# Modelling and Simulation of Rotorcraft

CONTROL Y PROGRAMACIÓN DE ROBOTS

**Grado en Electrónica, Robótica y Mecatrónica**

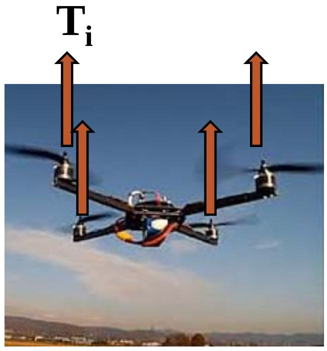
## Outline

1. Introduction
2. Kinematic model
3. Dynamic model
4. Simulations

# Introduction

Thrusts as result of wings rotation:

- Wings with variable pitch angle:
  - Constant rotation speed
  - Magnitude and direction of the thrust depends on:
    - Longitudinal and lateral cyclic
    - Collective
- Wings with fixed pitch angle:
  - Variable rotation speed
  - Thrust perpendicular to the rotation axis.


# Introduction

## Some features:

Nonlinear, high-coupled multivariable systems

Unstable systems

Usually, underactuated mechanical systems

- 6 DOF: 3D location and attitude
- Number of actuators: depends on the specific rotorcraft

Capable of hovering motion

Narrow speed range: up to 240 knots (≈120 m/s)

# Introduction

## Necessity of modelling:

For control purposes:

- Control laws based on models
- Simplified models to obtain main features
- Nonlinear models for control needed if attraction basis must be enlarged (e.g., for acrobatic motions)

For simulation purposes:

- Mechanical design (out of the scope of this course)
- Evaluation of control performance
- Training

***FMS Flight Simulator – DraganFly Quadrotor***

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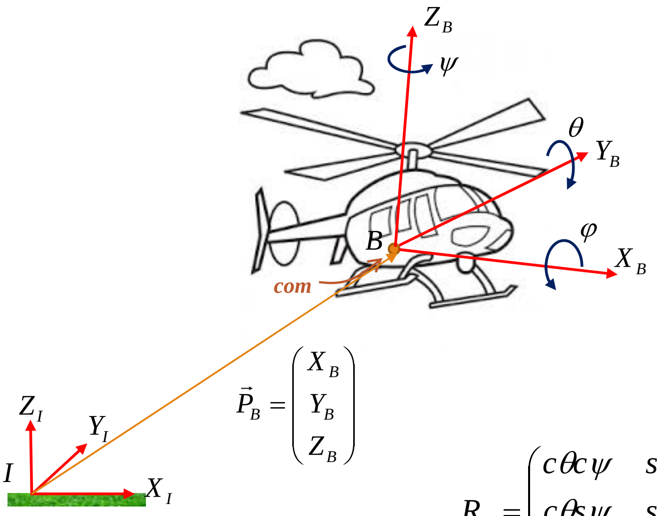
# Rotorcraft kinematics

## Coordinates frames:




$$\vec{P}_B = \begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix}$$

### Degrees of freedom:

- **Translation:**

$$\vec{P}_B = \begin{pmatrix} X_B & Y_B & Z_B \end{pmatrix}^T$$

- **Rotation:**

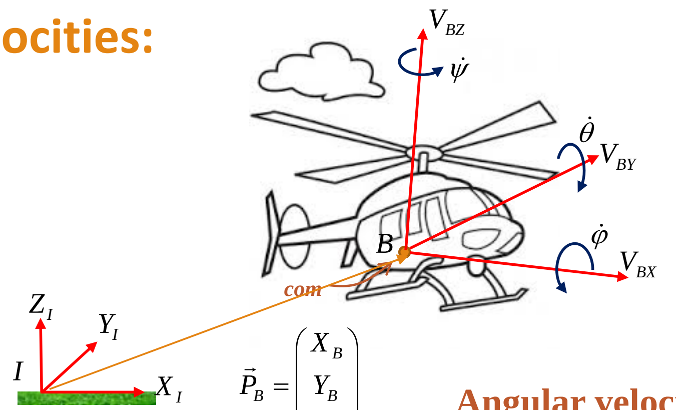
$$\vec{\eta} = \begin{pmatrix} \varphi & \theta & \psi \end{pmatrix}^T$$

