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& ΜΗΧΑΝΙΚΩΝ ΥΠΟΛΟΓΙΣΤΩΝ
ΑΡΙΣΤΟΤΕΛΕΙΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΟΝΙΚΗΣ

Adaptive Control and Autonomous Systems

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Contents

1 Introduction

The goal of this project is to control a system with many unknown parameters, using Model Reference Adaptive Control. The idea is to use different types of adaptive controllers, each one of them solving the control problem using the system as it is or after linearization. First we use the mathematical analysis to create the control law and then we create simulations in Matlab to verify the mathematical models we found earlier. We use multiple inputs as test cases in order to see the behaviour of our system to a variety of situations. Finally, to make sure that system is being controlled under all circumstances, we add disturbances and we see how our system and the control law are dealing with it. We use Matlab to simulate the system's behaviour and plot the results for better understanding of what is going on.

2 Analysis

The system we will try to control is:

$$M\ddot{q} + Gsin(q) + Cq = u \tag{1}$$

Where $q \in \mathbb{R}$ is the angle of rotation in rad, $\dot{q}$ is angular velocity in rad/s. The input of the system is $u \in \mathbb{R}$, it represents the torque and is measured in $N \cdot m$. The output of the system is the angle of rotation. The M, G, C represent unknown, constant purely positive parameters.

2.1 System Linearization

First, we want to linearize the system in the neighbor of zero. To do that we transform the system from the form above, to this one

$$\begin{aligned} x_1 &= q \\ x_2 &= \dot{q} \end{aligned}$$

and their derivatives will be

$$\dot{x_1} = \dot{q} = x_2 \tag{2}$$

$$\dot{x_2} = \ddot{q} = \frac{1}{M}(u - G\sin(x_1) - Cx_2) \tag{3}$$

Now we solve $\dot{x_1} = 0$ and $\dot{x_2} = 0$ to find the balance points. Balance points are $(x_1^\star, x_2^\star)$ =($\kappa\pi$, 0), where $\kappa = 0, 1, 2, ...$ While we are in the neighbor of $(0, 0)$ we assume that $sin(x_1) = x_1$

$$\begin{aligned} A &= \frac{dF}{dx} = \begin{bmatrix} 0 & 1 \\ -\frac{G}{M} & -\frac{C}{M} \end{bmatrix} \\ B &= \frac{dF}{du} = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \end{aligned}$$

so finally we have

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -\frac{G}{M} & -\frac{C}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

which is a linear system

2.2 Control Law on Linear System

In this part of the analysis we will use the linear system in order to create a control law using output feedback. First of all we check if the system is controllable, so

$$w = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{M} \\ \frac{1}{M} & -\frac{C}{M} \end{bmatrix}$$
$$det(w) = -\frac{1}{M^2} \neq 0$$

and $rank(w) = 2$, so, we end up that the system is controllable. So our next step is to decide the reference model. The transfer function for our linear system is

$$G_p(s) = \frac{y(s)}{u(s)} = \frac{1}{M} \frac{1}{s^2 + \frac{C}{M}s + \frac{G}{M}} \tag{4}$$

so we have ${n_p}^* = 2$ and we want ${n_m}^* = 2$ too. Our reference model is

$$\dot{x_m} = A_m x + B_m r, x_m(0) = x_{m0} \tag{5}$$
$$y_m = {C_m}^T x_m \tag{6}$$

our transfer function $G_p(s)$ satisfies every hypothesis needed to continue for a MRAC mathematical analysis. For our reference model we use $G_m(s) = \frac{1}{(s+1)^2}$ with ${n_m}^* = 2$ as needed. Again this transfer function satisfies the hypothesis needed, but it also has to be Strictly Positive Real (SPR). For this to happen $G_m(s)$ has to satisfy three more prerequisites:

1) $G_m(s)$ has to be derivable to $\sigma \geq 0$
2) $\Re(G(j\omega)) > 0$, for every $\omega \in \mathbb{R}$
3) $lim_{|x|->+\infty} \omega^2 \Re(G(j\omega)) > 0$, when $n^* = 1$

In our case the first prerequisite is checked, the second one ends up to the limitation $\rho_0 > -\frac{2\omega^2}{1-\omega^2}$ and the third one ends up to the limitation $\rho_0 < 2$. For the last one we used $(s + \rho_0)G_m(s)$ as $G(s)$, because we used the $\Lambda(s) = \Lambda_0(s)Z_m(s) = s + \lambda_o$ filter and $Z_m(s) = 1$ in our case, in order to go from ${n_m}^* = 2$ to ${n_m}^* = 1$ and use the analysis technique for ${n_m}^* = 1$. So we know that $\Lambda_0(s) == s + \lambda_o$ Of course, we keep in mind that $\rho_0 > 0$

Our reference model now is

$$\dot{x_m} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_m + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$
$$y_m = \begin{bmatrix} 1 & 0 \end{bmatrix} x_m$$

After a mathematical we end up to the following control law and equations, that will help us solve the control problem

$$u = \theta^T \omega + \dot{\theta^T} \phi \tag{7}$$
$$\dot{\omega_1} = F\omega_1 + gu, \omega_1(0) = 0 \tag{8}$$
$$\dot{\omega_2} = F\omega_2 + gy, \omega_2(0) = 0 \tag{9}$$
$$\dot{\phi} = -\rho_0 \phi + \omega \tag{10}$$
$$\dot{\theta} = -\Gamma \epsilon \phi sgn(\frac{k_p}{k_m}) \tag{11}$$

where

$$\epsilon = y - y_m \tag{12}$$
$$\omega = \begin{bmatrix} {\omega_1}^T & {\omega_2}^T & y & t \end{bmatrix}^T \tag{13}$$
$$\theta^* = \begin{bmatrix} {\theta_1}^* & {\theta_2}^* & \theta_3^* & {c_0}^* \end{bmatrix} \tag{14}$$
$$\phi = \begin{bmatrix} (SI - F)^{-1} g G_p(s) y \\ (SI - F)^{-1} g y \\ y \\ r \end{bmatrix} \frac{1}{s + \rho_0}, \rho_0 > 0 \tag{15}$$

We know that $g = 1$ and $F = -\lambda_0$, where $\lambda_0 > 0$ in order to have a stable filter. In our case both of these parameters are scalars.

2.3 Control Law on Non Linear System

Now, we will take the initial system, before linearization, in order to create a control law using state feedback. Our system in the original for is

$$M\ddot{q} + Gsin(q) + Cq = u$$

and after transforming in into state equations we have

$$x_1 = q$$
$$x_2 = \dot{q}$$

and their derivatives will be

$$\dot{x_1} = \dot{q} = x_2 \tag{16}$$
$$\dot{x_2} = \ddot{q} = \frac{1}{M}(u - G\sin(x_1) - Cx_2) \tag{17}$$

and we make it more compact like this

$$\dot{x} = Ax + B\Lambda(u - f(x)) \tag{18}$$
$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{C}{M} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{M}(u - Gsin(x_1)) \tag{19}$$

we have that $A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{C}{M} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\Lambda = \frac{1}{M}$ and $f(x) = \theta^T \Phi(x)$, where $\theta^T = \begin{bmatrix} 0 & -G \end{bmatrix}$ and $\Phi(x) = \begin{bmatrix} 0 \\ sin(x_1) \end{bmatrix}$.

Before we start analyzing the initial system, let's see the reference model. We want a reference model like that

$$\dot{x_m} = A_m x + B_m r \tag{20}$$

we want $\omega_n = 1$ and $\zeta = 0.7$. We will take the formula for a second class system and we will end up to $\frac{x_m}{r} = \frac{1}{s^2+1.4s+1}$ and let's transform this to state equations

$$\begin{bmatrix} \dot{x_{m1}} \\ \dot{x_{m2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \tag{21}$$

where $A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix}$, $B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Now it is time to see if the target is feasible, let's start with the reference model

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix} \Rightarrow det(A_m - sI) = det \begin{pmatrix} -s & 1 \\ -1 & -1.4 - s \end{pmatrix} \Rightarrow$$
$$det(A_m - sI) = s^2 + 1.4s + 1 = 0$$

this equation has two solutions $s_1 = -0.7 + i0.714$ and $s_2 = -0.7 - i0.714$. As we see here the real part of s_1 and s_2 is less than zero, so the A_m is stable.

Furthermore, in order to continue and analyze if the system is able to follow the reference model, we have to see if the pair of matrices $(A, B\Lambda)$ of the real system satisfies the criteria of controllability

$$w = \begin{bmatrix} B\Lambda & A \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{M} \\ \frac{1}{M} & -\frac{C}{M^2} \end{bmatrix}$$

so $rank(w) = 2$ and $det(w) = -\frac{1}{M^2} \neq 0$,because $M > 0$ and the pair $(A, B\Lambda)$ is controllable.

So, let's dive into the system analysis. If we new everything about the system, we would used the control law

$$u = -k^* x - L^* r \tag{22}$$

if we replace this u to the formula (18), we end up to

$$\dot{x} = (A - B\Lambda k^*) - B\Lambda L^* r + B\Lambda \theta^T \Phi(x) \tag{23}$$

and we want this to take the form of (20). So first of all, we change the control law to $u = -k^* x - L^* r - \theta^T \Phi(x)$, so $\dot{x} = (A - B\Lambda k^T)x - B\Lambda L^T r$ and we conclude to

$$A_m = A - B\Lambda k^* \tag{24}$$
$$B_m = -B\Lambda L^* \tag{25}$$

but the problem here is that we don't know k^* or L^* or θ, so in the control law we will use their estimations $\hat{k}$, $\hat{L}$ and $\hat{\theta}$. The new control law is

$$u = -\hat{k}x - \hat{L}r - \hat{\theta}^T\Phi(x) \tag{26}$$

so now, we take the initial formula (18) and we add these two mathematical tricks:

1) we add and subtract the volume $A_m x + B_m r$

2) from (24) and (25) we have $A - A_m = B\Lambda k^*$ and $B_m = -B\Lambda L^*$

Using these tricks and replacing u from the formula (26), we end up to

$$\dot{x} = A_m x + B_m r + B\Lambda(-\bar{k}x - \bar{L}r - \bar{\theta}\Phi(x)) \tag{27}$$

$$\tag{28}$$

where $\bar{k} = \hat{k} - k^*$, $\bar{L} = \hat{L} - L^*$ and $\bar{\theta} = \hat{\theta} - \theta^*$

Now let's

$$e = x - x_m \tag{29}$$

$$\dot{e} = \dot{x} - \dot{x_m} = A_m e + B\Lambda(-\bar{k}x - \bar{L}r - \bar{\theta}\Phi(x)) \tag{30}$$

Reaching this point means that it is time to assume a good Lyapunov function

$$V = e^t P e + tr\left\{\frac{\bar{k}^T\bar{k}}{\gamma_1}\right\} + tr\left\{\frac{\bar{L}^T\bar{L}}{\gamma_2}\right\} + tr\left\{\frac{\bar{\theta}^T\bar{\theta}}{\gamma_3}\right\} \tag{31}$$

where $\gamma_1, \gamma_2, \gamma_3 > 0$ and $P = P^T > 0$, P is a matrix that is the solution of the Lyapunov equation $PA_m + {A_m}^T P = -Q$, $Q = Q^T > 0$. We continue with $\dot{V}$

