4.1 The Basic Model

This model is similar to the Asynchronous VI and does not require all components to be updated at each iteration,

$$Hr = r,$$

The solution $Hr^* = r^*$ is the fixed point of H.

$$Hr = \left((Hr)(1), \dots (Hr)(n) \right) \quad , \forall r \in \mathbb{R}^n.$$

Let $T^{(i)}$ be an infinite set of integers,

$$r_{t+1}(i) = r_t(i) \quad , \quad t \notin T^i.$$

and $r_{t+1}(i) = \left(1-\gamma_t(i)\right) r_t(i) + \gamma_t(i)\Big(\underbrace{(Hr_t)(i) + w_t(i)}_{\text{stochastic approximate}}\Big). \quad , t \in T^i$

Thus, $(Hr_t)(i) - r_t(i) + w_t(i)$ is the step direction.

$$\boxed{r_{t+1}(i) = r_t(i) + \gamma_t(i)\left[(Hr_t)(i) + w_t(i) - r_t(i)\right]}$$

Assumption 1: The stepsizes $\gamma_t(i)$ are nonnegative and $\gamma_t(i) = 0$ for $t \in T^i$. Furthermore, the following hold with probability 1:

(a) $\forall i, \quad \sum_{t=0}^{\infty} \gamma_t(i) = \infty.$

(b) $\forall i, \quad \sum_{t=0}^{\infty} \gamma_t^2(i) < \infty.$