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Simulation of the Quadcopter Dynamics with LQR based Control

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Abstract

The present paper deals with simulation of an unmanned aerial vehicle (UAV) called quadcopter. An optimal control is developed for the position and yaw control of the quadcopter, based on linear quadratic regulator (LQR). The quadcopter dynamics describe its behaviour in three dimensional spaces. The developed LQR based controller applied on the quadcopter positions in longitudinal, lateral, and vertical directions, and orientation in yaw direction. Simulation studies are performed on the dynamic model of quadcopter, while applying the developed control strategy. Furthermore, to improve the obtained results, an integral compensation is induced for the position control in vertical direction.

Keywords: Quadcopter, Modeling, LQR, LQI, Control;

1. Introduction

In the last decades, the research in the area of drones is increased due to their interesting application in various fields. This paper discusses the modeling and control of a quadcopter UAV (drone). A Quadcopter has four propellers with four D.C. motor mounted at the end of each arm of its cross section frame. It also has on-board microcontroller and battery which provide required control and power to the motors. These motors provide the angular speeds to the respective propellers. Quadcopter is an under-actuated system, where six degrees of freedom are controlled by the four input actuators. Furthermore, rotation of propellers generates upward thrust forces, which enables the motion of quadcopter in three dimensional spaces.

Many researchers have contributed in the field of modeling and controlling of quadcopter. The existing literature provides the motivation to analyze the quadcopter dynamics with control to improve its performance. The

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common control strategies for quadcopter include proportional-derivative PD [1], proportional-integral-derivative PID [2-4], LQR [5], and Fuzzy [6, 7]. Among these control strategies, LQR provides optimal control, where a quadratic cost function is minimized. Hence, it is interesting to apply LQR based control on a quadcopter. L.M. Argentim et. al present the comparison between ITAE tuned PID, classical LQR and PID tuned LQR controller for vertical speed step response [5]. M. pena et. al introduce the LQR controller with integral effect to control the roll, pitch and yaw angle. Furthermore the linear system model was compared by the non linear system [8]. N.A. Ismail et. al controlled the attitude of quadrotor using the LQR controller furthermore the performance of the LQR controller was compared by the PD controller [9].

This paper applied LQR based control on positions and yaw orientation of the quadcopter. Simulation of the quadcopter dynamics with LQR control is performed to observe the performance of control, and an integral feedback is applied on the altitude to improve the response. Quadcopter is extremely complex because of its motion in 3-D space; LQR with integral term would help to reduce the steady state error [10]. The rest of paper is organized in the following sections. The detailed mathematical modeling with kinematics and dynamics of quadcopter is presented in Section 2. LQR based control is described in Section 3.In Section 4, simulation results are presented. Section 5 concludes the paper.

2. Modelling

In this section the mathematical model is obtained considering the kinematics and dynamics of the quadcopter. The schematic representation of the Quadcopter is shown in Figure 1.

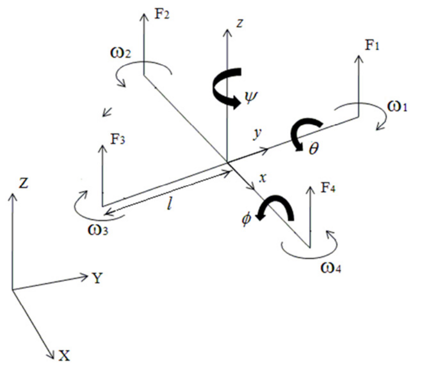

Fig. 1. Schematic representation of the quadcopter [11].

In Fig. 1, the four arms of the quadcopter are symmetric and at the end of each arm one DC motor is mounted to rotate the propeller. F_1, F_2, F_3 and F_4 are the upward thrust forces generated by the angular speeds ω_1, ω_2, ω_3, and ω_4 of DC motors, respectively. Roll, pitch and yaw angles about the *x*, *y* and *z* axes are denoted by ϕ, θ, *and* ψ, respectively. The arm length of the quadcopter frame is denoted by *l*.

2.1 Kinematics

In Fig. 1, the body fixed frame is represented by *x-y-z*; while the inertial frame is represented by *X-Y-Z*. The transformation between these two frames is represented by the transformation matrix *R* [11-13], which is obtained by the Euler angles (ϕ, θ, *and* ψ) about *x*, *y* and *z* axes.

$$[R]=\begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi - c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{1}$$

Where, $c\theta = cos\theta$ and $s\theta = sin\theta$.