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} + tr\left\{\frac{\bar{k}^T\dot{\hat{k}}}{\gamma_1}\right\} + tr\left\{\frac{\bar{L}^T\dot{\hat{L}}}{\gamma_2}\right\} + tr\left\{\frac{\bar{\theta}^T\dot{\hat{\theta}}}{\gamma_3}\right\} \tag{32}$$

so we have that

$$\begin{aligned} \dot{e}^T P e + e^T P \dot{e} &= \dot{e}^T(PA_m + {A_m}^T P)e \\ -2e^T PB\Lambda\bar{k}x - 2e^T PB\Lambda\bar{L}r &- 2e^T PB\Lambda\bar{\theta}\Phi(x) \end{aligned}$$

let's analyze this part by part

$$\begin{aligned} \dot{e}^T(PA_m + {A_m}^T P)e &= -e^T Q e \\ -2e^T PB\Lambda\bar{k}x &= -2tr\left\{xe^T PB\Lambda\bar{k}\right\} = -2tr\left\{\bar{k}^T\Lambda B^T P e x^T\right\} \\ -2e^T PB\Lambda\bar{L}r &= -2tr\left\{re^T PB\Lambda\bar{L}\right\} = -2tr\left\{\bar{L}^T\Lambda B^T P e r^T\right\} \\ -2e^T PB\Lambda\bar{\theta}\Phi(x) &= -2tr\left\{\Phi(x)e^T PB\Lambda\bar{\theta}\right\} = -2tr\left\{\bar{\theta}^T\Lambda B^T P e \Phi(x)^T\right\} \end{aligned}$$

so we conclude that we want

$$\dot{\hat{k}} = \gamma_1\Lambda B^T P e x^T \tag{33}$$

$$\dot{\hat{L}} = \gamma_2\Lambda B^T P e r^T \tag{34}$$

$$\dot{\hat{\theta}} = \gamma_3\Lambda B^T P e \Phi(x)^T \tag{35}$$

but the problem is that Λ is unknown so we can't use it in order to find the derivatives of our controller's estimations. That's why we add the Λ inside the Lyapunov function

$$V = e^t Pe + tr\left\{\frac{\bar{k}^T \Lambda \bar{k}}{\gamma_1}\right\} + tr\left\{\frac{\bar{L}^T \Lambda \bar{L}}{\gamma_2}\right\} + tr\left\{\frac{\bar{\theta}^T \Lambda \bar{\theta}}{\gamma_3}\right\} \tag{36}$$

so now we have

$$\begin{aligned}\dot{V} = -e^T Qe &+ tr\left\{\frac{\bar{k}^T \Lambda \dot{\hat{k}}}{\gamma_1} - \bar{k}^T \Lambda B^T Pex^T\right\} \\ &+ tr\left\{\frac{\bar{L}^T \Lambda \dot{\hat{L}}}{\gamma_2} - \bar{L}^T \Lambda B^T Per^T\right\} \\ &+ tr\left\{\frac{\bar{\theta}^T \Lambda \dot{\hat{\theta}}}{\gamma_3} - \bar{\theta}^T \Lambda B^T Pe\Phi(x)^T\right\}\end{aligned}$$

so the result of these of these change is that we can define the derivatives of the estimations as

$$\dot{\hat{k}} = \gamma_1 B^T Pex^T \tag{37}$$

$$\dot{\hat{L}} = \gamma_2 B^T Per^T \tag{38}$$

$$\dot{\hat{\theta}} = \gamma_3 B^T Pe\Phi(x)^T \tag{39}$$

in these equations everything is known and as a result

$$\dot{V} = -e^T Qe \leq 0 \tag{40}$$

Using the Lyapunov's theorem, we know that every signal in our Lyapunov function is bounded, so $e, \bar{k}, \bar{L}, \bar{\theta} \in L_\infty$, from $e = x - x_m$, $\bar{k} = \hat{k} - k^*$, $\bar{L} = \hat{L} - L^*$ and $\bar{\theta} = \hat{\theta} - \theta^*$, we conclude that $x, \hat{k}, \hat{L}, \hat{\theta} \in L_\infty$ (of course we know that $x_m, k^*, L^*, \theta^* \in L_\infty$). Combining these we see that $u \in L_\infty$, so $\dot{e} \in L_\infty$. We also know that $\lambda_{min}(Q)||e||^2 \leq e^T Qe \leq \lambda_{max}(Q)||e||^2$. If we use the last formula of $\dot{V}$ and we integrate it we have

$$\int_0^t V dt = \int_0^t -e^T Qedt = V_0 - V_\infty \tag{41}$$

so from (40) we have

$$\dot{V} leq \lambda_{min}(Q) \Rightarrow -\int_0^t e^T Qedt \leq \lambda_{min}(Q) \int_0^t e^T edt \Rightarrow \int_0^t e^T edt = \frac{V_0 - V_\infty}{\lambda_{min}(Q)} \tag{42}$$

so we proved that $e \in L_2$

Now we have proved that $e \in L_\infty \cap L_2$ and $\dot{e} \in L_\infty$. Utilizing the theorem of Barbalat we conclude that

$$lim_{t \to +\infty} e(t) = 0 \tag{43}$$

which means that $x \to x_m$ when $t \to +\infty$

3 Simulation

For the simulations we used Matlab and the main target was to test if the above two control laws we created are able to control the system. We also, use the values $M = 1\ N \cdot m \cdot s^2/rad$, $G = 10\ N \cdot m$ and $C = 1\ N \cdot m \cdot s/rad$

3.1 Linear System Controller

We will start with the simulations of the controller, which is based on the linear system. We have selected $\rho_0 = 1$ and $\lambda_0 = 5$. We will simulate the behaviour of the system for two values of diagonal elements of Γ, 500 and 1000. Also, we will use a as input the signal $r(t) = Asin(\omega t)$ and we will test how the behaviour changes based on the input signal.

For the first four plots, we will use $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 500$ and four different input signals.

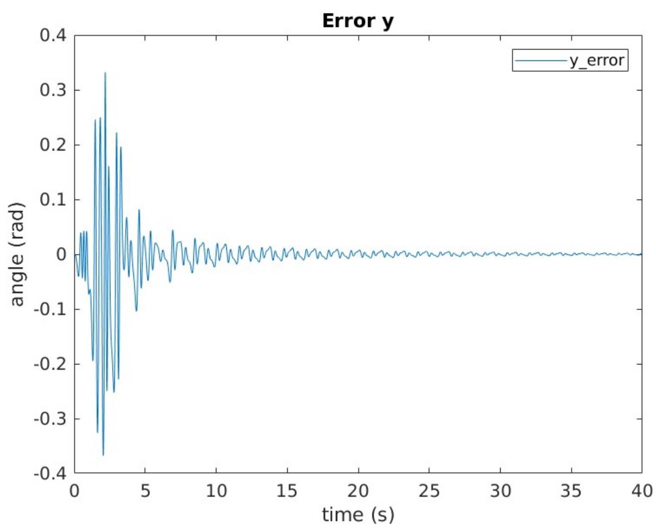

Figure 1: $r(t) = 4sin(4t)$

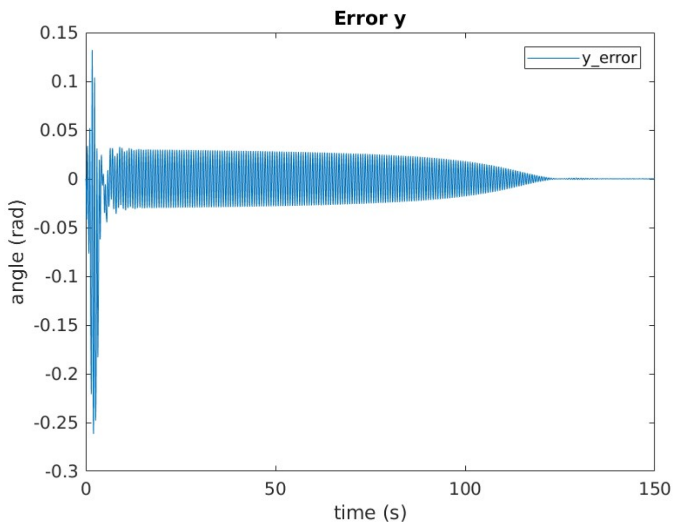

Figure 2: $r(t) = 4sin(10t)$

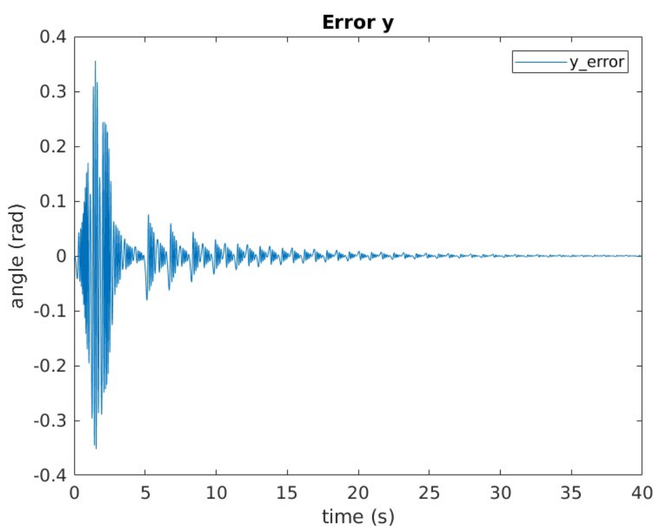

Figure 3: $r(t) = 10sin(4t)$

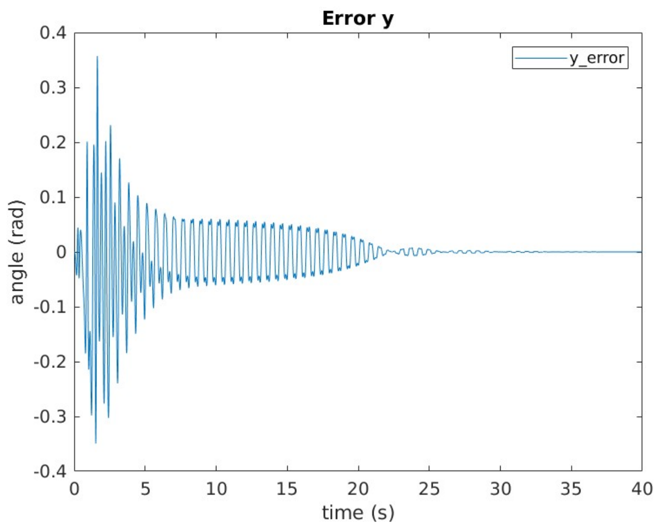

Figure 4: $r(t) = 10sin(10t)$

For the next four plots, we will use $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1000$ and the four different input signals we used above.

