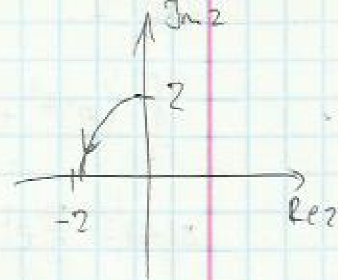
[Z9]

zad. 1.

b) $\int_L z\cdot|z|\,dz = \int_{\frac{\pi}{2}}^{\pi} 2e^{it}\cdot 2\cdot 2ie^{it}\,dt =$

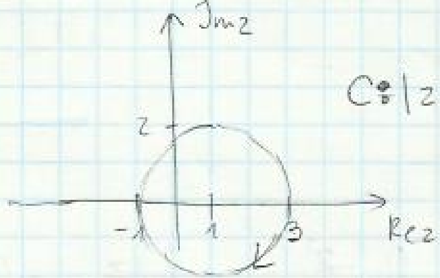


$= 8i\cdot\int_{\frac{\pi}{2}}^{\pi} e^{2it}\,dt = 8i\left(\frac{e^{2it}}{2i}\right)\Big|_{\frac{\pi}{2}}^{\pi} =$

$L = \begin{cases} t\in[\frac{\pi}{2},\pi] \\ z = \underset{0}{z_0} + \underset{2}{r}e^{it} = 2e^{it} \end{cases}$

$= 4\cdot(\cos 2t + i\sin 2t)\Big|_{\frac{\pi}{2}}^{\pi} = 4(1-(-1)) = 8$

$dz = i2e^{it}\,dt$



c) $\oint_C \frac{|z|^2}{z-1}\,dz = (*)$

$C: |z-1| = 2$

$z = 1+2e^{it}$

$z = \underbrace{1+2\cos t}_{Re\,z} + \underbrace{2\sin t}_{Im\,z}\cdot i$

$C = \begin{cases} t\in[2\pi,0] \\ z = 1+2e^{it} \end{cases}$

$dz = i2e^{it}\,dt$

$|z| = \sqrt{(1+2\cos t)^2+(2\sin t)^2} =$

$= \sqrt{1+4\cos t+4\cos^2 t+4\sin^2 t} =$

$= \sqrt{1+4\cos t+4} = \sqrt{5+4\cos t}$

← nie używamy Tw. Cauchy'ego, bo $|z|^2$ nie jest holomorficzna

$(*)\ \int_{2\pi}^{0} \frac{5+4\cos t}{2e^{it}}\,2ie^{it}\,dt = i\int_{2\pi}^{0}(5+4\cos t)\,dt =$

$= i(5t+4\sin t)\Big|_{2\pi}^{0} = -10\pi i$

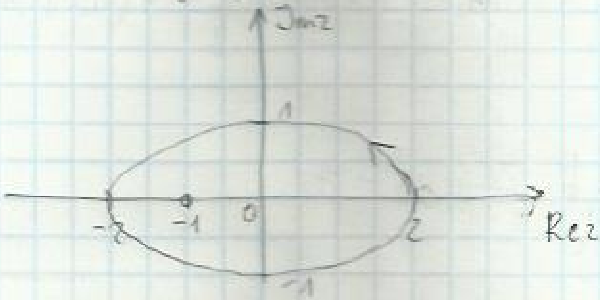
zad. 2.

$$\oint_C \left(e^{-z^2} + \frac{z}{z+1}\right) dz = (*) \qquad C = \{z : z(t) = \frac{3}{2}e^{it} + \frac{1}{2}e^{-it}, t\in[0,2\pi]\}$$

$$z(t) = \frac{3}{2}e^{it} + \frac{1}{2}e^{-it} = \frac{3}{2}\cos t + i\frac{3}{2}\sin t + \frac{1}{2}\cos(-t) + i\frac{1}{2}\sin(-t) =$$

$$= 2\cos t + i\sin t$$

$$\begin{cases} x(t) = 2\cos t \\ y(t) = \sin t \end{cases}$$ – zapis parametryczny elipsy



$$(*) = \oint_C \underbrace{e^{-z^2}}_{\text{holomorficzna}} dz + \oint_C \frac{z}{z+1} dz \overset{\text{Tw. C.}}{=} 0 + \oint_C \frac{z}{z+1} dz = (**)$$