2.2 Dynamics

Refer to Fig. 1, ω_1 and ω_3 are rotate in clockwise direction, while ω_2 and ω_4 rotate in anticlockwise direction to balance the torque about the z axis. The difference between ω_1 and ω_3 provide the torque T_x about the x axis.

$$T_x = (\omega_1^2 - \omega_3^2)K_t l \tag{2}$$

The difference between ω_2 and ω_4 provide the torque T_y about the y axis.

$$T_y = (\omega_2^2 - \omega_4^2)K_t l \tag{3}$$

Similarly, the yaw motion is obtained when there is difference in angular speeds of two pairs of propellers (one pair in clockwise rotation and other pair in anticlockwise rotation). Mathematically, it is given by:

$$T_z = (\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)K_d \tag{4}$$

Where, K_t and K_d are the coefficients which depend on air density, propellers geometry and blade radius.
According to Euler equations, the torques about x, y and z axes are represented by the following equation.

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} I_x\ddot{\phi} \\ I_y\ddot{\theta} \\ I_z\ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} I_x\dot{\phi} \\ I_y\dot{\theta} \\ I_z\dot{\psi} \end{bmatrix} \tag{5}$$

According to Newton's equation:

$$m\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = [R]\begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{6}$$

Where, $F = F_1 + F_2 + F_3 + F_4$ and $F_i = K_t\omega_i^2$ (i=1, 2, 3, and 4). The parameters m and g represent mass of the quadcopter and acceleration due to gravity, respectively. Using equation 1 to 6 following equation of the motion is obtained [11, 13].

$$m\ddot{X} = (F_1 + F_2 + F_3 + F_4)(c\phi s\theta c\psi + s\phi s\psi) \tag{7}$$

$$m\ddot{Y} = (F_1 + F_2 + F_3 + F_4)(c\phi s\theta s\psi + s\phi c\psi) \tag{8}$$

$$m\ddot{Z} = (F_1 + F_2 + F_3 + F_4)(c\phi c\theta) - mg \tag{9}$$

$$I_x\ddot{\phi} = (F_1 - F_3)l + \dot{\theta}\dot{\psi}(I_y - I_z) \tag{10}$$

$$I_y\ddot{\theta} = (F_2 - F_4)l + \dot{\psi}\dot{\phi}(I_z - I_x) \tag{11}$$

$$I_z\ddot{\psi} = (M_2 + M_4 - M_1 - M_3) + \dot{\phi}\dot{\theta}(I_x - I_y) \tag{12}$$

These equations represent the non-linear dynamic model of the quadcopter.

2.3 State space model

The model is linearized about the equilibrium hovering point, and the linear system of equation is given as follows.

$$\ddot{X} = \frac{F}{m}\theta \tag{13}$$

$$\ddot{Y} = \frac{F}{m}\phi \tag{14}$$

$$\ddot{Z} = \frac{F}{m} - g \tag{15}$$

$$\ddot{\phi} = \frac{T_x}{I_x} \tag{16}$$

$$\ddot{\theta} = \frac{T_y}{I_y} \tag{17}$$

$$\ddot{\psi} = \frac{T_z}{I_z} \tag{18}$$

Let

$$\begin{aligned}
x_1 &= X & x_2 &= \dot{x}_1 = \dot{X} \\
x_3 &= Y & x_4 &= \dot{x}_3 = \dot{Y} \\
x_5 &= Z & x_6 &= \dot{x}_5 = \dot{Z} \\
x_7 &= \phi & x_8 &= \dot{x}_7 = \dot{\phi} \\
x_9 &= \theta & x_{10} &= \dot{x}_9 = \dot{\theta} \\
x_{11} &= \psi & x_{12} &= \dot{x}_{11} = \dot{\psi}
\end{aligned} \tag{19}$$

To apply LQR control; the model need to be in state space form as given in the equation(20).

$$\begin{aligned}
\dot{x}_v &= Ax_v + Bu \\
y_v &= Cx_v + Du
\end{aligned} \tag{20}$$