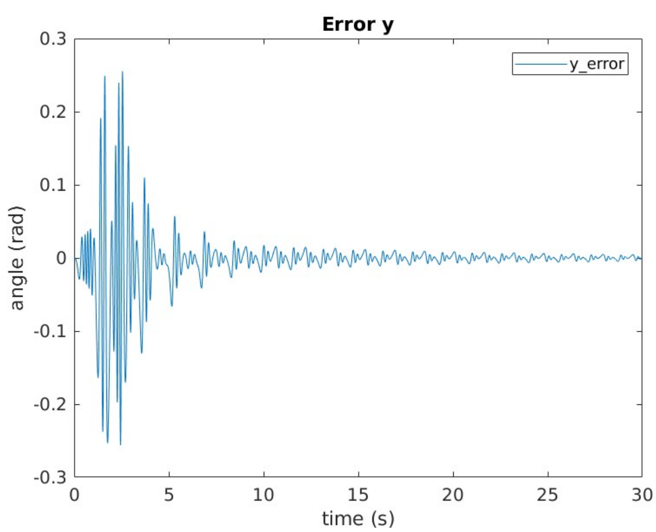

Figure 5: $r(t) = 4sin(4t)$

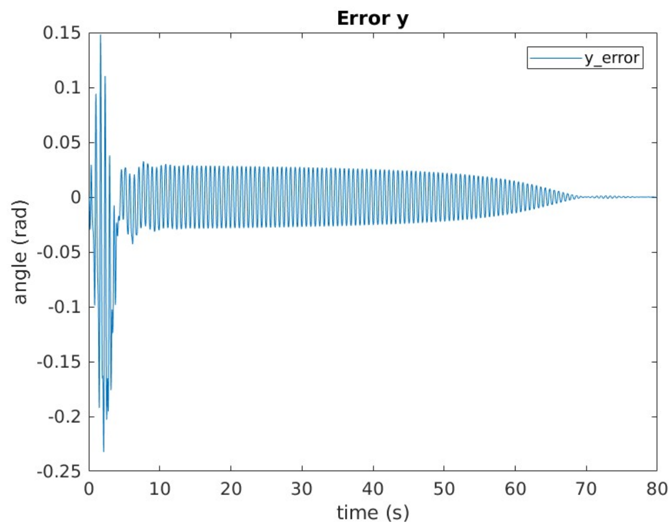

Figure 6: $r(t) = 4sin(10t)$

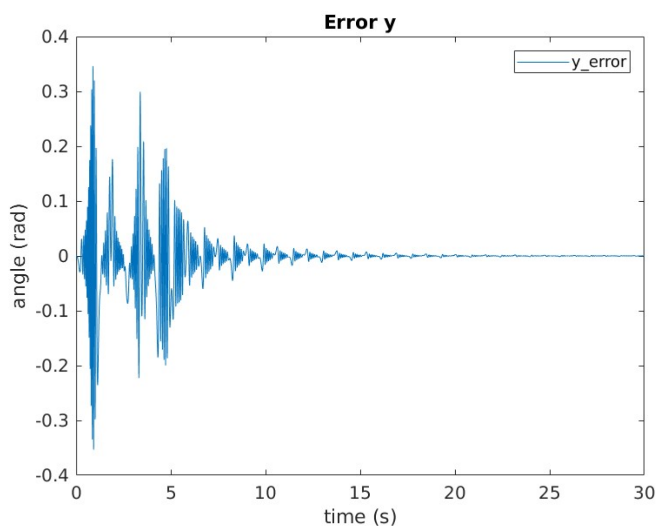

Figure 7: $r(t) = 10sin(4t)$

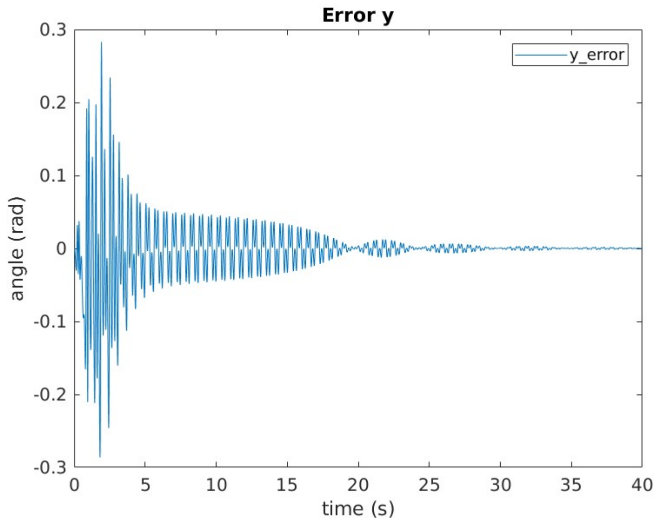

Figure 8: $r(t) = 10sin(10t)$

From the above diagrams, we can conclude three things:

- If input's amplitude or frequency increases while the other one is stable, then system needs more time to reach the stable state, but the delay added, to reach the stable state, is not a worrying event.
- Γ parameter is really important for convergence, because it reduces drastically the time needed for convergence.
- This type of control has an obvious problem when we try increase input signal's frequency. This event make the convergence much more slower than how the increase in amplitude act on convergence.

3.2 Non Linear System Controller

We will continue with the simulations of the controller, which is based on the non linear system. We have selected $Q = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$, which set the values for P matrix, solving the Lyapunov equation. We will simulate the behaviour of the system for two values of γ parameters. Also, we will use a as input the signal $r(t) = Asin(\omega t)$ and we will test how the behaviour changes based on the input signal.

For the these four plots, we will use $\gamma_1 = \gamma_2 = \gamma_3 = 500$ and four different input signals.

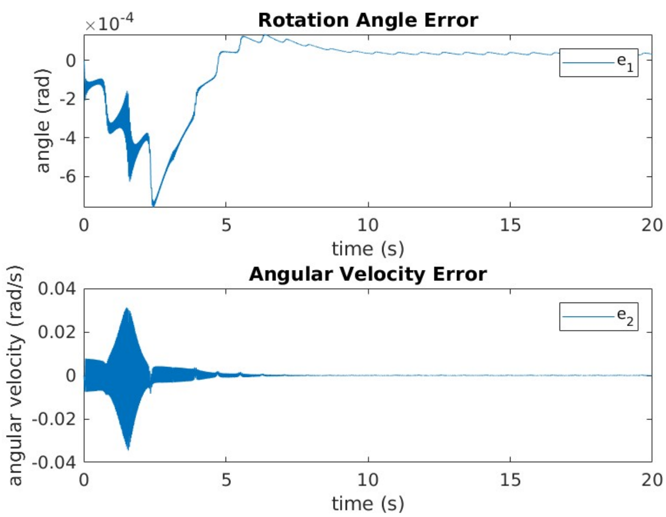

Figure 9: $r(t) = 4sin(4t)$

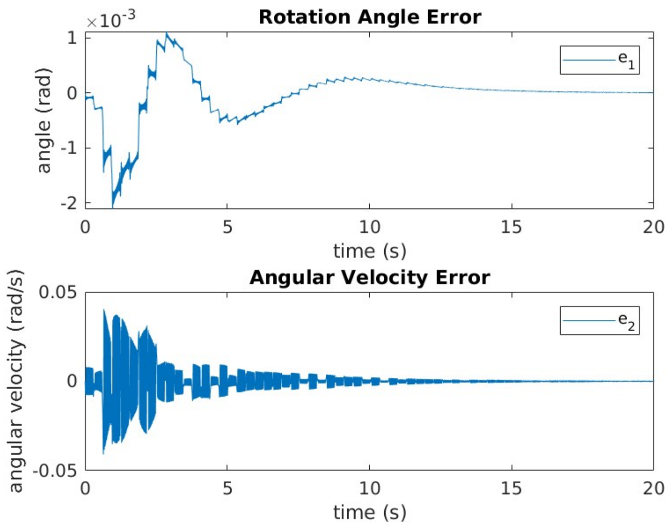

Figure 10: $r(t) = 4sin(10t)$

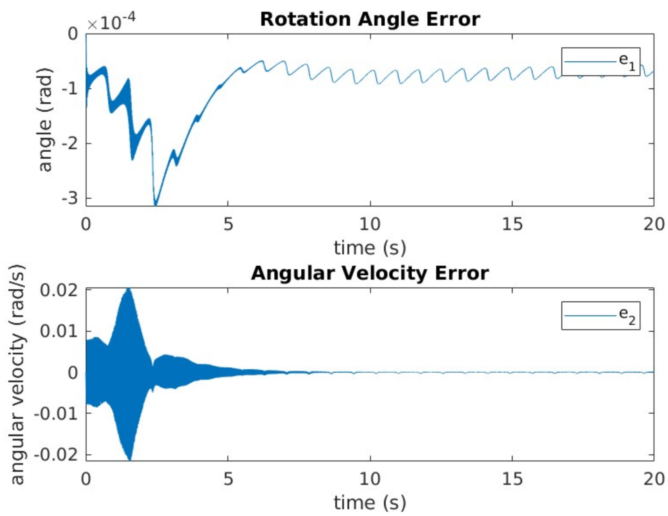

Figure 11: $r(t) = 10sin(4t)$

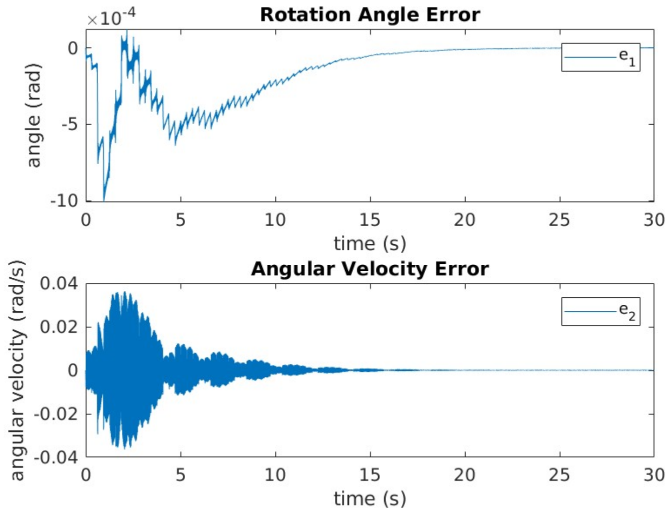

Figure 12: $r(t) = 10sin(10t)$

For the next four plots, we will use $\gamma_1 = \gamma_2 = \gamma_3 = 1000$ and the four different input signals we used above.

