Chapter-5

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5.5 $\min\ c^Tx \quad \text{s.t.}\quad Gx \preceq h,\ Ax = b$

$$L(x,\lambda,v) = c^Tx + \lambda(Gx-h) + v(Ax-b)$$

$$G(x,\lambda,v) = -(\lambda h + vb) + (c + \lambda G + vA)x.$$

Thus, $\max\ -(\lambda h + vb) \quad \text{s.t.}\quad c + \lambda G + vA = 0,\ \lambda \succeq 0.$

5.10 (a) $\min\ \log\det\left(\sum_{i=1}^{p} x_i v_i v_i^T\right)^{-1} \quad \text{s.t.}\quad x \succeq 0,\ \mathbf{1}^Tx = 1$

$$L(x,\lambda,v) = \log\det\left(\sum_{i=1}^{p} x_i v_i v_i^T\right)^{-1} - \lambda x + v(\mathbf{1}^Tx - 1).$$

$$= \log\det(X^{-1}) + tr(ZX) + \sum_{i=1}^{p} x_i(-v_i^TZv_i - z_i + v)$$

$$g(Z,z) = \begin{cases} \log\det Z + n - v, & v - v_i^TZv_i = z_i \\ -\infty, & o.w. \end{cases}$$

Thus, $\max\ \log\det Z + n - v \quad \text{s.t.}\quad v_i^TZv_i \leq v.$

5.11 $\min \sum_{i=1}^{N} \|A_ix + b_i\|_2 + \frac{1}{2}\|x - x_0\|_2^2.$

Introducing new variables $y_i = A_ix + b_i$,

$$\min \sum_{i=1}^{N} \|y_i\|_2 + \frac{1}{2}\|x - x_0\|_2^2 \quad \text{s.t.}\ y_i = A_ix + b_i$$

$$\Rightarrow \min \sum_{i=1}^{n} \|y_i\|_2 + \frac{1}{2}\|x - x_0\|_2^2 - \sum_{i=1}^{n} z_i^T(y_i - A_ix - b_i).$$

$$\inf_{y_i}(\|y_i\|_2 + z_i^Ty_i) = \begin{cases} 0, & \|z_i\|_2 \leq 1 \\ \infty, & o.w. \end{cases}$$

Min. over x,

$$x - x_0 - ZAx = 0$$

$$\Rightarrow x = ZAx + x_0$$

$$\Rightarrow g(z_1,\dots,z_N) = \begin{cases} \sum_{i=1}^{N}(A_ix_0 + b_i)^Tz_i - \frac{1}{2}\|\sum_{i=1}^{N} A_i^Tz_i\|_2^2, & \|z_i\|^2 \leq 1. \\ -\infty, & o.w. \end{cases}$$

$$\max \sum_{i=1}^{N}(A_ix_0 + b_i)^Tz_i - \frac{1}{2}\|\sum_{i=1}^{N} A_i^Tz_i\|^2 \quad \text{s.t.}\ \|z_i\|_2 \leq 1.$$

X ——— X ———→ X