

## 5.4 Optimistic Policy Iteration

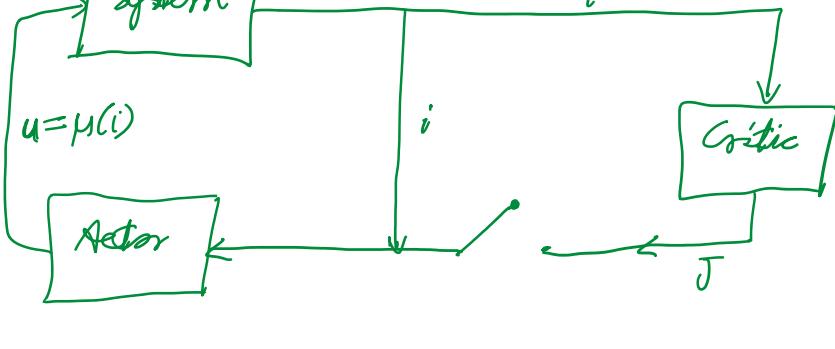
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The algorithm fixes a policy  $\mu$ , evaluates  $J^\mu$ , and then perform a policy update.

Updates are carried out using Actor-Critic system wherein actor uses policy  $\mu$  to control the system and critic computes  $J^\mu$ .

Actions are chosen to minimize,

$$\sum_j p_{ij}(u) (g(i, u, j) + \lambda J(j)).$$



We can perform a policy update corresponding to every update of policy evaluation algorithm (critic). Thus, we perform updates at each simulation step. These type of methods are called **optimistic policy iteration**.

Visualizing Policy Iteration -

We say that  $\mu$  is greedy w.r.t  $J$  if  $\mu$  attains the minimum,

$$\sum_j p_{ij}(\mu(i)) (g(i, \mu(i), j) + \lambda J(j)) = \min_{u \in U(i)} \sum_j p_{ij}(u) (g(i, u, j) + \lambda J(j)).$$

$$\Rightarrow T_\mu J = TJ.$$

Let  $R_\mu$  be the set of vectors  $J$  that lead to greedy policy,

$$R_\mu = \{ J \mid u \text{ is greedy w.r.t } J \}.$$

$$\Rightarrow J \in R_\mu \text{ iff } T_\mu J = TJ.$$

$$\Rightarrow \sum_j p_{ij}(\mu(i)) (g(i, \mu(i), j) + \lambda J(j)) \leq \sum_j p_{ij}(u) (g(i, u, j) + \lambda J(j)).$$

Since this is a set of linear inequalities,  $R_\mu$  is a polyhedron.

Optimistic TD(0).

Consider the online TD(0) algorithm,

$$\begin{aligned} J(i) &= J^0 + \gamma [g(i, \mu(i), j) + \lambda J(j) - J^0] \\ &= (1-\gamma) J^0 + \gamma [g(i, \mu(i), j) + \lambda J(j)]. \end{aligned}$$

This is of the form,

$$J(i) = (1-\gamma) J^0 + \gamma (T_\mu J)(i) + \gamma \omega. \quad \begin{matrix} \text{zero mean} \\ \text{noise term} \end{matrix}$$

With a diminishing step size,

$$J(i) = (1-\gamma) J^0 + \gamma (T_\mu J)(i).$$

In the case of optimistic TD(0), policy  $\mu$  updated at each update is a greedy policy w.r.t current vector  $J$  and satisfies  $T_\mu J \leq TJ$ .

$$J(i) = (1-\gamma) J^0 + \gamma (T_\mu J)(i) + \gamma \omega.$$

Optimistic TD(1).

Consider the case when TD(1) is used for policy evaluation and  $\gamma < 1$  (discounted problem).

The cumulative cost for every state  $i_k$ ,

$$g(i_k, i_{k+1}) + \gamma g(i_{k+1}, i_{k+2}) + \dots$$

This yields an unbiased estimate of  $J^\mu(i_k)$ .

In theory, the synchronous version of TD(1) is guaranteed to converge to  $J^*$  with probability 1.