### Rotation matrix

$$R_B = \begin{pmatrix} c\theta c\psi & s\varphi s\theta c\psi - c\varphi s\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ c\theta s\psi & s\varphi s\theta s\psi + c\varphi c\psi & c\varphi s\theta s\psi - s\varphi c\psi \\ -s\theta & s\varphi c\theta & c\varphi c\theta \end{pmatrix}$$

# Rotorcraft kinematics

## Velocities:




$$\vec{P}_B = \begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix}$$

**Linear velocities:** $\vec{V}_B = \dfrac{d\vec{P}_B}{dt}$

- In the Body frame:

$${}^B\vec{V}_B = \begin{pmatrix} V_{BX} \\ V_{BY} \\ V_{BZ} \end{pmatrix}$$

- In the Inertial frame:

$${}^I\vec{V}_B = {}^I R_B \, {}^B\vec{V}_B$$

**Angular velocities**

### Euler matrix:

- In the Body frame: $\vec{\Omega}_B = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

$$\underbrace{\begin{pmatrix} p \\ q \\ r \end{pmatrix}}_{\vec{\Omega}_B} = \underbrace{\begin{pmatrix} 1 & 0 & -s\theta \\ 0 & c\varphi & s\varphi c\theta \\ 0 & -s\varphi & c\varphi c\theta \end{pmatrix}}_{W_\eta} \underbrace{\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}}_{\dot{\vec{\eta}}}$$

$$\dot{\vec{\eta}} = W_\eta^{-1} \vec{\Omega}_B$$

$$W_\eta^{-1} = \begin{pmatrix} 1 & s\varphi tg\theta & c\varphi tg\theta \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi \sec\theta & c\varphi \sec\theta \end{pmatrix}$$

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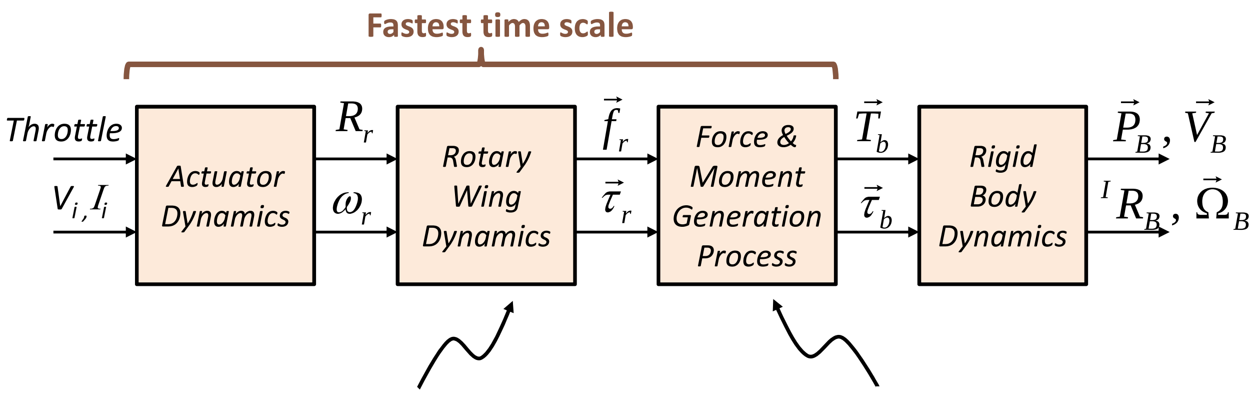
# Rotorcraft dynamics

**Modules for simulation purposes:**

$$\vec{T}_b = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \qquad \vec{\tau}_b = \begin{bmatrix} \tau_\varphi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

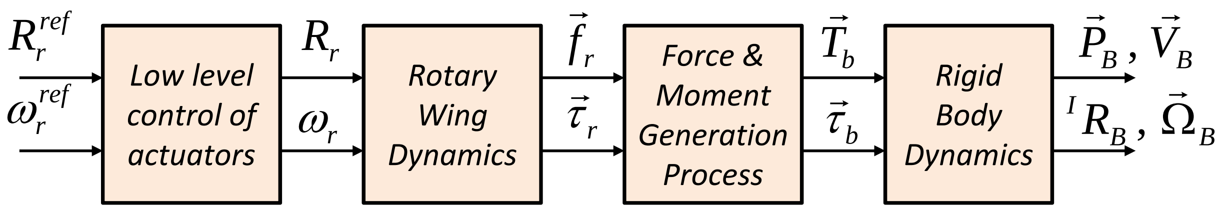



**Depends on rotor properties (fixed-pitch-angle blades, …)**

**Depends on the helicopter configuration**

# Rotorcraft dynamics

## **Low level control of actuators** included for simulation purposes:




**Depends on rotor properties (fixed-pitch-angle blades, …)**

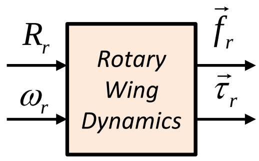
**Depends on the helicopter configuration**