Where, x_v, u, and y_v are state, input, and output vectors. The quadcopter dynamics is described by the twelve states as given in the equation (19).Hence, the state space model for the considered quadcopter dynamics is given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{Y} \\ \ddot{Y} \\ \dot{Z} \\ \ddot{Z} \\ \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ Y \\ \dot{Y} \\ Z \\ \dot{Z} \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{m} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & \frac{1}{I_x} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{I_y} & 0 \\
0 & 0 & 0 & \frac{1}{I_z}
\end{bmatrix} \begin{bmatrix} F \\ T_x \\ T_y \\ T_z \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ Y \\ \dot{Y} \\ Z \\ \dot{Z} \\ \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} F \\ T_x \\ T_y \\ T_z \end{bmatrix} \tag{22}$$

3. LQR control

LQR is an optimal control approach as shown in Fig.2, in which a quadratic cost function J is minimized.

$$J = \int_0^\infty (x_v^T Q x_v + u^T R u) dt \tag{23}$$

Where, Q(size $n \times n$, n is number of states)and R (size $m \times m$, m is number of inputs) are positive definite symmetric matrices.

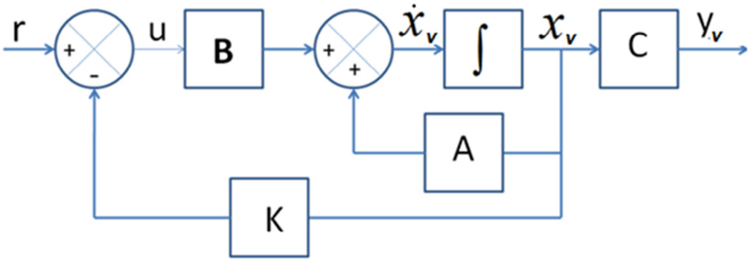

Fig. 2. LQR control diagram.

$$u = -Kx_v + r \tag{24}$$

Where, u is the control input and r is the reference input, and the LQR gain vector K is represented as:

$$K = R^{-1} B^T P \tag{25}$$

Where, P is a positive definite symmetric constant matrix. P is obtained from the Reccati equation as given below.

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{26}$$

Matrices Q and R are taken as follows: $Q = C' * C$ and $R = diag([1 \quad 1 \quad 1 \quad 1])$.

Furthermore, an integral feedback can be added to LQR in order to improve the performance of control by minimizing the steady-state error. LQR with integral feedback is shown in Fig. 3.

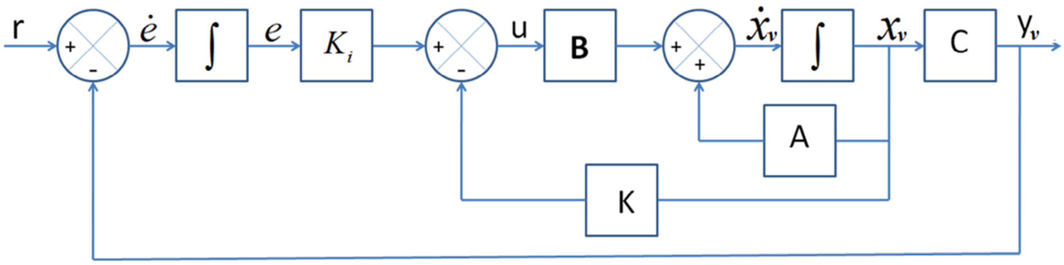

Fig. 3. LQR control with integral feedback.

The controlled input is given by:

$$u = -Kx_v + K_i e \tag{27}$$

Where, $\dot{e} = r - y_v$ and and K_i is the integral gain. Now, the new state space model can be given as follows:

$$\begin{bmatrix} \dot{x}_v \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_v \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$
$$y_v = [C \quad 0] \begin{bmatrix} x_v \\ e \end{bmatrix} \tag{28}$$

4. Simulation results

In this section, the developed model is simulated in Matlab. The results are obtained for the position (*X*, *Y*, and *Z*) and orientation (yaw ψ) of the quadcopter. The controlled input vector *u* develops necessary roll φ and pitch θ to obtain the desired positions (*X* and *Y*). The parameters values for simulation are given in Table 1.

Table 1. Parameters for simulation.

Parameter	Symbol	Numerical value
Distance between the center of Quadcopter to the center of propeller (m)	l	0.2
Thrust coefficient	K_t	3×10^{-6}
Mass of the Quadcopter (Kg)	m	1
Gravitational acceleration (m/s^2)	g	9.81
Body moment of inertia about the *x*-axis (Kg-m^2)	I_x	0.11
Body moment of inertia about the *y*-axis (Kg-m^2)	I_y	0.11
Body moment of inertia about the *z*-axis (Kg-m^2)	I_z	0.04
Drag coefficient	K_d	4×10^{-9}

The aim is to control position and orientations of the quadcopter i.e., *X*, *Y*, *Z*, and ψ. The desired values of parameters for controlled position of the quadcopter are as follows: *X*=5 m, *Y*=7 m, *Z*=10 m, and ψ=0 rad. It is desirable that the quadcopter achieve the desired positions in less than 2 s rise time. Using LQR based controller, we get the following results.

