POEL C15: Liniowe układy transmisyjne z czasem ciągłym i dyskretnym – odpowiedzi

1. a) $H(s) = \frac{sRC}{1+sRC}$

   $h(t) = L^{-1}[H(s)] = \delta(t) - \frac{1}{RC} e^{-\frac{t}{RC}} \cdot \mathbb{1}(t)$

   $r(t) = L^{-1}[\frac{H(s)}{s}] = e^{-\frac{t}{RC}} \cdot \mathbb{1}(t)$

   $A(\omega) = \frac{1}{\sqrt{1+\frac{1}{\omega^2 R^2 C^2}}}$

   b) $H(s) = \frac{R}{R+sL}$

   $h(t) = L^{-1}[H(s)] = \frac{R}{L} \cdot e^{-t\frac{R}{L}} \cdot \mathbb{1}(t)$

   $r(t) = L^{-1}[\frac{H(s)}{s}] = (1 - e^{-t\frac{R}{L}}) \cdot \mathbb{1}(t)$

   $A(\omega) = \frac{1}{\sqrt{1+\frac{1}{\frac{\omega^2 L^2}{R^2}}}}$

2. $H(s) = \frac{1+s\alpha RC}{s^2R^2C^2+s(3RC-\alpha RC)+1}$

   $\alpha_g = 3$

   - $A(\omega) = \frac{\sqrt{1+36\omega^2}}{\sqrt{1-4\omega^2}}$
   - $\varphi(\omega) = \begin{cases} \arctan(6\omega), & 0 < \omega < \frac{1}{2} \\ \arctan(6\omega) - \pi, & \omega > \frac{1}{2} \end{cases}$

   $\alpha = 2$

   - $A(\omega) = \frac{\sqrt{1+16\omega^2}}{\sqrt{1-4\omega^2+16\omega^4}}$
   - $\varphi(\omega) = \begin{cases} \arctan(4\omega) - \arctan(\frac{2\omega}{1-4\omega^2}), & 0 < \omega < \frac{1}{2} \\ \arctan(4\omega) - \arctan(\frac{2\omega}{1-4\omega^2}) - \pi, & \omega > \frac{1}{2} \end{cases}$

3. $H(s) = \frac{s}{s^2+(3-k)s+1}$

   $k = 1$

   - $A(\omega) = \frac{|\omega|}{1+\omega^2}$
   - $\varphi(\omega) = \frac{\pi}{2} - \arctan(\frac{2\omega}{1-\omega^2})$

   $k = 3$

   - $A(\omega) = \frac{|\omega|}{1-\omega^2}$
   - $\varphi(\omega) = \begin{cases} \frac{\pi}{2}, & 0 < \omega < 1 \\ -\frac{\pi}{2}, & \omega > 1 \end{cases}$

4. $H(z) = \frac{z^2}{(z-1)^2}$

   $h[n] = (n+1) \cdot \mathbb{1}[n]$

5. $y[n] = \frac{3}{4}x[n] - \frac{1}{8}x[n-1] + \frac{1}{4}y[n-1]$

   $h[n] = \frac{3}{4}(\frac{1}{4})^n \cdot \mathbb{1}[n] - \frac{1}{8}(\frac{1}{4})^{n-1} \cdot \mathbb{1}[n-1]$