MECE 6374: Fun Work #1

Eric Eldridge (1561585)

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1

Write the complete response of the following unforced scalar linear system. Confirm that it represents a solution.

$$\dot{x} = -4x, \qquad x(0) = 2$$

Solution

$$\frac{\dot{x}}{x} = -4 \tag{1.1}$$
$$\int \frac{\dot{x}}{x} dt = \int -4dt \tag{1.2}$$
$$ln(x) = -4t + C \tag{1.3}$$
$$x = e^{-4t+C} = e^{-4t}e^{C} \tag{1.4}$$
$$x = Ce^{-4t} \tag{1.5}$$
$$x(0) = Ce^{-4*0} = 2 \tag{1.6}$$
$$C = 2 \tag{1.7}$$
$$x(t) = 2e^{-4t} \tag{1.8}$$

To confirm that this represents a solution, let's plug x(t) into our system and confirm that the system holds.

$$\frac{\dot{x}}{x} = -4 \tag{1.1}$$
$$x(t) = 2e^{-4t} \tag{1.8}$$
$$\dot{x}(t) = -8e^{-4t} \tag{1.9}$$

Plugging into

$$\frac{-8e^{-4t}}{2e^{-4t}} = -4 \tag{1.10}$$

and we can see that the equation holds.

2

Write a state-space representation of the following dynamic systems:

a) $4\ddot{q} + 5\dot{q} + 8q = 0$

Let $q_1 = q$ and $q_2 = \dot{q}$ We can plug q_1 and q_2 into equation (a) to get

$$4\dot{q_2} + 5q_2 + 8q_1 = 0 \quad (2.1)$$
$$4\dot{q_2} = -5q_2 - 8q_1 \quad (2.2)$$
$$\dot{q_2} = \frac{-5}{4}q_2 - 2q_1 \quad (2.3)$$

We solved for q_2 above and we know $\dot{q_1} = q_2$
Let $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$. Then

$$\dot{q_1} = q_2 \quad (2.4)$$
$$\dot{q_2} = -\frac{5}{4}q_2 - 2q_1 \quad (2.5)$$

$$\begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -\frac{5}{4} \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad (2.6)$$

.

b) $4\ddot{q} + 5\dot{q} + 8q^3 = 0$

Let $q_1 = q$ and $q_2 = \dot{q}$ We can plug q_1 and q_2 into equation (b) to get

$$4\dot{q_2} + 5q_2 + 8q_1^3 = 0 \quad (2.7)$$
$$4\dot{q_2} = -5q_2 - 8q_1^3 \quad (2.8)$$
$$\dot{q_2} = -\frac{5}{4}q_2 - 2q_1^3 \quad (2.9)$$

$$\dot{q_1} = q_2 \quad (2.10)$$
$$\dot{q_2} = -\frac{5}{4}q_2 - 2q_1^3 \quad (2.11)$$

This can not be split up any further as in part a) because of the non-linearity of the system.

c) $2\dddot{q} + 4\ddot{q} + 5\dot{q} + 8q = 0$

Let $q_1 = q$ and $q_2 = \dot{q} and q_3 = \ddot{q}$ We can plug q_1, q_2, and q_3 into equation (c) to get

$$2\dot{q_3} + 4q_3 + 5q_2 + 8q_1 = 0 \tag{2.12}$$
$$2\dot{q_3} = -4q_3 - 5q_2 - 8q_1 \tag{2.13}$$
$$\dot{q_3} = -2q_3 - \frac{5}{2}q_2 - 4q_1 \tag{2.14}$$

$$\dot{q_1} = q_2 \tag{2.15}$$
$$\dot{q_2} = q_3 \tag{2.16}$$
$$\dot{q_3} = -4q_1 - \frac{5}{2}q_2 - 2q_3 \tag{2.17}$$

$$\begin{bmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -\frac{5}{2} & -2 \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{2.18}$$

3

Find the equilibrium points of the following systems. Show the equilibrium points on the phase plane.

a)

$$\begin{aligned} \dot{x_1} &= -2x_1 + x_1 x_2 \\ \dot{x_2} &= x_2 - 2x_1 x_2 \end{aligned}$$