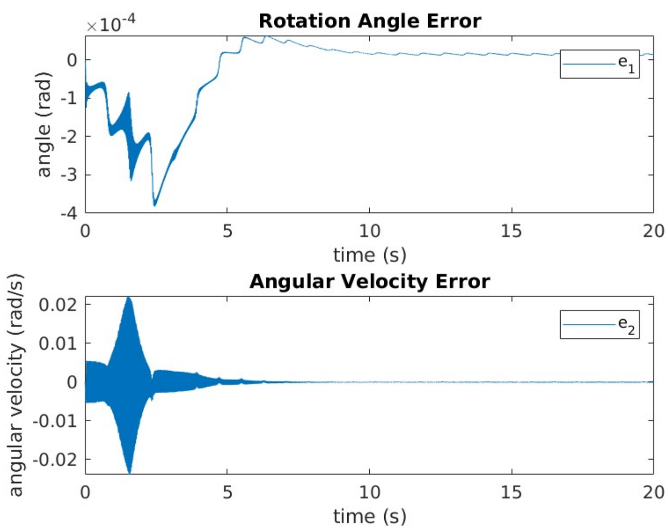

Figure 13: $r(t) = 4sin(4t)$

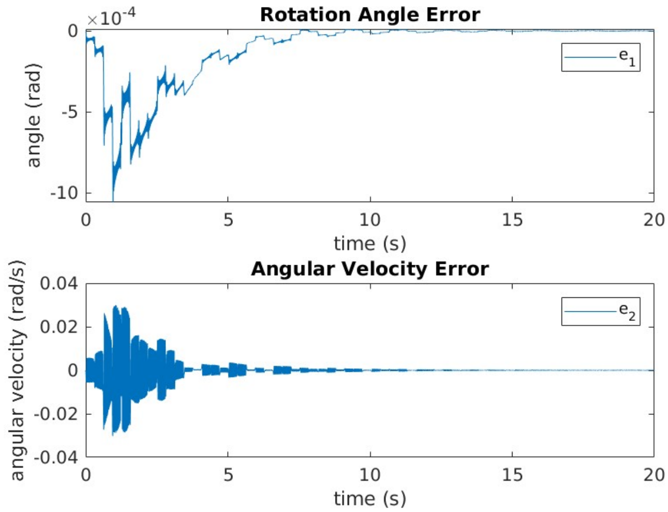

Figure 14: $r(t) = 4sin(10t)$

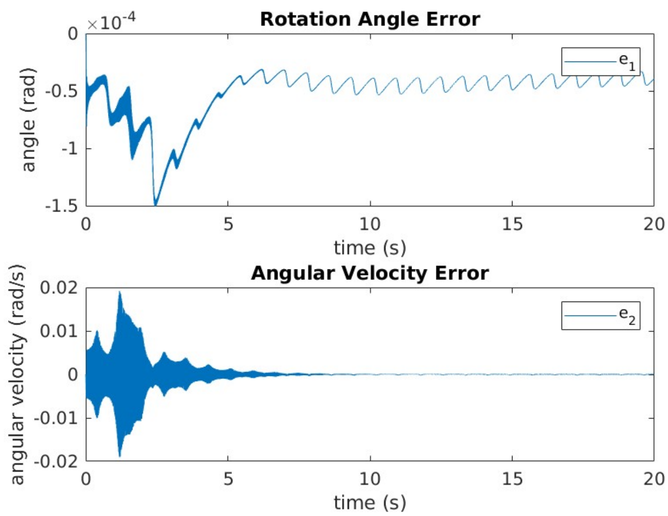

Figure 15: $r(t) = 10sin(4t)$

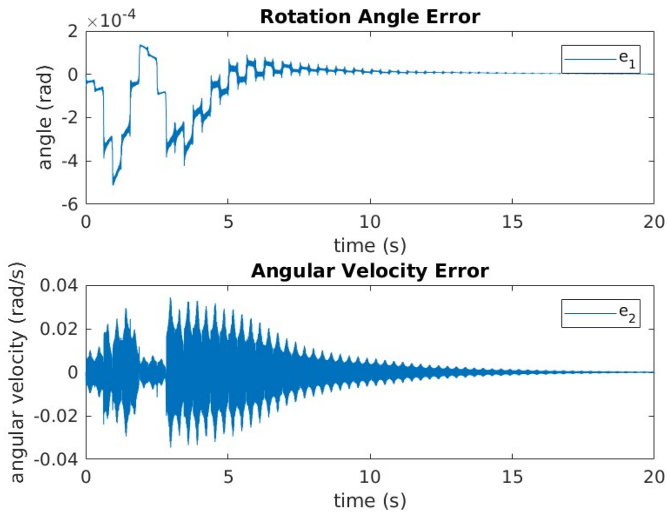

Figure 16: $r(t) = 10sin(10t)$

From the above diagrams, we can conclude three things:

- If input's amplitude or frequency increases while the other one is stable, then system needs more time to reach the stable state, but the delay added, to reach the stable state, is not a worrying event.
- Γ parameter is really important for convergence, because it reduces drastically the time needed for convergence.
- This type of control solves the problem that we saw on the case of the linear controller. It does not matter if we increase the frequency of the input signal, the delay added to reach the stable state is not a worrying event.

3.3 Linear System Controller with Disturbances

We will continue with the simulations of the controller, which is based on the linear system, but this time we will add disturbances. When our system manage to reach the stable state, we will add a disturbance for 5 seconds. We have selected $\rho_0 = 1$, $\lambda_0 = 5$ and diagonal values of Γ equals to 1000. Also, we will use a as input the signal $r(t) = 2sin(t)$ and we will test how the behaviour changes based on the amplitude of the disturbance.

We will test for different values of amplitude [10, 30, 50, 70, 90, 110, 200].

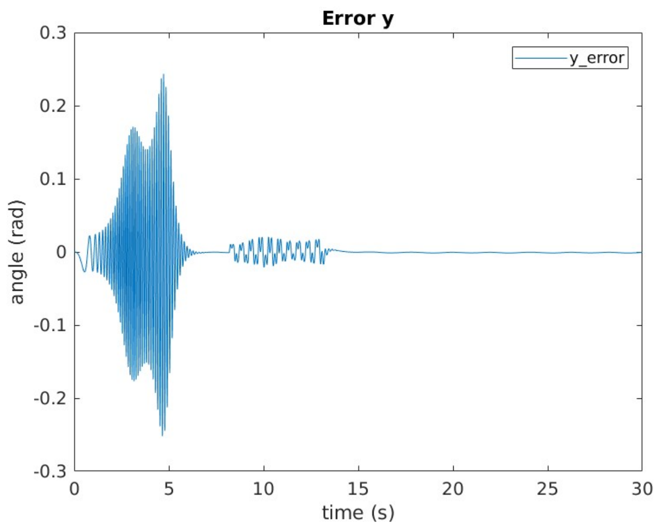

Figure 17: $d(t) = 10sign(sin(2\pi 2t)), f = 2$

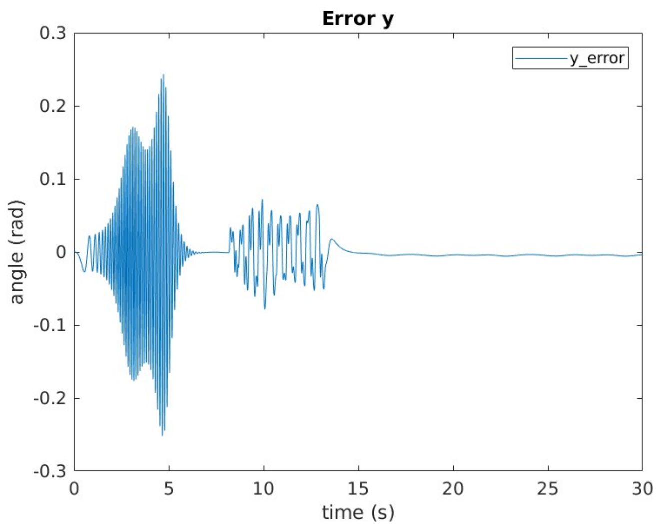

Figure 18: $d(t) = 30sign(sin(2\pi 2t)), f = 2$

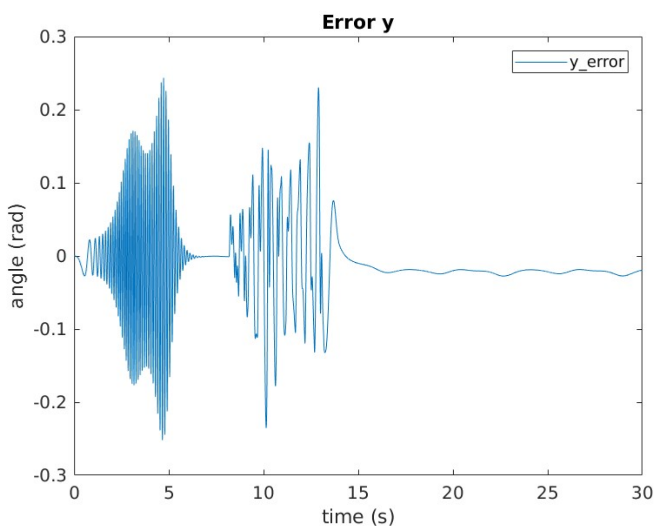

Figure 19: $d(t) = 50sign(sin(2\pi 2t)), f = 2$

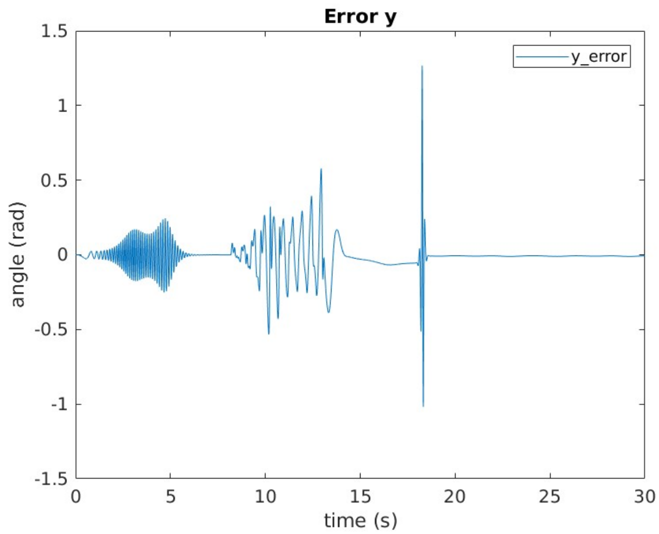

Figure 20: $d(t) = 70sign(sin(2\pi 2t)), f = 2$

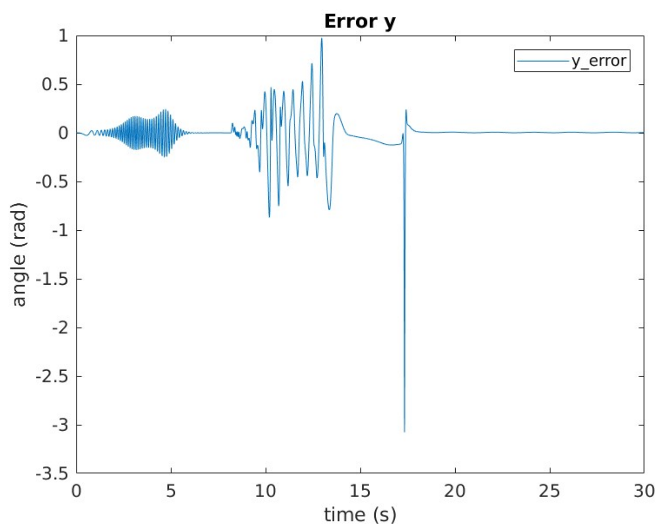

Figure 21: $d(t) = 90sign(sin(2\pi 2t)), f = 2$

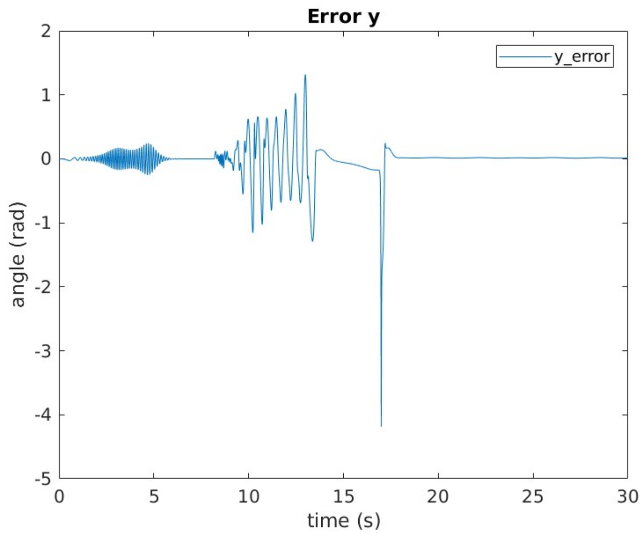

Figure 22: $d(t) = 110sign(sin(2\pi 2t)), f = 2$

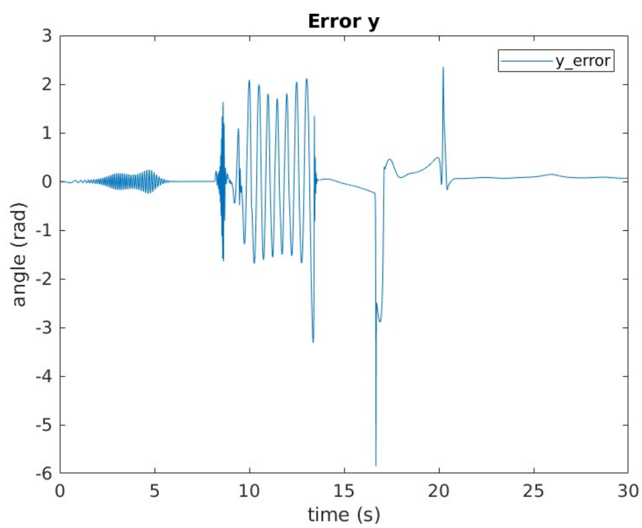

Figure 23: $d(t) = 200sign(sin(2\pi 2t)), f = 2$

From the above diagrams, we can conclude that the linear controller does not perform well above a certain value of disturbance's amplitude and this causes problems to the system even after the end of the 5 second disturbance's period and sometimes we see that it is hard for the controller to put the back to the stable state. The bigger the amplitude is the more time the system needs to recover and the more unusual behaviour (check the random error spikes) is being unveiled.