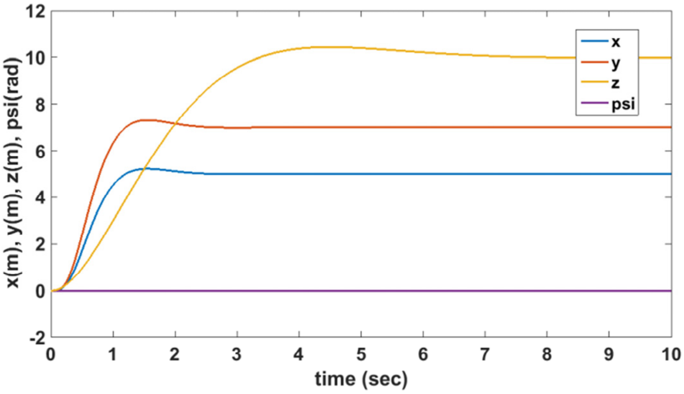

Fig. 4. Response of LQR control.

In the above results from Fig.4, quadcopter obtains desired *X* any *Y* positions and orientation within settling time of 2.5 s, and rise time of less than 1.5 s. But, the position *Z* takes large rise time more than 3.5 s, which is not desirable. Therefore, to reduce the rise time of *Z*, we modified numerical value in *Q* matrix corresponding to element (5, 5) which represents weight on *Z* position.

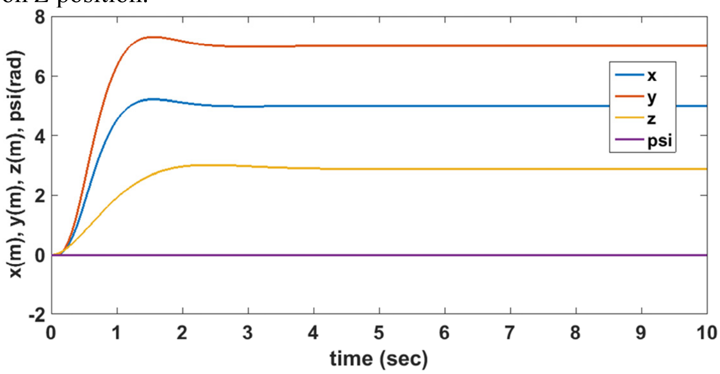

Fig. 5. Response after modifying the weight on *Z* position in *Q* matrix.

In Fig. 5, the rise time for Z is less than 2 s. But, there is big steady-state error in Z position. It is required to solve this problem; therefore, an integral feedback is applied on the Z position only.

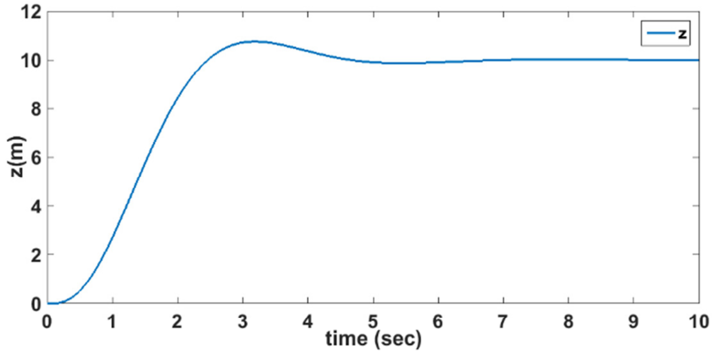

Fig. 6. Response after applying the integral feedback on Z position.

In Fig. 6, Z position (altitude) of quadcopter is shown after applying the integral action. It can be observed that the steady-state error in altitude is eliminated.

5. Conclusion

This paper presents detailed model of the quadcopter dynamics. LQR based controller is developed to control the positions and yaw orientation of the quadcopter. Simulation was performed to analyze the performance of the developed controller. It is observed that steady-state error in altitude can be eliminated by applying an integral feedback in LQR control. For future work, it will be interesting to develop detailed modeling of actuators and aerodynamic effects.

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