**Usually, modelled as fast second order dynamic systems**

*State space description to provide initial conditions*

# Rotorcraft dynamics

## Rotary wing dynamics:




- Visited in the preceding block.
- Use of complex/simplified expressions depending on the required degree of complexity, operation conditions, …
- Complex expressions may affect simulation times of computation significantly.
- Static models for simulations.
- Examples:

**Fixed-pitch-angle blades**

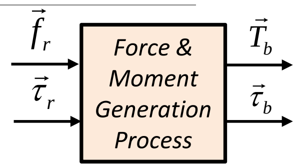
$$\vec{f}_r \approx c_t \rho \omega_r^2 \vec{n}$$
$$\vec{\tau}_r \approx c_{drag} \rho \omega_r^2 \vec{n}$$
$$\vec{n} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$$

**Regulated-speed rotors**

$$\vec{f}_r \approx \left( c_{t1} \rho \omega_r^2 \theta_0 + c_{t2} \omega_r \right) \vec{n}$$
$$\vec{\tau}_r \approx \left( c_{drag} \rho \omega_r^2 + c_v \omega_r \theta_0 \right) \vec{n}$$
$$\vec{n} = \begin{pmatrix} s\theta_{1s} c\theta_{1c} & c\theta_{1s} s\theta_{1c} & c\theta_{1s} c\theta_{1c} \end{pmatrix}^T$$

$\theta_0, \theta_{1s}, \theta_{1c}$ : Collective angle, longitudinal pitch angle, and lateral pitch angle

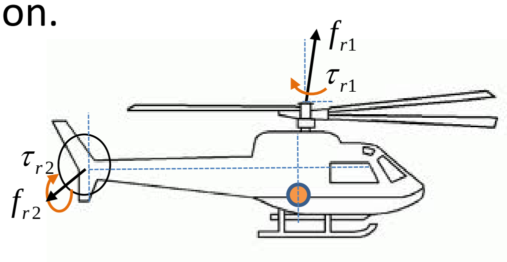
# Rotorcraft dynamics

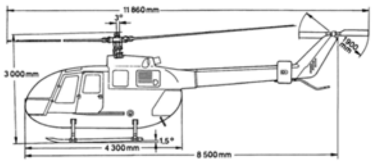


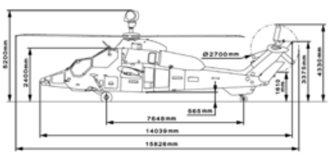

## Force and moment generation process:

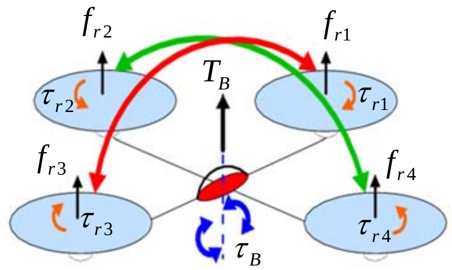
Resulting thrusts/torques ($f_{ri}$ / $\tau_{ri}$) cause forces/torques ($T_{bi}$ / $\tau_{bi}$) on the rotorcrafts center of mass, depending on the configuration.

- Single main rotor helicopters
- Tandem rotors helicopters
- Quad rotors helicopters
- Tilt rotor helicopters
- …



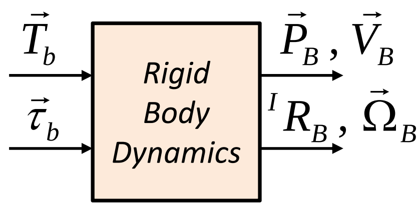








# Rotorcraft dynamics

## Rigid body dynamics:




### Newton-Euler formulation
**(writing with respect to Body coordinate frame)**

**Newton's equations (forces on com)**

$$m\,{}^B\dot{\vec{V}}_B + \vec{\Omega}_B \times m\,{}^B\vec{V}_B = \sum_i \vec{T}_{bi} + \sum \vec{F}_{Aerodynamics}$$