$f(z) = z \rightarrow$ holomorficzna, więc możemy tw. Cauchy'ego zatem

$$(**) = 2\pi i \cdot \underset{\substack{\shortparallel \\ -1}}{z_0} = -2\pi i$$

Tw. Cauchy'ego $\oint_C \frac{f(z)}{z-z_0} = (*)$ $\rightarrow$ holomorficzna w C, z_0 – punkt wewnętrzny

$(*) = 2\pi i f(z_0)$

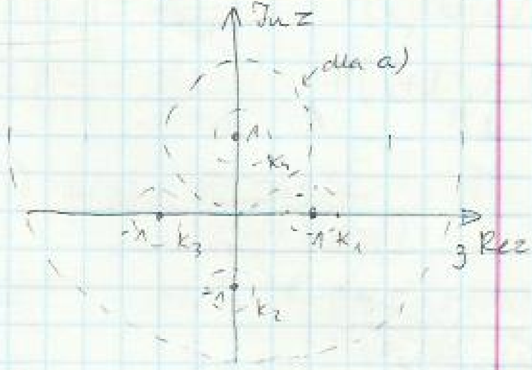
$\oint$

zad. 3.

$$\oint_C \frac{z}{z^4-1}\,dz = \oint_C \overbrace{\frac{z}{(z-1)(z+1)(z-i)(z+i)}}^{=f(z)}\,dz$$

a) $K(i;1)$

$$\oint_{K(i;1)} f(z) = \oint_{K(i;1)} \frac{\frac{z}{(z-1)(z+1)(z+i)}}{z-i} =$$



$$= 2\pi i \cdot \frac{\cancel{z}\, i}{(i-1)(i+1)(i+i)} =$$

$$= 2\pi i \cdot \frac{i}{-2\cdot 2i} = -\frac{1}{2}\pi i$$

b) $K(i;3)$

$$\oint_{K(i;3)} f(z)dz = \oint_{K_1} f(z) + \oint_{K_2} f(z) + \oint_{K_3} f(z) + \oint_{K_4} f(z) =$$

$$= \oint_{K_1(1;r_1)} \frac{\overbrace{\frac{z}{(z+1)(z-i)(z+i)}}^{\text{holomorf. w } K_1}}{z-1} + \dots =$$

c) $K(3;1)$

$$\oint_{K(3;1)} f(z)dz \overset{\text{Tw. p.C.}}{=} 0$$

d) (jak w b)

zad. 4. next time
(uogólnione tw. Cauchy'ego)

01.06.2015 Zad. 4.

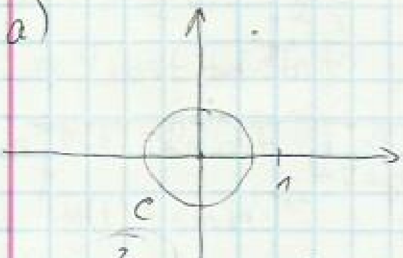
$$\oint_C \frac{e^z}{z^2(1-z)^3}\,dz$$

holomorficzne w C

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

$$\oint_C \frac{f(z)}{(z-z_0)^{1}}\,dz = 2\pi i \cdot f(z_0)$$

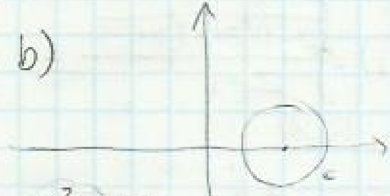
a)



holo w C

$$\oint_C \frac{\frac{e^z}{(1-z)^3}}{(z-0)^2}\,dz \underset{(n=1)}{=} \frac{2\pi i}{n!} f'(0) =$$

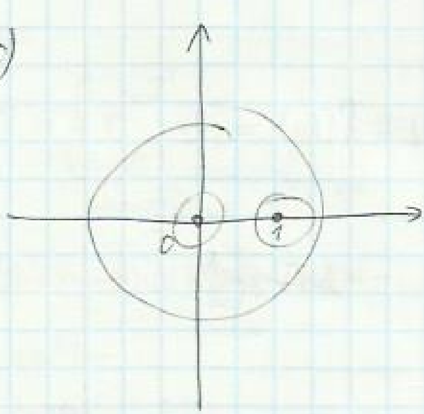
b)



holo w C

$$\oint \frac{\frac{-e^z}{z^2}}{(z-1)^3}\,dz \underset{n=2}{=} \frac{2\pi i}{2!} f''(1)$$

c)



suuuma

a) + b)