Hashing Technique

Keys - 8, 3, 6, 10, 15, 18, 4

| 8 | 3 | 6 | 10 | 15 | 18 | 4 | Linear Search O(n)

| 3 | 4 | 6 | 8 | 10 | 15 | 18 | Binary Search O(log n)

| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
0   1   2   3   4   5   6   7   8   9 10 11 12 13 14 15 16 17 18 19

O(1) time!

Drawback - Space complexity proportional to max elements.

Mappings:

One - one   — $h(x) = x$ (ideal hashing)
One - many   —
Many - one   — $h(x) = x \% 10$
Many - many —
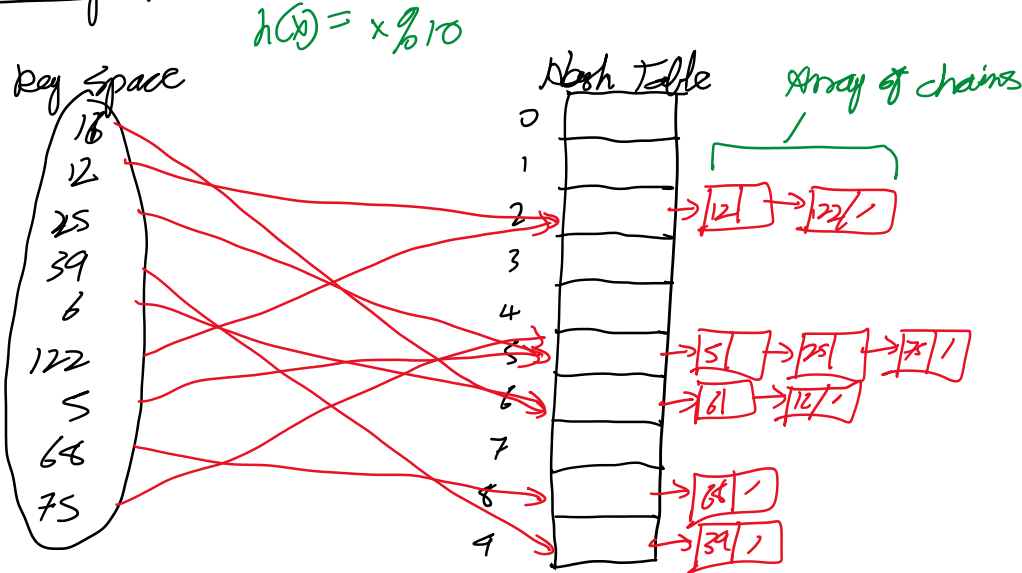
Many-one mapping results in collisions!

Open Hashing:
    Chaining

Closed Hashing:
    Open Addressing
    1. Linear Probing
    2. Quadratic Probing
    3. Double Hashing

Chaining. $h(x) = x \% 10$

Key Space / Hash Table / Array of chains

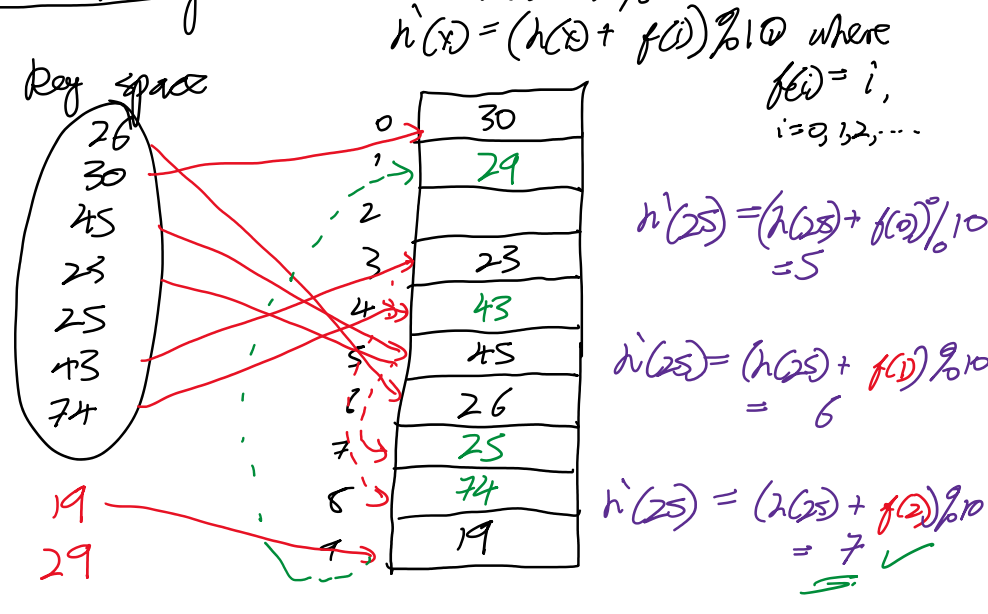Loading factor $(\lambda) = \dfrac{n}{size}$.

for n = 100,
$\lambda = \dfrac{100}{10} = 10$

λ indicates the average number of keys for each location.