The equilibrium point is found by setting $\dot{x_1} = \dot{x_2} = 0$

$$\begin{aligned} \dot{x_1} &= 0 \\ -2x_1 + x_1 x_2 &= 0 \\ x_1(-2 + x_2) &= 0 \\ x_1 = 0 \ \ or \ \ x_2 &= 2 \end{aligned} \tag{3.1}$$

$$\begin{aligned} \dot{x_2} &= 0 \\ x_2 - 2x_1 x_2 &= 0 \\ x_2(1 - 2x_1) &= 0 \\ x_2 = 0 \ \ or \ \ x_1 &= \frac{1}{2} \end{aligned} \tag{3.2}$$

The only intersection between these two sets of solutions is the point (0,0)

b)

$$\begin{aligned} \dot{x_1} &= x_1 - x_2 \\ \dot{x_2} &= 1 - x_1 x_2 \end{aligned}$$

The equilibrium point is found by setting $\dot{x_1} = \dot{x_2} = 0$

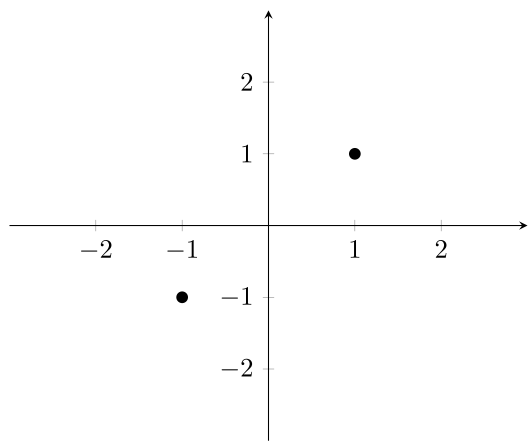
$$\begin{aligned} \dot{x_1} &= 0 \\ x_1 - x_2 &= 0 \\ x_1 &= x_2 \end{aligned} \tag{3.3}$$

$$\begin{aligned} \dot{x_2} &= 0 \\ 1 - x_1 x_2 &= 0 \\ x_1 x_2 &= 1 \end{aligned} \tag{3.4}$$

Plugging in (3.3) into (3.4)

$$\begin{aligned} x_1^2 &= 1 \\ x_1 &= \pm 1 \\ x_1 = x_2 \rightarrow x_2 &= \pm 1 \end{aligned} \tag{3.5}$$

There are two equilibrium points: (-1, -1) and (1, 1)



c)

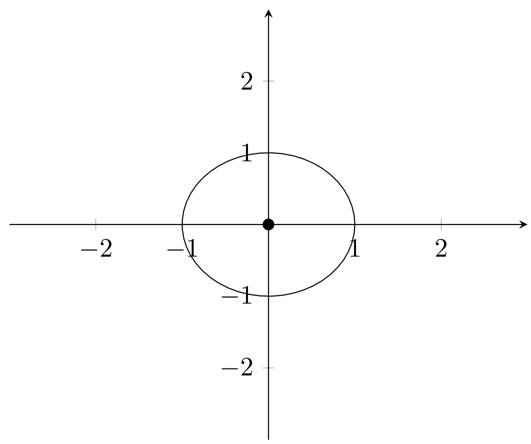
$$\dot{x_1} = x_2(x_1^2 + x_2^2 - 1)$$
$$\dot{x_2} = x_1(x_1^2 + x_2^2 - 1)$$

The equilibrium point is found by setting $\dot{x_1} = \dot{x_2} = 0$

$$\begin{aligned} \dot{x_1} &= 0 \\ x_2 = 0 \;\; or \;\; x_1^2 + x_2^2 &= 1 \end{aligned} \tag{3.6}$$

$$\begin{aligned} \dot{x_2} &= 0 \\ x_1 = 0 \;\; or \;\; x_1^2 + x_2^2 &= 1 \end{aligned} \tag{3.7}$$

The equilibrium points lie at the point $(0, \quad 0)$ and on the unit circle.