**Euler's equations (torques on com)**

$$I\dot{\vec{\Omega}}_B + \vec{\Omega}_B \times I\,\vec{\Omega}_B = \sum_i \vec{\tau}_{bi} + \sum \vec{\tau}_{Aerodynamics}$$

$\vec{F}_{Aerodynamics}$
$\vec{\tau}_{Aerodynamics}$

- **Vertical stabilizer**
- **Horizontal stabilizer**
- **Fuselage**
- **…**

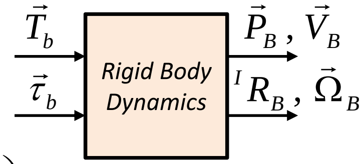
**Inertia matrix with respect to body frame**

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}$$

$$I_{ij} = \int_{V_h} \rho_h \left( \delta_{ij} \left( \sum_i d_i^2 \right) - d_i d_j \right) dV_h$$

# Rotorcraft dynamics

## Rigid body dynamics:

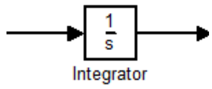



### State space equations:

$$\begin{pmatrix} \dot{\vec{P}}_B \\ {}^{I}\dot{\vec{V}}_B \\ \dot{\vec{\eta}} \\ \dot{\vec{\Omega}}_B \end{pmatrix} = \begin{pmatrix} {}^{I}\vec{V}_B \\ \frac{1}{m}\,{}^{I}R_B\left(\sum_i \vec{T}_{bi} + \sum \vec{F}_{Aerodynamics}\right) \\ W_\eta^{-1}\,{}^{B}\vec{\Omega}_B \\ I^{-1}\left(\sum_i \vec{\tau}_{bi} + \sum \vec{\tau}_{Aerodynamics} - \vec{\Omega}_B \times I\,\vec{\Omega}_B\right) \end{pmatrix}$$

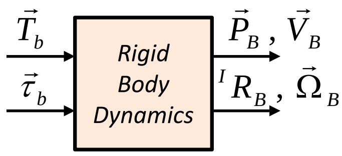
**By "numerical" integration of accelerations, velocities and positions are computed**

**(Simulink on Matlab)**




# Rotorcraft dynamics

## Structure of the rigid body dynamics:




**By defining:** $\vec{q} = \begin{pmatrix} \vec{P}_B \\ \vec{\eta} \end{pmatrix} = \begin{pmatrix} X_B & Y_B & Z_B & \varphi & \theta & \psi \end{pmatrix}^T$

**Dynamics equations can we written as:**

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\Gamma + \Gamma_{Aerodynamics}$$

$M(q)$ : Dynamic matrix
$C(q,\dot{q})$ : Centrifugal and Coriolis matrix
$G(q)$ : Gravitational vector
$B(q)$ : Input matrix
$\Gamma$ : Generalized torques