Avg. time for successful search $= 1 + \dfrac{\lambda}{2}$.

Avg time for unsuccessful search $= 1 + \lambda$

Linear Probing. $h(x) = x \% 10$

$h'(x) = (h(x) + f(i)) \% 10$ where
$f(i) = i$,
$i = 0, 1, 2, \cdots$

$h'(25) = (h(25) + f(0)) \% 10 = 5$

$h'(25) = (h(25) + f(1)) \% 10 = 6$

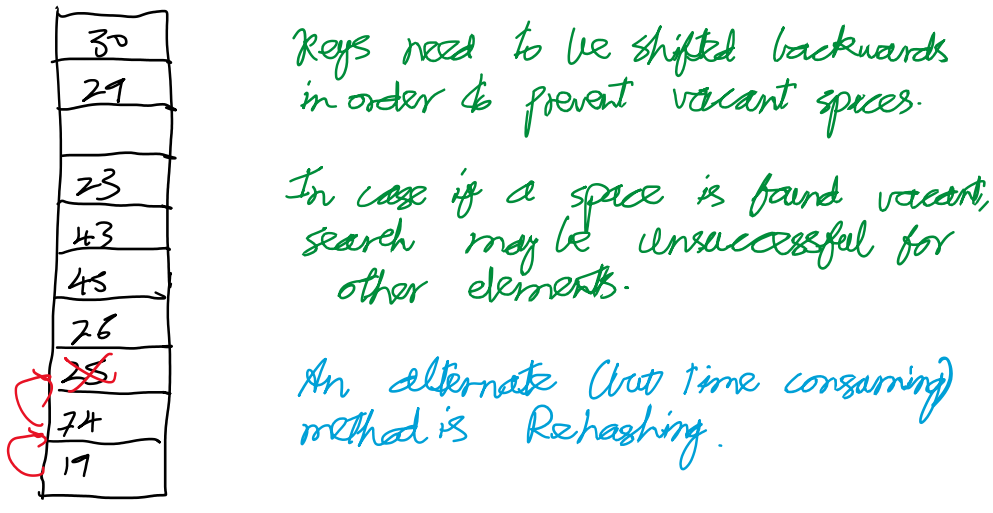$h'(25) = (h(25) + f(2)) \% 10 = 7$ ✓

$h'(29) = (h(29) + f(0)) \% 10 = 9$

$h'(29) = (h(29) + f(1)) \% 10 = 0$

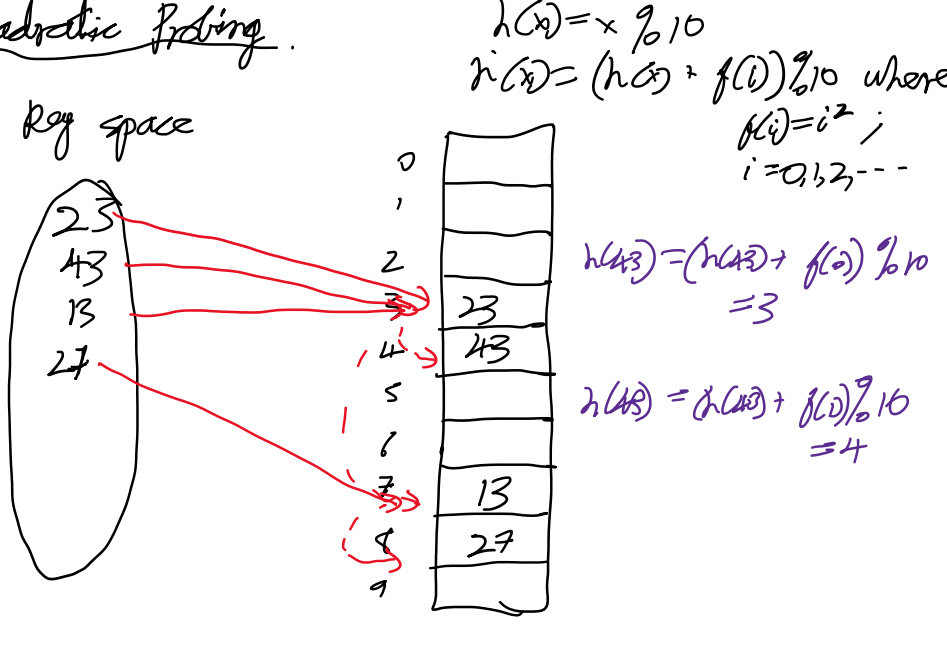$h'(29) = (h(29) + f(2)) \% 10 = 1$ ✓

Time for successful search $= \dfrac{1}{\lambda} \ln\left(\dfrac{1}{1-\lambda}\right)$.