4

Consider the Van der Pol equation:

$$\ddot{y} - (1 - y^2)\dot{y} + y = 0$$

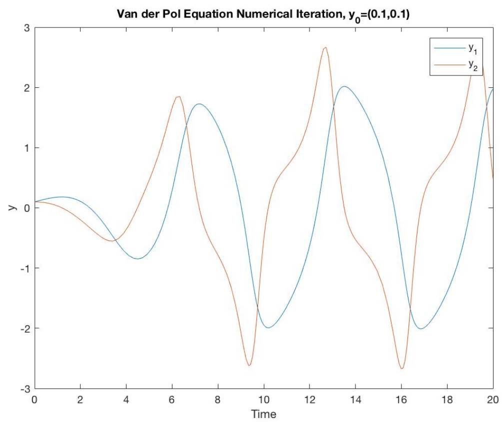
where $y = y(t)$. A state space representation of this equation is

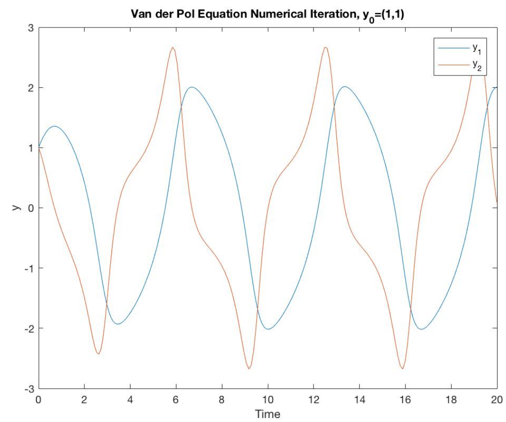
$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -x_1 + (1 - x_1^2)x_2 \end{aligned}$$

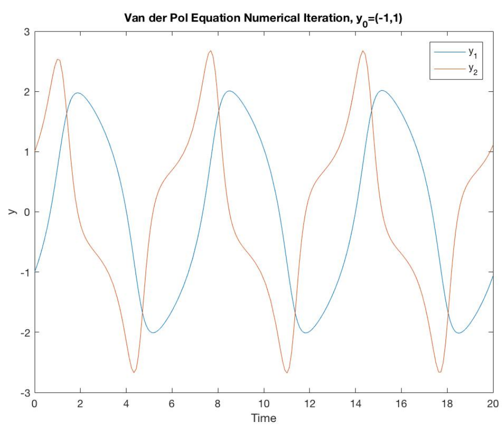
where $x_1 = y$ and $x_2 = \dot{y}$. Integrate numerically the state space equations with initial conditions

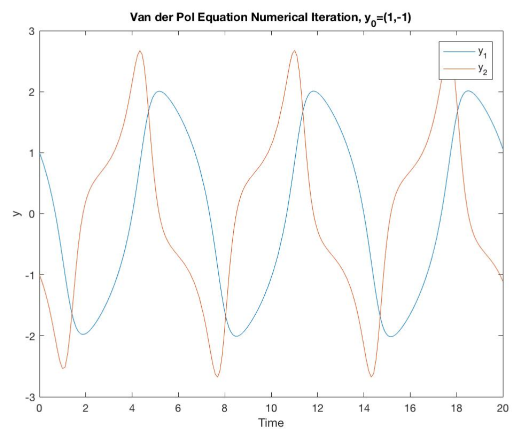
$$(x_1(0), x_2(0)) = (0.1, 0.1), (1, 1), (-1, 1), (1, -1), (-1, -1), (3, 3), (-3, 3), (3, -3), (-3, -3)$$

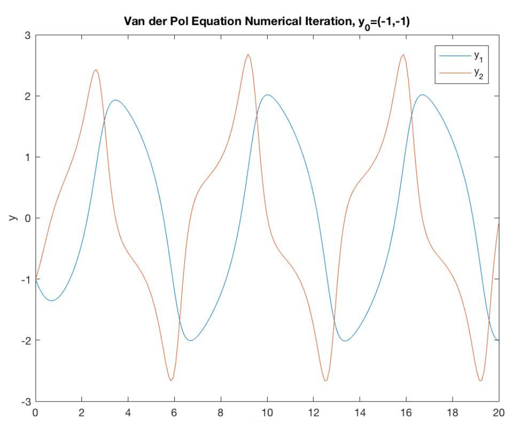
respectively to obtain a representation of the phase plane portrait.