**Important properties for controllers synthesis!**

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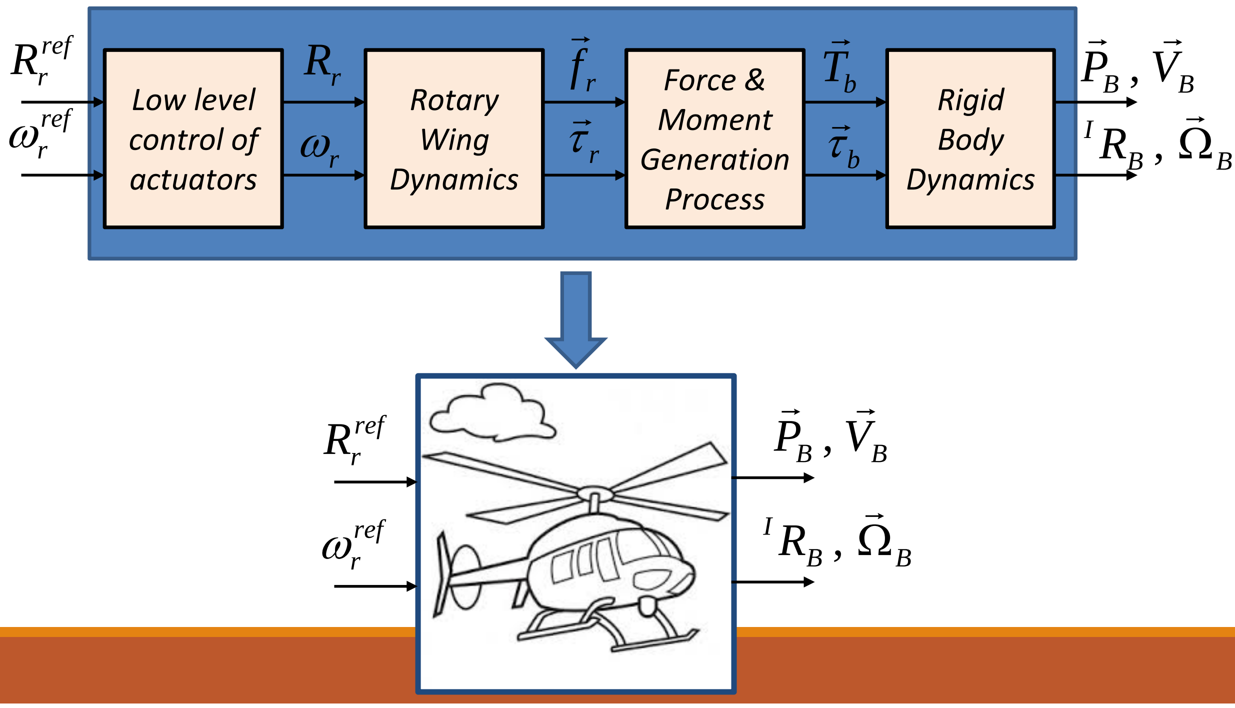
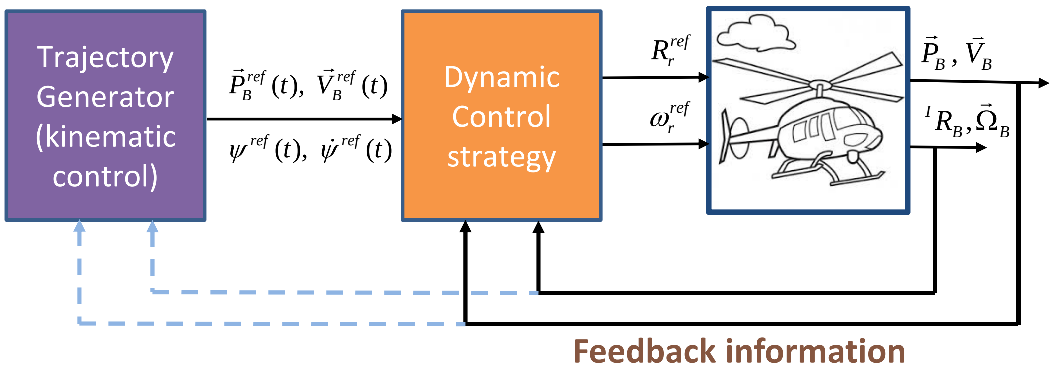
# Simulations

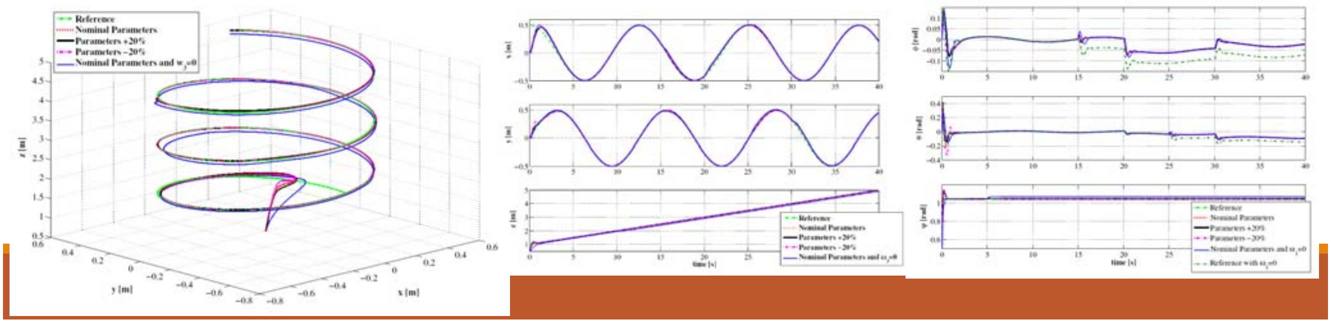
## Dynamic model as a unique system:



# Simulations

## Simulation including control:

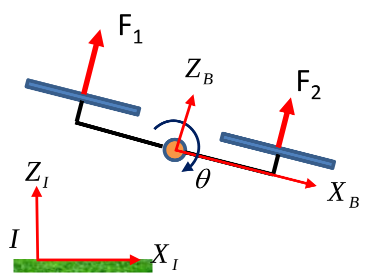


# Simulations

Case study: **PLANAR TWIN-ROTOR HELICOPTER (balanced)**

| Parameter description | Parameter | Value |
|---|---|---|
| Mass of the helicopter | $m$ | 2.24 kg |
| Distance between mass center and rotors | $l$ | 0.332 m |
| Thrust coefficient of the rotors | $b$ | 9.5e-6 N.s$^2$ |
| Drag coefficient of the rotors | $k_t$ | 1,7e-7 Nm.s$^2$ |
| Moment of inertia | $I_{yy}$ | 0,0363 kg.m$^2$ |




**Control signals dynamics and saturations:**

$\omega_{r\max} = 10.000\, rpm$

$\omega_{r\min} = 100\, rpm$

$$\omega_r^{ref} = \frac{1}{(0.04s+1)^2}\omega_r$$

**Aerodynamics frictions:**

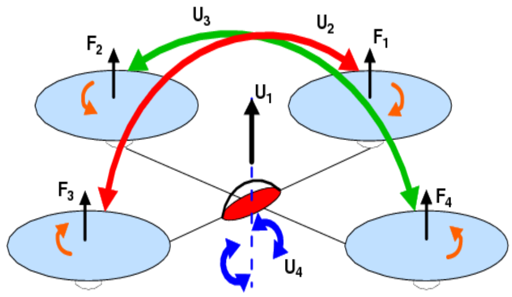
$$\left|\vec{F}_{Aerodynamics}\right| = \frac{1}{2}\rho f \left|\vec{V}_B\right| \qquad \rho \approx 1.2\, kg/m^3$$

***Equivalent flat-plate area:*** $f \approx 0.1m$

# Simulations

## Case study: QUAD-ROTOR HELICOPTER (balanced)

| Parameter Description | Parameter | Value |
|---|---|---|
| Mass of the *QuadRotor* helicopter | $m$ | $2.24\ kg$ |
| Distance between the mass center and the rotors | $l$ | $0.332\ m$ |
| Thrust coefficient of the rotors | $b$ | $9.5e-6\ Ns^2$ |
| Drag coefficient of the rotors | $k_\tau$ | $1.7e-7\ Nms^2$ |
| Gravitational acceleration | $g$ | $9.81\ m/s^2$ |
| Moment of inertia around the $x$-axis | $I_{xx}$ | $0.0363\ Kg.m^2$ |
| Moment of inertia around the $y$-axis | $I_{yy}$ | $0.0363\ Kg.m^2$ |
| Moment of inertia around the $z$-axis | $I_{zz}$ | $0.0615\ Kg.m^2$ |




### Control signals dynamics and saturations:

$$\omega_{r\max} = 10.000\, rpm$$
$$\omega_{r\min} = 100\, rpm$$
$$\omega_r^{ref} = \frac{1}{(0.04s+1)^2}\omega_r$$

### Aerodynamics frictions:

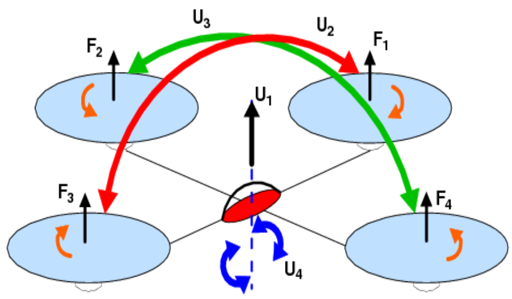
$$\left|\vec{F}_{Aerodynamics}\right| = \frac{1}{2}\rho f \left|\vec{V}_B\right| \qquad \rho \approx 1.2\, kg/m^3$$