3.4 Non Linear System Controller with Disturbances

We will continue with the simulations of the controller, which is based on the non linear system, but this time we will add disturbances. When our system manage to reach the stable state, we will add a disturbance for 5 seconds. We have selected $Q = \begin{bmatrix} 150 & 0 \\ 0 & 150 \end{bmatrix}$, which set the values for P matrix, solving the Lyapunov equation, and γ parameters are equal to 1000. Also, we will use a as input the signal $r(t) = 2sin(t)$ and we will test how the behaviour changes based on the amplitude of the disturbance.

We will test for different values of amplitude [10, 30, 50, 70, 90, 110, 225].

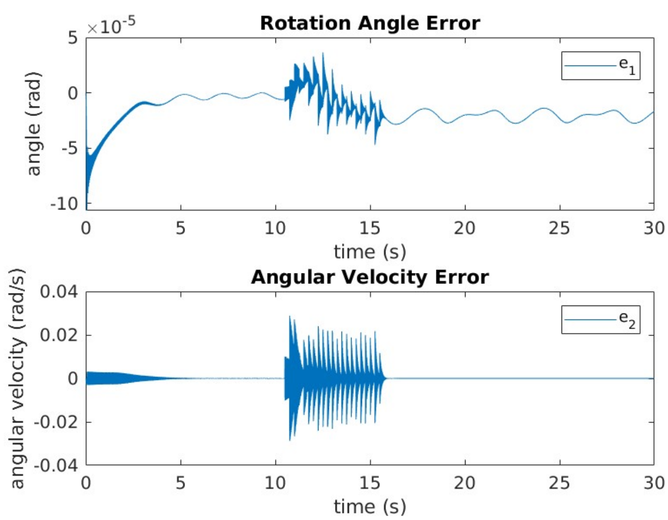

Figure 24: $d(t) = 10sign(sin(2\pi 2t)), f = 2$

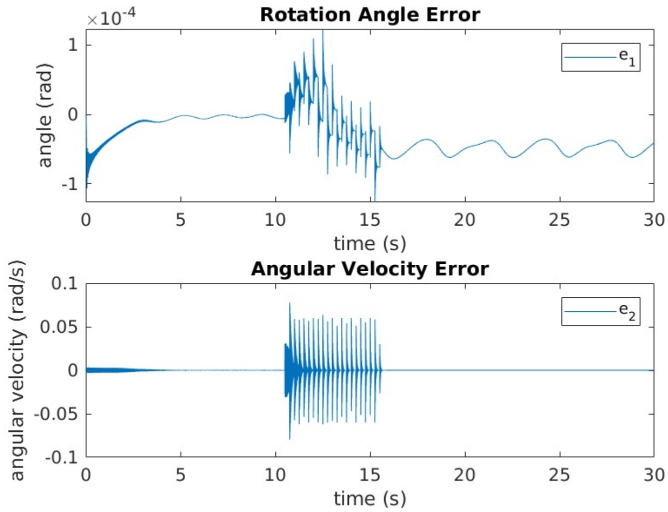

Figure 25: $d(t) = 30sign(sin(2\pi 2t)), f = 2$

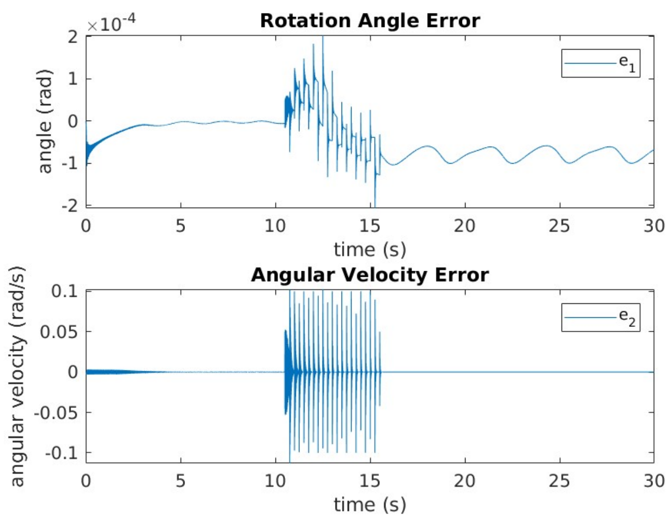

Figure 26: $d(t) = 50sign(sin(2\pi 2t)), f = 2$

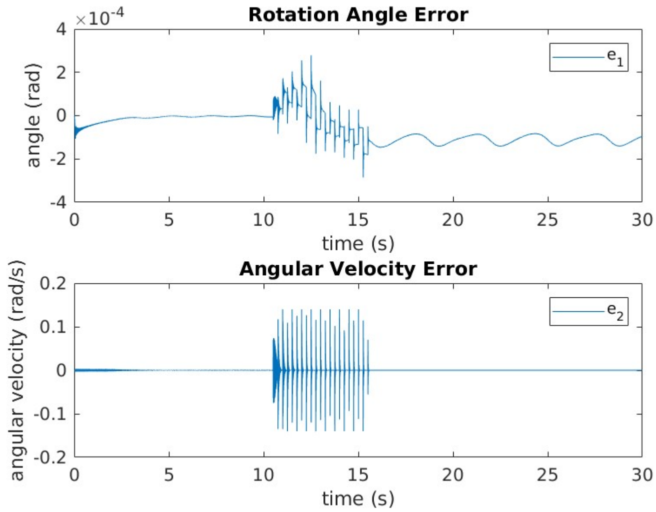

Figure 27: $d(t) = 70sign(sin(2\pi 2t)), f = 2$

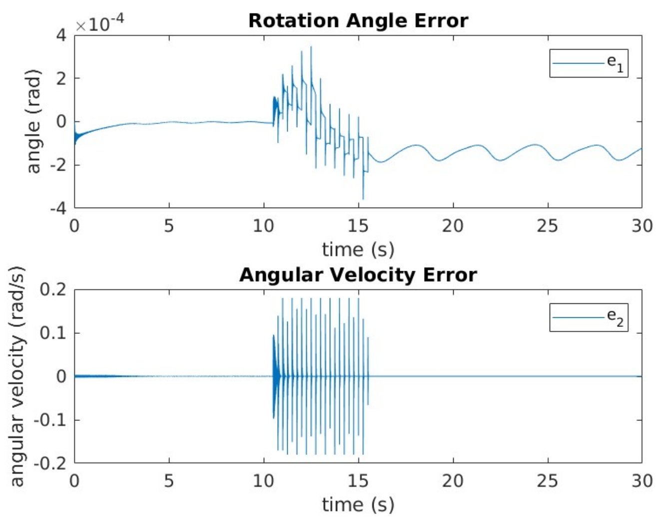

Figure 28: $d(t) = 90sign(sin(2\pi 2t)), f = 2$

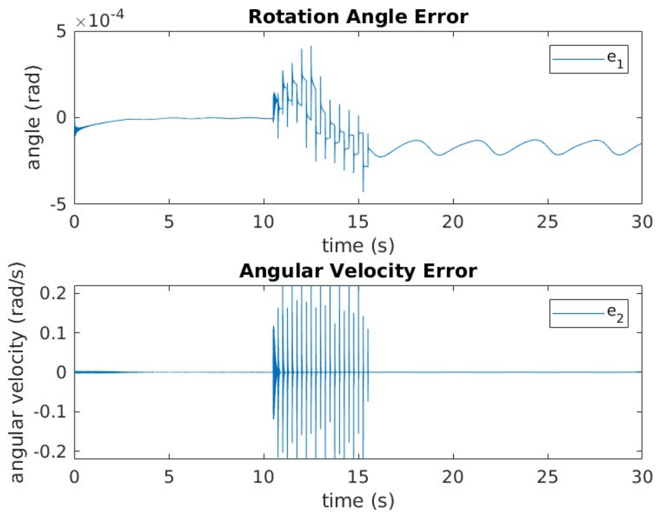

Figure 29: $d(t) = 110 sign(sin(2\pi 2t)), f = 2$

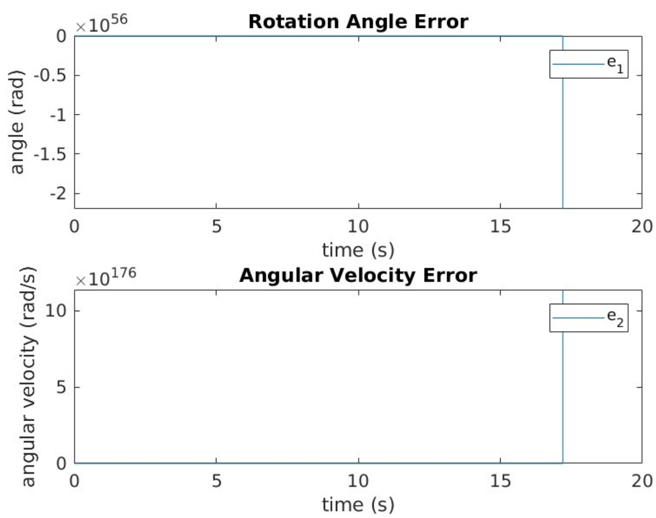

Figure 30: $d(t) = 225 sign(sin(2\pi 2t)), f = 2$

From the above diagrams, we can conclude that the non linear controller performs pretty well despite the amplitude of the disturbance. Of course, if we use a high enough value we get values to infinity and NaN values and the simulation stops. There is no unusual behaviour after the disturbance ends, except from the fact that the specific value of error, when the disturbance ends, becomes the new value around which, the following values of error will be.

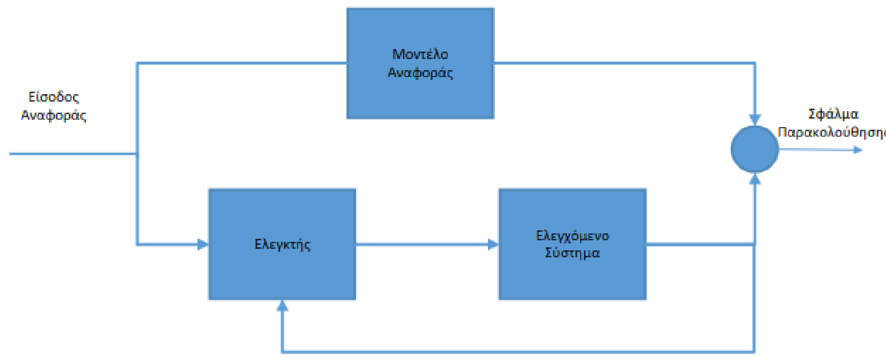

Figure 31: MRAC Block Diagram

3.5 Linear System Controller with Disturbances, d is constant

We will run the same simulation, but this time d is constant.