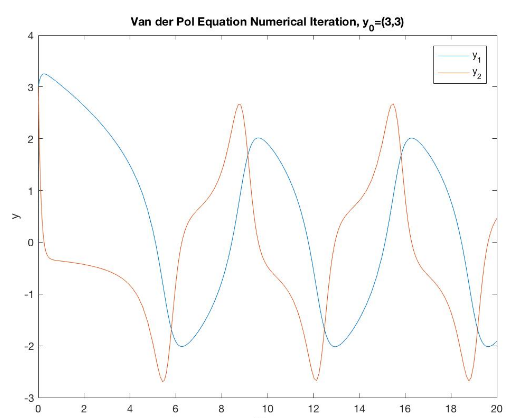

Van der Pol Equation Numerical Iteration, y_0=(0.1,0.1)
y_1
y_2
y
Time

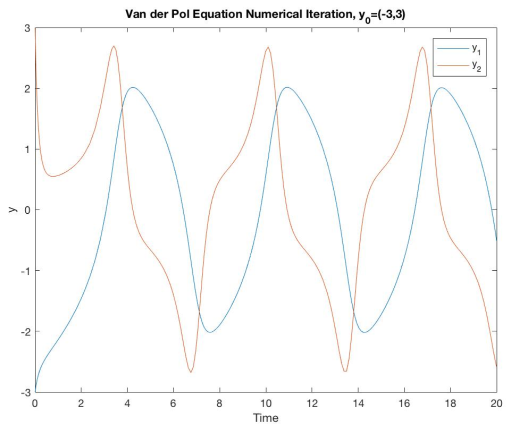

Van der Pol Equation Numerical Iteration, y_0=(1,1)
y_1
y_2
y
Time

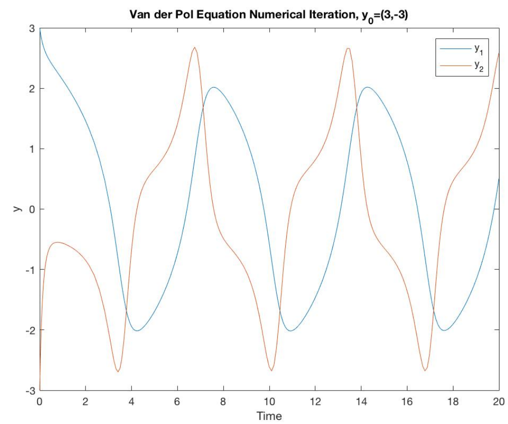

Van der Pol Equation Numerical Iteration, y_0=(-1,1)
y_1
y_2
y
Time

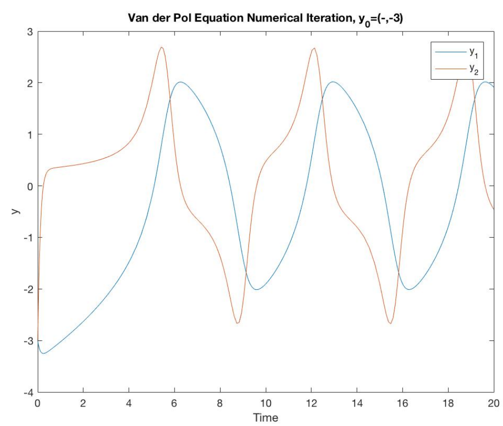

Van der Pol Equation Numerical Iteration, y_0=(1,-1)
y_1
y_2
y
Time


Van der Pol Equation Numerical Iteration, y_0=(-1,-1)
y_1
y_2
y
Time


Van der Pol Equation Numerical Iteration, y_0=(3,3)
y_1
y_2
y
Time


Van der Pol Equation Numerical Iteration, y_0=(-3,3)
y_1
y_2
y
Time


Van der Pol Equation Numerical Iteration, y_0=(3,-3)
y_1
y_2
y
Time


Van der Pol Equation Numerical Iteration, y_0=(-,-3)
y_1
y_2
y
Time

MATLAB Code

```
%HW#1 Problem 4

clear all
clc
close all

t = [0 20];
y_0 = [0.1, 0.1];
y_0 = [1,1];
y_0 = [-1,1];
y_0 = [1,-1];
y_0 = [-1,-1];
y_0 = [3,3];
%y_0 = [-3,3];
%y_0 = [3,-3];
%y_0 = [-3,-3];
%sol = ode45([y(2); (1-y(1)^2)*y(2)-y(1)],tspan,y_0)
[t,y] = ode45(@vdp_eqn,t,y_0);

plot(t,y(:,1),t,y(:,2))
title('Van der Pol Equation Numerical Iteration, y_0=(3,3)');
xlabel('Time');
ylabel('y');
legend('y_1','y_2')


function ydot = vdp_eqn(t,y)

ydot = [y(2); (1-y(1)^2)*y(2)-y(1)];
end
```