Time for unsuccessful search $= \dfrac{1}{1-\lambda}$

Deletion:-

Keys need to be shifted backwards in order to prevent vacant spaces.

In case if a space is found vacant, search may be unsuccessful for other elements.

An alternate (but time consuming) method is Rehashing.

Quadratic Probing. $h(x) = x \% 10$

$h'(x) = (h(x) + f(i)) \% 10$ where
$f(i) = i^2$,
$i = 0, 1, 2, \cdots$

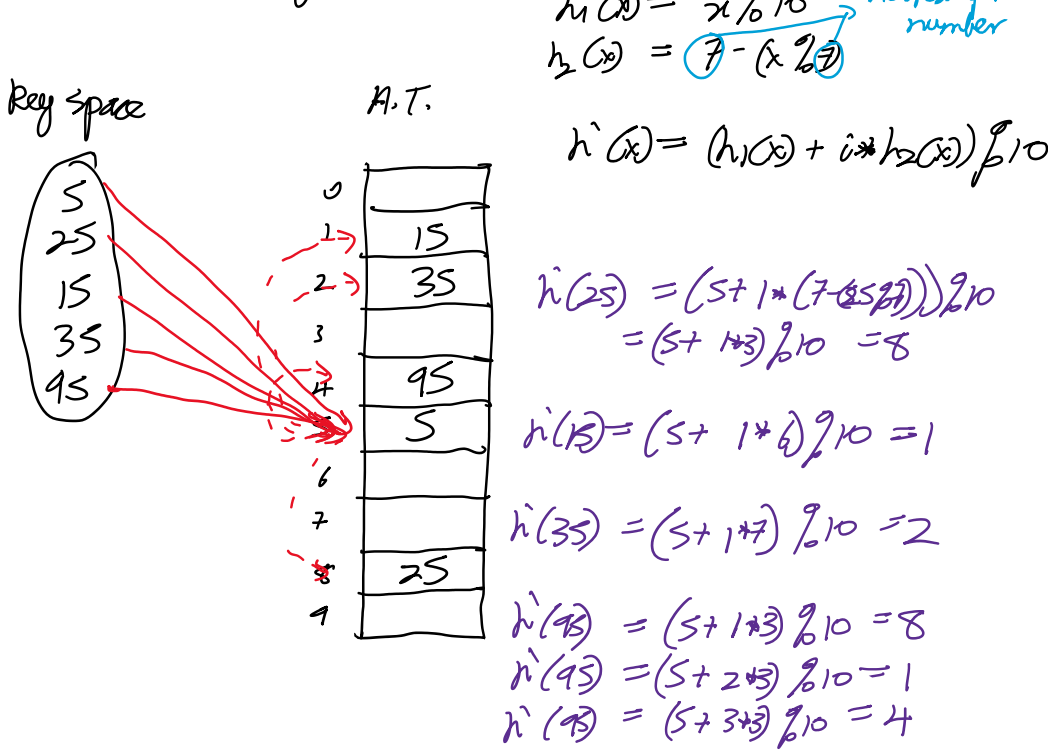$h'(43) = (h(43) + f(0)) \% 10 = 3$

$h'(43) = (h(43) + f(1)) \% 10 = 4$

$h'(13) = (h(13) + f(0)) \% 10 = 3$

$h'(13) = (h(13) + f(1)) \% 10 = 4$

$h'(13) = (h(13) + f(2)) \% 10 = 7$

Double Hashing.

$h_1(x) = x \% 10$
$h_2(x) = (7 - (x \% 7))$ nearest prime number

$h'(x) = (h_1(x) + i * h_2(x)) \% 10$

Key space / A.T.

$h'(25) = (5 + 1 * (7 - (25 \% 7))) \% 10$
$= (5 + 1 * 3) \% 10 = 8$

$h'(15) = (5 + 1 * 6) \% 10 = 1$

$h'(35) = (5 + 1 * 7) \% 10 = 2$

$h'(45) = (5 + 1 * 3) \% 10 = 8$
$h'(45) = (5 + 2 * 3) \% 10 = 1$
$h'(45) = (5 + 3 * 3) \% 10 = 4$