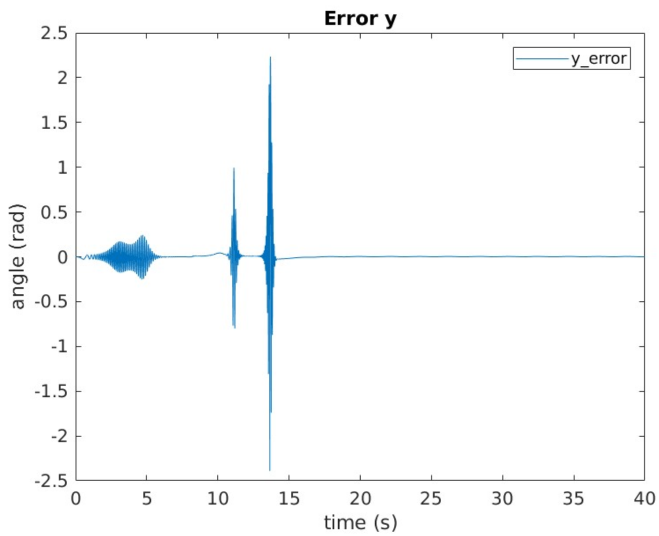

Figure 32: $d = 10$

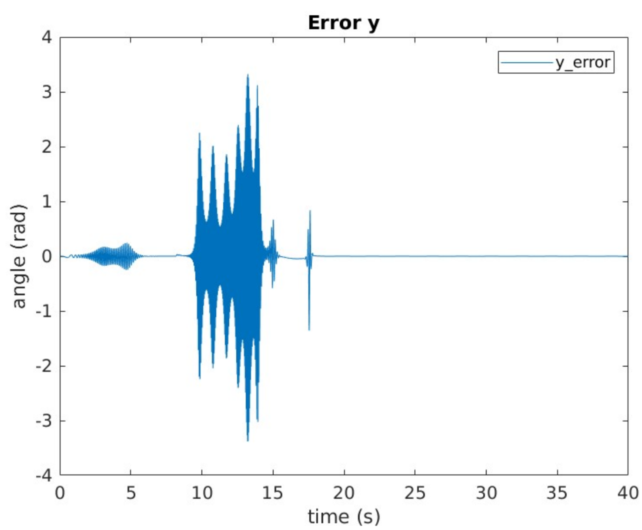

Figure 33: $d = 30$

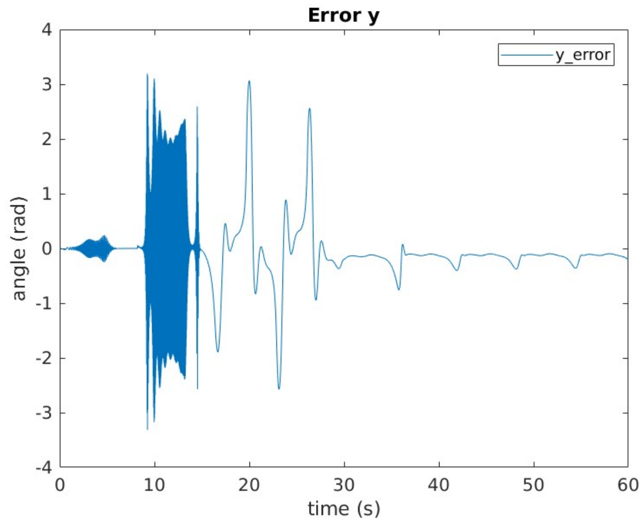

Figure 34: $d = 50$

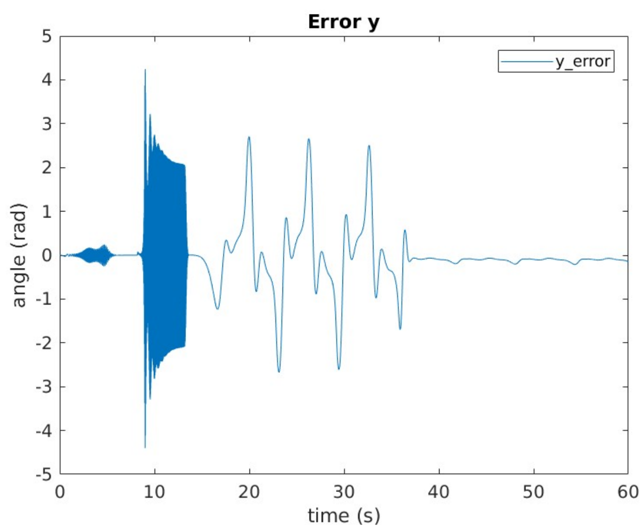

Figure 35: $d = 70$

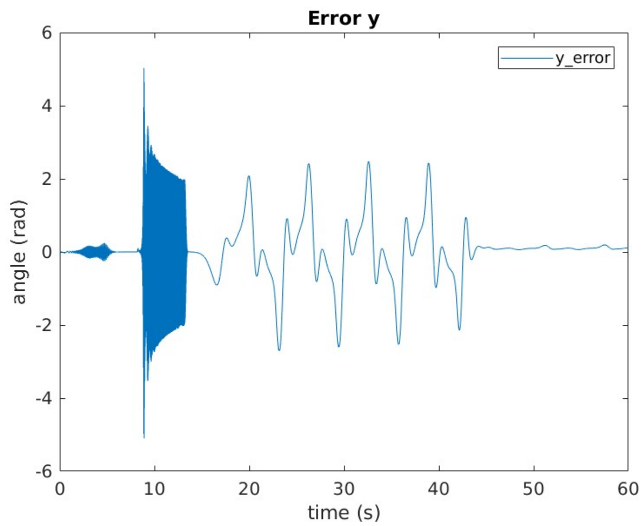

Figure 36: $d = 90$

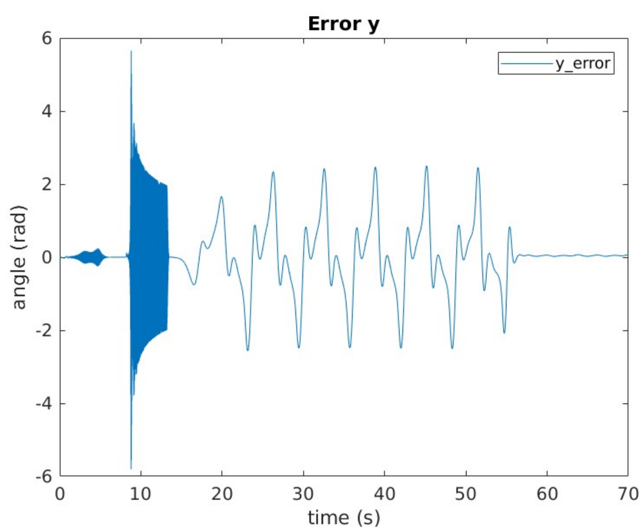

Figure 37: $d = 110$

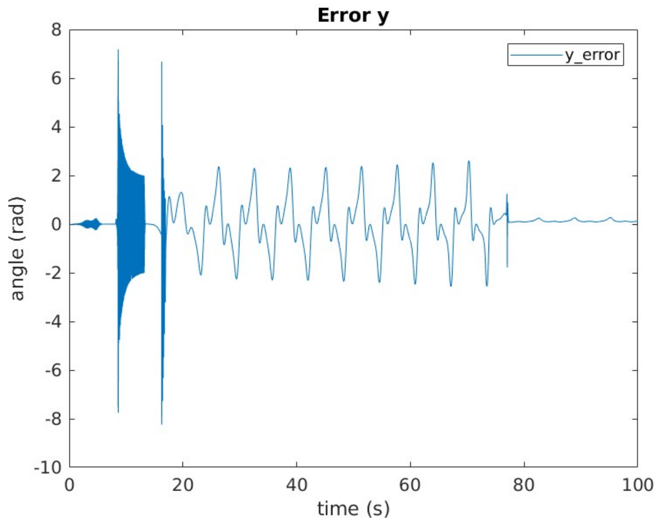

Figure 38: $d = 200$

We can see the same problem still exist. The main points of the simulation that the controller has to adjust is when the disturbance starts and when the disturbance ends. In between, even with the disturbance active the controller tries to control the system. But again when the amplitude is too high, after the disturbance the controller needs a significant amount of time in order to be able to drive the system to the stable state.

3.6 Non Linear System Controller with Disturbances, d is constant

Again, We will run the same simulation, but this time d is constant.