*Equivalent flat-plate area:* $f \approx 0.1m$

# Simulations

## Case study: QUAD-ROTOR HELICOPTER (unbalanced)

| Parameter Description | Parameter | Value |
|---|---|---|
| Mass of the *QuadRotor* helicopter | $m$ | $2.24\ kg$ |
| Distance between the mass center and the rotors | $l$ | $0.332\ m$ |
| Thrust coefficient of the rotors | $b$ | $9.5e-6\ Ns^2$ |
| Drag coefficient of the rotors | $k_\tau$ | $1.7e-7\ Nms^2$ |
| Gravitational acceleration | $g$ | $9.81\ m/s^2$ |
| Moment of inertia around the $x$-axis | $I_{xx}$ | $0.0363\ Kg.m^2$ |
| Moment of inertia around the $y$-axis | $I_{yy}$ | $0.0363\ Kg.m^2$ |
| Moment of inertia around the $z$-axis | $I_{zz}$ | $0.0615\ Kg.m^2$ |
| Position of the center of mass in $x$ from the body-fixed frames | $r_x$ | $-0.00069\ m$ |
| Position of the center of mass in $y$ from the body-fixed frames | $r_y$ | $-0.0014\ m$ |
| Position of the center of mass in $z$ from the body-fixed frames | $r_z$ | $-0.0311\ m$ |




### Control signals dynamics and saturations:

$$\omega_{r\max} = 10.000\, rpm$$
$$\omega_{r\min} = 100\, rpm$$
$$\omega_r^{ref} = \frac{1}{(0.04s+1)^2}\omega_r$$

### Aerodynamics frictions:

$$\left|\vec{F}_{Aerodynamics}\right| = \frac{1}{2}\rho f \left|\vec{V}_B\right| \qquad \rho \approx 1.2\, kg/m^3$$

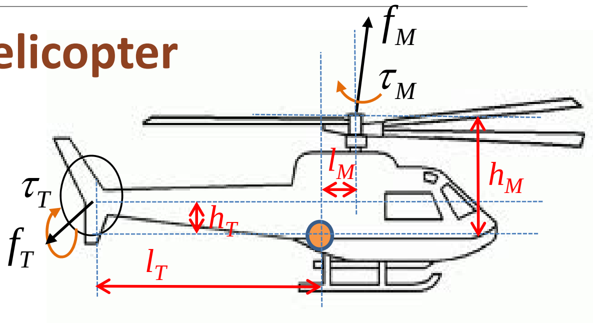
$f \approx 0.1m$

*Equivalent flat-plate area:*

# Simulations

Case study: **Single main rotor helicopter**

| Param. | Value |
|---|---|
| $M$ | 4,9 kg |
| $I_x$ | 0,1424 kg.m² |
| $I_y$ | 0,2712 kg.m² |
| $I_z$ | 0,2715 kg.m² |
| $l_M$ | 0,015 m |
| $h_M$ | 0,294 m |
| $y_M$ | 0 m |
| $l_T$ | 0,871 m |
| $h_T$ | 0,115 m |




**Propulsion models:**

$$\left|\vec{f}_M\right| = \left(6.4578\theta_{M0} + 100.3752\theta_{M0}^3\right)$$

$$\left|\vec{f}_T\right| = \left(1.837\theta_{T0} + 1.545\theta_{T0}^3\right)$$

$$\tau_{MDrag} = 0.004452\left|\vec{f}_M\right|^{1.5} + 0.6304$$

$$\tau_{TDrag} = 0.005066\left|\vec{f}_T\right|^{1.5} + 0.008488$$

$$\vec{\tau}_M = \begin{pmatrix} 25.23\theta_{M1s} - \tau_{MDrag}\sin(\theta_{M1c}) \\ 25.23\theta_{M1c} + \tau_{MDrag}\sin(\theta_{M1s}) \\ -\tau_{MDrag}\cos(\theta_{M1c})\cos(\theta_{M1c}) \end{pmatrix}$$

$$\vec{\tau}_T = \begin{pmatrix} 0 \\ \tau_{TDrag} \\ 0 \end{pmatrix}$$

**Control signals dynamics:**

$$\theta_i^{ref} = \frac{1}{(0.01s+1)^2}\theta_i \quad , i = 0, 1s, 1c$$

**Control signals saturations:**

$$|\theta_{M1s}| \le 0.4363\,rad \qquad -0.5556\,rad \le \theta_{M0} \le 0.8604\,rad$$

$$|\theta_{M1c}| \le 0.3491\,rad \qquad -1.240\,rad \le \theta_{T0} \le 1.240\,rad$$