MECE 6374: Fun Work #7

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Problem 1

Consider the scalar system described by

$$\dot{x} = -x^3 + \frac{x^3}{2} sin(t)$$

Use a Lyapunov function $V(x) = \frac{1}{2}x^2$ to show global uniform asymptotic stability of the origin (show all details of work).

Solution

To show global uniform asymptotic stability, we must choose V(x,t) s.t.

(i) V(x,t) is decrescent

(ii) V(x,t) is gpd and R.U.

(iii) $\dot{V}(x,t)$ is gnd

A function will be called *decrescent* if $V(0,t) = 0$ and $\exists V_1(x)$ gpd s.t. $V(x,t) \leq V_1(x)$.
If we choose $V(x,t) = \frac{1}{4}(1 + sin^2(t))x^2$ we can see that

$$\begin{aligned} V(0,t) &= 0 \\ V(x,t) &= \frac{1}{4}(1 + sin^2(t))x^2 \leq V_1(x) = \frac{1}{2}x^2 \\ \implies & V(x,t) \textit{ is decrescent} \end{aligned}$$

We can see from observation that $V(x,t)$ is gpd and R.U
We now wish to show that $\dot{V}(x,t)$ is gnd

$$\begin{aligned} V(x,t) &= \frac{1}{4}(1 + sin^2(t))x^2 \\ \dot{V}(x,t) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\frac{dx}{dt} \\ \dot{V}(x,t) &= \frac{1}{2}sin(t)cos(t)x^2 + \frac{1}{2}(1 + sin^2(t))x \end{aligned}$$

Problem 2

Consider the non-autonomous linear system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - (2 + sin(t))x_1$$

Use $V(x) = x_1^2 + x_2^2/(2 + sin(t))$ as a Lyapunov function candidate show that the origin is a uniformly stable equilibrium point.

Solution