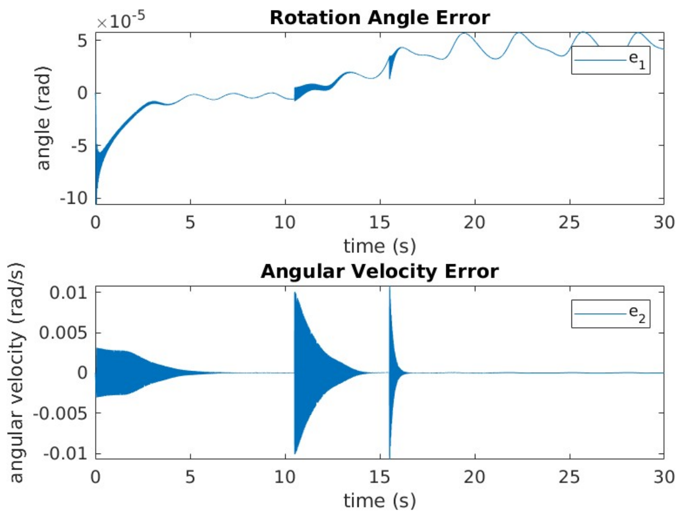

Figure 39: $d = 10$

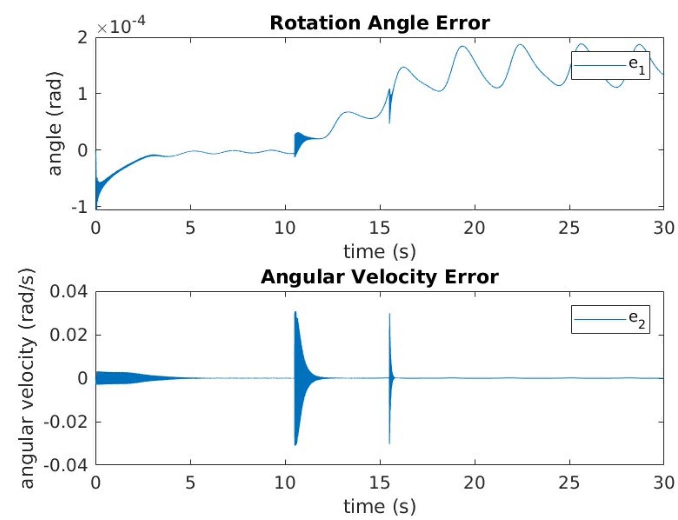

Figure 40: $d = 30$

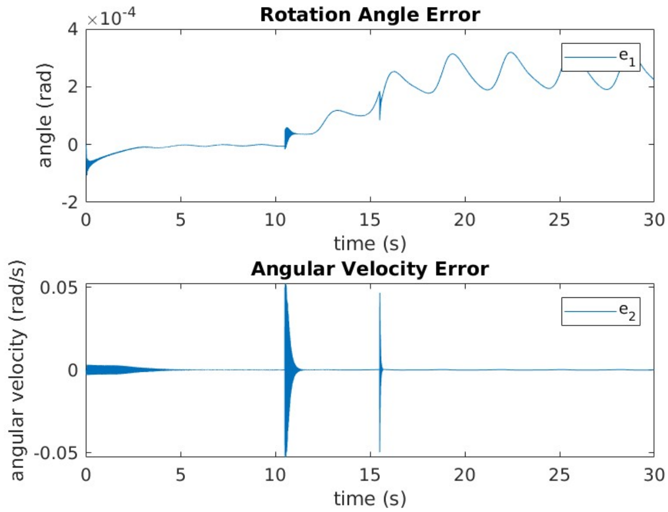

Figure 41: $d = 50$

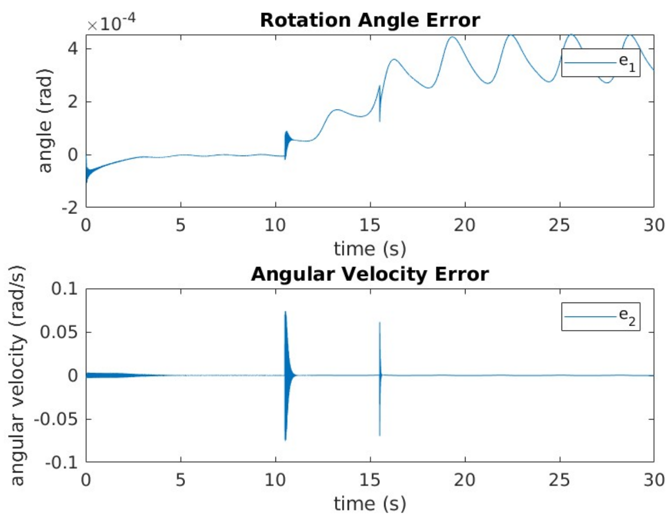

Figure 42: $d = 70$

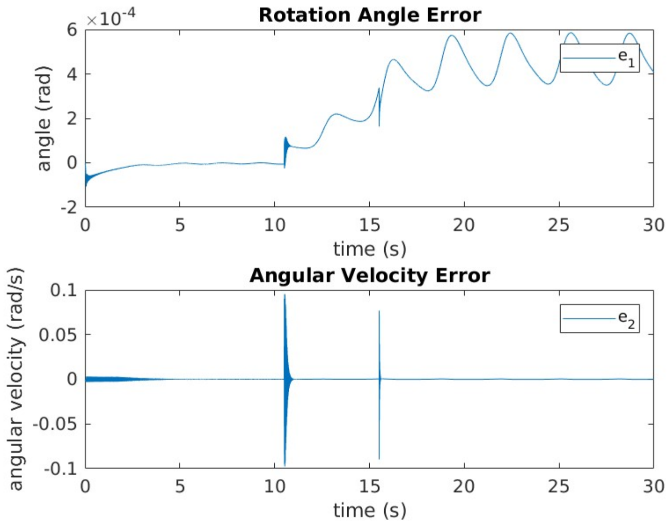

Figure 43: $d = 90$

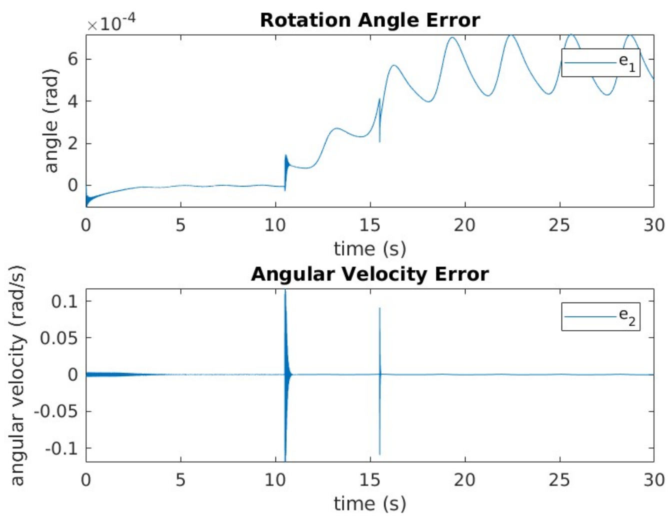

Figure 44: $d = 110$

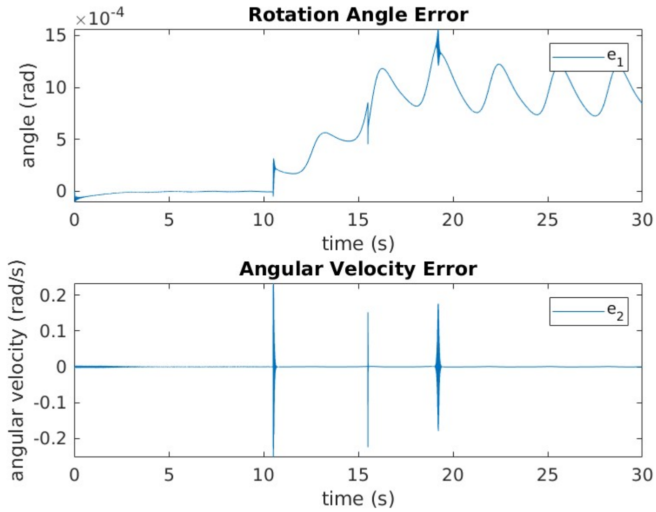

Figure 45: $d = 225$

We can see that non linear controller is not affected from disturbances again, of course there area some minor ups and downs but nothing important. We actually see two spikes, that is because when the disturbance starts, the system has some anomalies for less than one second and then it get stabilized again until the disturbance ends where the anomalies are back for less than one second again and the new stable state is e bit different from the initial one. Basically, almost the same thing as before, when the d was not constant. We also, have to mention that if we increase the amplitude enough, we see another spike about 5 minutes after the ending of the disturbance, but the controller does a good job again and after one second the system is back to its stable state.

3.7 Linear System Controller with Disturbances, $d = asin(\omega t)$

We will run the same simulation, but this time $d = asin(\omega t)$.

