2 \cdot (x-iy)^2}{(x+iy)^2 \cdot (x-iy)^2} = \frac{(x^2-y^2)(x^2-y^2-2ixy)}{(x^2+y^2)^2} =$$

$$= \frac{(x^2-y^2)^2}{(x^2+y^2)^2} - \frac{2ixy(x^2-y^2)}{(x^2+y^2)^2}$$

$$\lim_{z\to 0} f(z) = \lim_{\substack{x\to 0\\ y\to 0}} \left(\frac{(x^2-y^2)^2}{(x^2+y^2)^2} - 2i \frac{xy(x^2-y^2)}{(x^2+y^2)} \right)$$

$$\frac{(x^2-y^2)^2}{(x^2+y^2)^2} \qquad \begin{matrix} x_n = \frac{1}{n} \\ y_n = 0 \end{matrix} \to \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = 1$$

$$\neq$$

$$\begin{matrix} x_n = \frac{1}{n} \\ y_n = \frac{1}{n} \end{matrix} \to \frac{0}{\left(\frac{2}{n^2}\right)^2} = 0$$

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zad. 4.

a) $$f(z) = \frac{1}{z^2} = \frac{(x-iy)^2}{(x+iy)^2(x-iy)^2} = \frac{(x-iy)^2}{(x^2+y^2)^2} =$$

$$= \underbrace{\frac{x^2-y^2}{(x^2+y^2)^2}}_{Re(f(z))} - i \underbrace{\frac{2xy}{(x^2+y^2)^2}}_{Im(f(z))}$$

c) $$f(z) = \cos z = \frac{e^{iz} + e^{-iz}}{2} = \frac{e^{i(x+iy)} + e^{-i(x+iy)}}{2} =$$

$$= \frac{e^{ix} \cdot e^{-y} + e^{-ix} \cdot e^{y}}{2} = \frac{(\cos x + i \sin x)e^{-y} + (\cos(-x) + i\sin(-x)) \cdot e^{y}}{2} =$$

$$= \frac{e^{-y} \cos x + i \cdot e^{-y} \sin x + e^{y} \cos x - i e^{y} \sin x}{2} =$$

$$= \underbrace{\frac{\cos x\,(e^{y}+e^{-y})}{2}}_{Re(f(z))} + i\cdot \underbrace{\frac{\sin x\,(e^{-y}-e^{y})}{2}}_{Im(f(z))}$$

b) $$f(z)=\sin z = \frac{e^{ix}-e^{-ix}}{2i} = \frac{e^{i(x+iy)}-e^{-i(x+iy)}}{2i} =$$

$$= \frac{e^{ix}\cdot e^{-y} - e^{-ix}\cdot e^{y}}{2y} = \frac{(\cos x + i\sin x)\,e^{-y}\cdot i - (\cos(-x)+i\sin(-x))\,e^{y}\cdot i}{-2} =$$

$$= \frac{i\cos x\cdot e^{-y} \cancel{+ i} \sin x\cdot e^{-y} - i\cdot\cos x\cdot e^{y} - \sin x\, e^{y}}{-2} =$$

$$\cancel{=} \sin x\,(e^{y}+e^{-y})$$

$=$