MECE 6374: Fun Work #10

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Problem 1

Consider the following state-space model of an inverted pendulum

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{g}{l} sin(x_1 + \frac{\pi}{2}) + \frac{1}{ml^2} u$$

where $l = 1m$, $m = 0.1kg$. The objective is to implement a sliding mode control that stabilizes the pendulum at the upright position.

(i) Show that the stability surface $\sigma(x) = x_1 + x_2$ is a good selection.

(ii) Simulate the sliding mode control

$$u(x) = -4sgn(x_1 + x_2)$$

for initial conditions $x_1 = 1$, $x_2 = 0$. Plot the states $x_1(t)$ and $x_2(t)$ as well as the control input u(t).

(iii) To eliminate chattering, modify the control law to

$$u(x) = -4sat(\frac{\sigma(x)}{\epsilon})$$

where ϵ is a small positive scalar. Repeat the closed-loop simulations of $x_1(t)$, $x_2(t)$, and $u(t)$.

Note: sat($\frac{\sigma(x)}{\epsilon}$) is the saturation function defined as:

$$\begin{cases} sgn(\sigma(x) & |\sigma(x)| \geq \epsilon \\ \frac{\sigma(x)}{\epsilon} & |\sigma(x)| \leq \epsilon \end{cases}$$

Solution

(i) To show that the stability surface $\sigma(x) = x_1 + x_2$ is a good selection we need to show that the surface $\sigma(x) = 0$ is stable.

$$\begin{aligned} & x_1 + x_2 = 0 \\ & x_2 = -x_1 \\ & \dot{x_2} = -\dot{x_1} = -x_2 \\ \implies & x_2(t) = x_2(0)e^{-t} \\ \implies & x_1(t) = -x_2(0)e^{-t} \end{aligned}$$

This shows that our stability surface is a good selection.

(ii) We want to simulate the sliding mode control $u(x) = -4sgn(x_1 + x_2)$ for our system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{l} sin(x_1 + \frac{\pi}{2}) + \frac{1}{ml^2} u \end{aligned}$$

This can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{l} sin(x_1 + \frac{\pi}{2}) + \frac{1}{ml^2}(-4sgn(x_1 + x_2)) \\ \dot{x}_2 &= -\frac{g}{l} cos(x_1) + \frac{1}{ml^2}(-4sgn(x_1 + x_2)) \end{aligned}$$