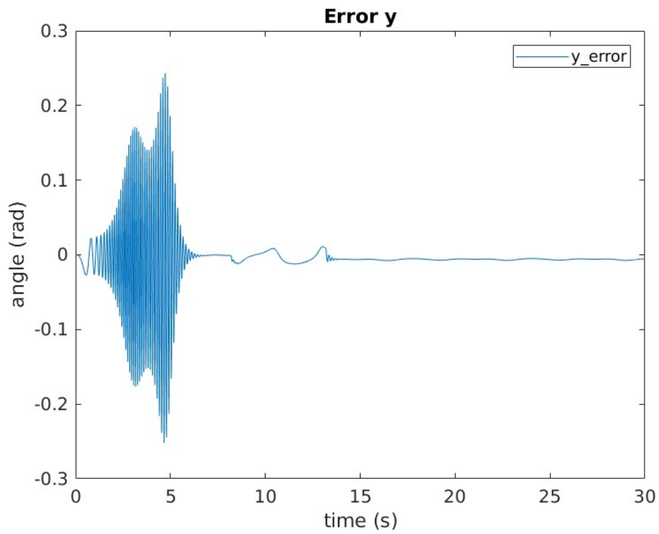

Figure 46: $d = 10$

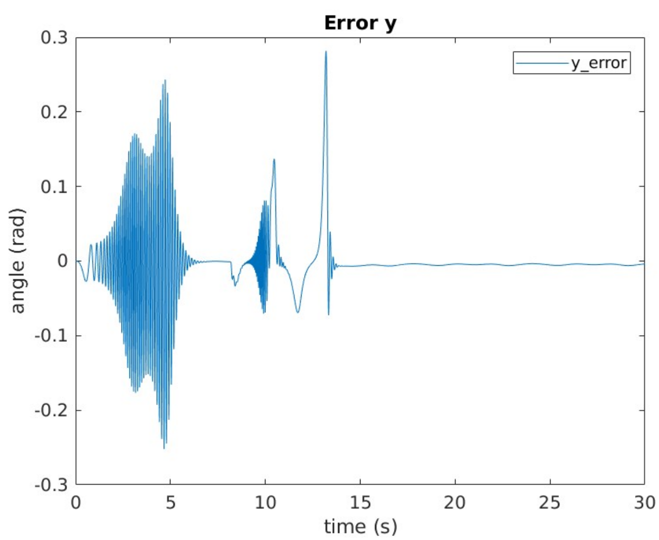

Figure 47: $d = 30$

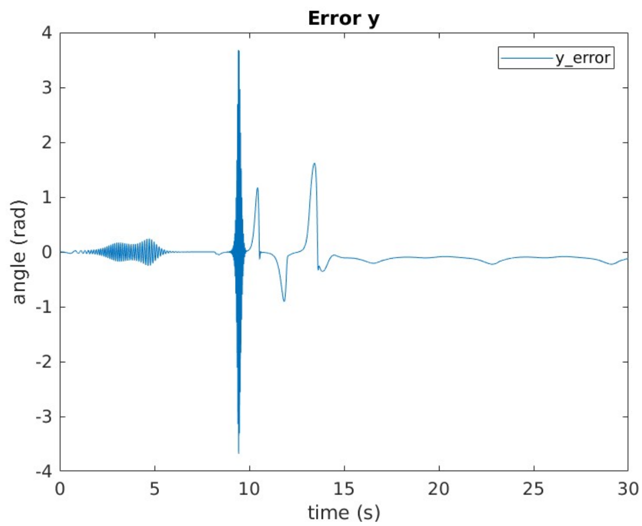

Figure 48: $d = 50$

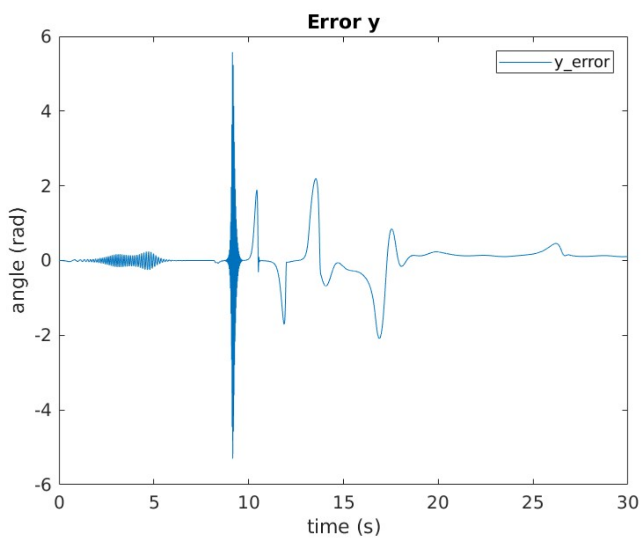

Figure 49: $d = 70$

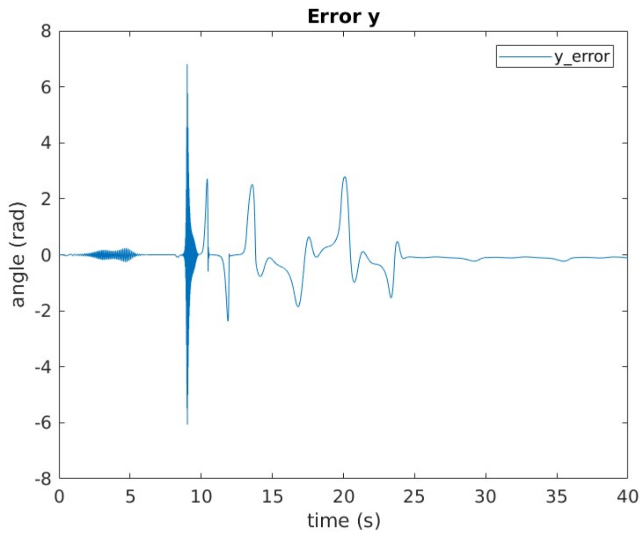

Figure 50: $d = 90$

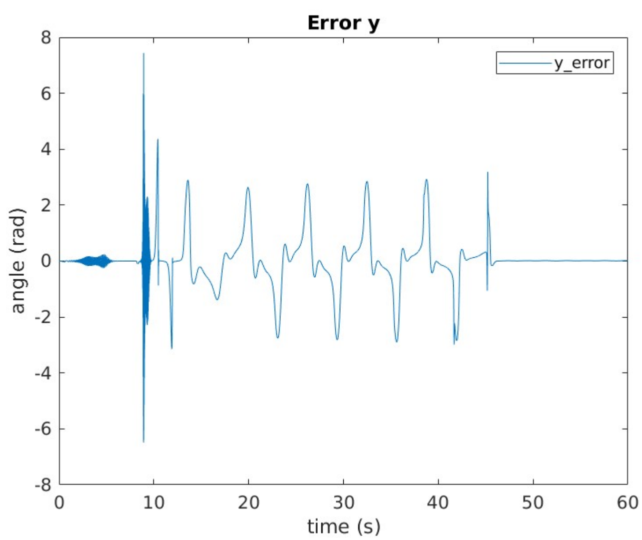

Figure 51: $d = 110$

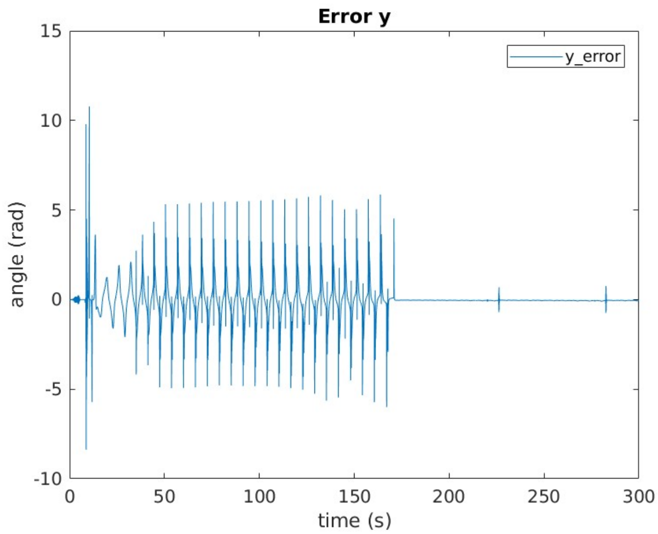

Figure 52: $d = 200$

We can see the same problem still exist. The disturbance still affects the system even after its ending and the controller needs some time to control the system, when the disturbance's amplitude is high enough.

3.8 Non Linear System Controller with Disturbances, $d = asin(\omega t)$

Again, we will run the same simulation, but this time $d = asin(\omega t)$.

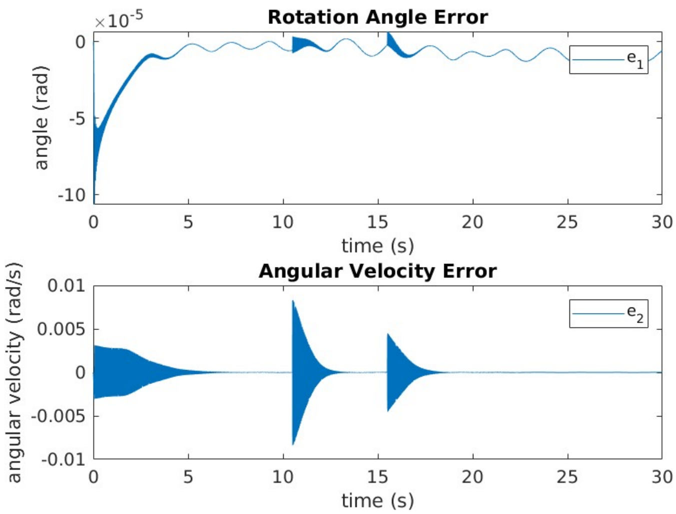

Figure 53: $d = 10$

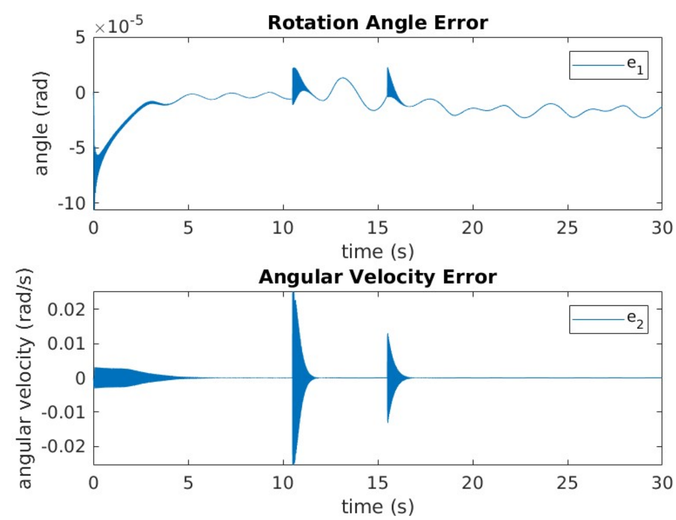

Figure 54: $d = 30$

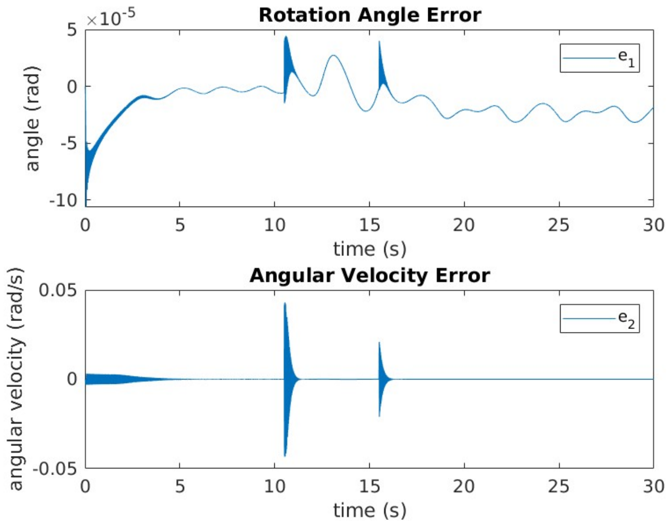

Figure 55: $d = 50$

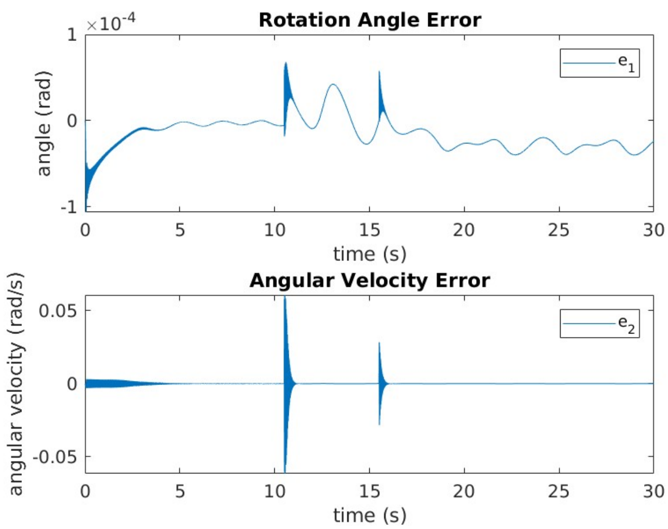

Figure 56: $d = 70$

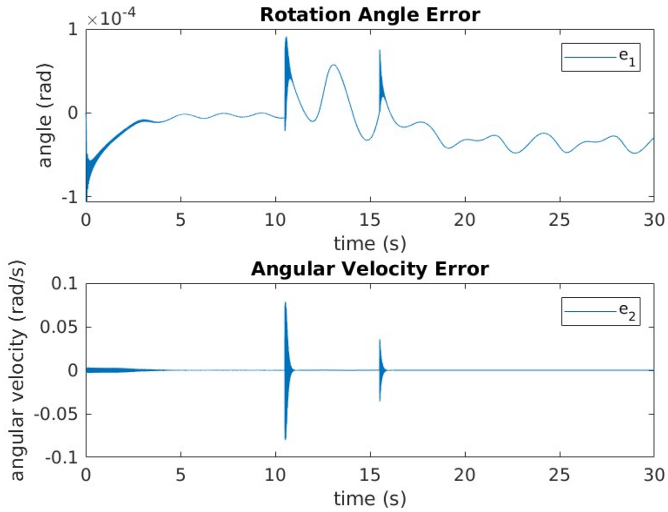

Figure 57: $d = 90$

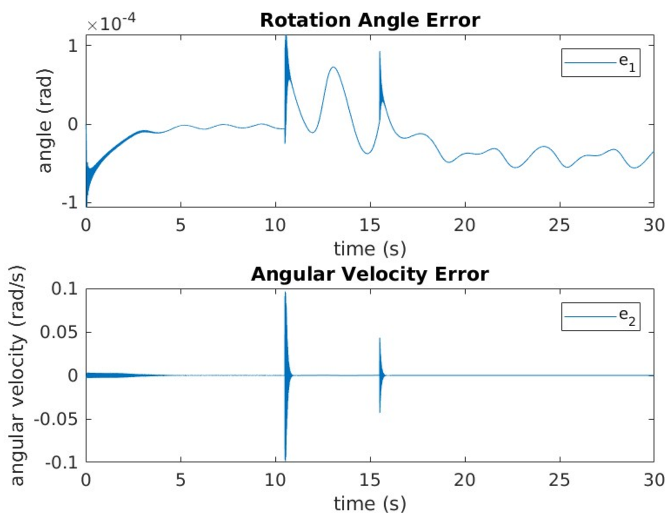

Figure 58: $d = 110$

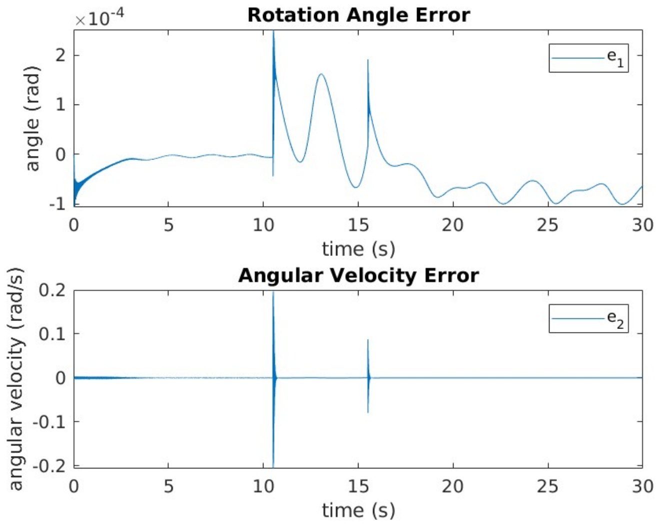

Figure 59: $d = 225$

Again, the non linear controller has no problem controlling the system on whatever value for disturbance's amplitude we use, and after the end of the disturbance, almost immediately the system reach a stable state again.

4 Conclusion

From the above results we can conclude that:

- Both controllers are able to drive the system to the stable state after a specific period of time.
- Linear system controller has troubles when we increase the frequency of the input signal and all comes down to the time needed to reach stable state.
- Non linear controllers are capable of keeping the errors' values low and drive the system to a more stable state (a state with lower error's values).
- Linear system controller can deal with disturbances, but they have a limit on amplitude. If disturbances pass that limit, we lose control of the system even after the ending of the disturbance and we need more time to control the system again, in some cases the stable state after the disturbance is not as stable as the stable state before the disturbance.
- Non linear controller, again has no problem dealing with the increase of the disturbance's amplitude. It performs pretty well, except from really high values of amplitude, when the calculation are not feasible, or we see some spikes for some milliseconds.
- It is worth to mention that after any of the disturbances, the system that uses the non linear controller reaches the stable state almost immediately, but this is a new stable state, not the same one as before. Although, the error is really low so this difference is not important.

In the zip file accompanying this report, you will find the plots that has been used to create this report.