

## 5.4 Optimistic Policy Iteration

Friday, February 12, 2021

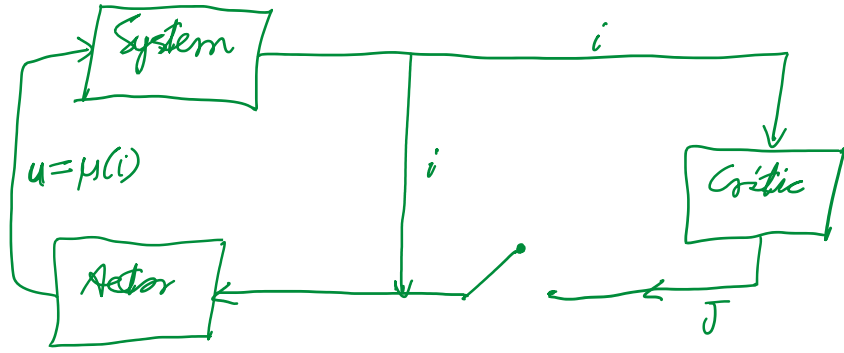
10:53 AM

The algorithm fixes a policy  $\mu$ , evaluates  $J^\mu$ , and then perform a policy update.

Updates are carried out using Actor-Critic system wherein actor uses policy  $\mu$  to control the system and critic computes  $J^\mu$ .

Actions are chosen to minimize,

$$\sum_j P_{ij}(u) (g(i, u, j) + J(j)).$$



We can perform a policy update corresponding to every update of policy evaluation algorithm (critic). Thus, we perform updates at each simulation step. These type of methods are called optimistic policy iteration.

Visualizing Policy Iteration -

We say that  $\mu$  is greedy w.r. to  $J$  if  $\mu$  attains the minimum,

$$\sum_j P_{ij}(\mu(i)) (g(i, \mu(i), j) + \gamma J(j)) = \min_{u \in U(i)} \sum_j P_{ij}(u) (g(i, u, j) + \gamma J(j)).$$

$$\Rightarrow T_\mu J = T J.$$

Let  $R_\mu$  be the set of vectors  $J$  that lead to greedy policy,

$$R_\mu = \{ J \mid \mu \text{ is greedy w.r. to } J \}.$$

$$\Rightarrow J \in R_\mu \text{ iff } T_\mu J = T J.$$

$$\Rightarrow \sum_j P_{ij}(\mu(i)) (g(i, \mu(i), j) + \gamma J(j)) \leq \sum_j P_{ij}(u) (g(i, u, j) + \gamma J(j)).$$

Since this is a set of linear inequalities,  $R_\mu$  is a polyhedron.

Optimistic TD(0).

Consider the online TD(0) algorithm,

$$\begin{aligned} J(i) &= J(i) + \gamma [g(i, \mu(i), j) + \gamma J(j) - J(i)] \\ &= (1-\gamma) J(i) + \gamma [g(i, \mu(i), j) + \gamma J(j)]. \end{aligned}$$

This is of the form,

$$J(i) = (1-\gamma) J(i) + \gamma (T_\mu J)(i) + \gamma w.$$

$\nwarrow$  zero mean noise term

With a diminishing step size,

$$J(i) = (1-\gamma) J(i) + \gamma (T_\mu J)(i).$$

In the case of optimistic TD(0), policy  $\mu$  updated at each update is a greedy policy w.r.t. current vector  $J$  and satisfies  $T_\mu J \leq T J$ .

$$J(i) = (1-\gamma) J(i) + \gamma (T J)(i) + \gamma w.$$

Optimistic TD(1).

Consider the case when TD(1) is used for policy evaluation and  $\gamma < 1$  (discounted problem).

The cumulative cost for every state  $i_k$ ,

$$g(i_k, i_{k+1}) + \gamma g(i_{k+1}, i_{k+2}) + \dots$$

This yields an unbiased estimate of  $J^\mu(i_k)$ .

In theory, the synchronous version of TD(1) is guaranteed to converge to  $J^*$  with probability 1.