5.1 Some Aspects of Monte Carlo Simulation

Thursday, February 11, 2021 2:03 PM

Form sample mean $M_N = \frac{1}{N}\sum_{k=1}^{N} v_k$.

$$M_{N+1} = M_N + \frac{1}{N+1}(v_{N+1} - M_N).$$

<u>The Case of iid samples-</u>

$$E(M_N) = \frac{1}{N}\sum_{k=1}^{N} E[v_k] = m.$$

M_N is said to be an unbiased estimator.

$$Var(M_N) = \frac{1}{N^2}\sum_{k=1}^{K} Var(v_k) = \frac{\sigma^2}{N}.$$

<u>The Case of a Random Sample Size-</u>

Mean of $v_1, \dots v_N$ conditioned on N is the same as unconditional mean m.

$$E[M_N] = E\left[E\left[\frac{1}{N}\sum_{k=1}^{N} v_k \,\Big|\, N\right]\right] = E[m] = m.$$

M_N is a consistent estimator as long as the sequence $v_1, \dots v_N$ satisfies a strong law of large numbers.

Consider $N_K \to \infty$ as parameter K increases,

then, $\lim_{K\to\infty} \frac{1}{N_K}\sum_{k=1}^{N_K} v_k = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} v_k = m.$

Suppose that $v_1, v_2, \dots v_N$ have a common mean, and

$$E[v_k \mid N \geq k] = E[v_1].$$

We can claim that $E\left[\sum_{k=1}^{N} v_k\right] = E[v_1]E[N]$.

This is known as Wald's Identity.

Proof: $E\left[\sum_{k=1}^{N} v_k\right] = \sum_{k=1}^{\infty} P(N \geq k) E[v_k \mid N \geq k]$.

$$= E[v_1]\sum_{k=1}^{\infty} P(N \geq k).$$

$$= E[v_1]\sum_{k=1}^{\infty}\sum_{n=k}^{\infty} P(N = n).$$

$$= E[v_1]\sum_{n=1}^{\infty} n\, P(N = n)$$

$$= E[v_1]\,E[N]. \quad \blacksquare$$