

# The Math Behind Escher

1°) Transform coordinate plane to complex plane

Use either of these:

↳ Rectangular Form:  $Z = x + iy$

↳ Polar " :  $Z = r \cdot \cos(\theta) + r \cdot \sin(\theta) i$

↳ Exponential :  $Z = r \cdot e^{i\theta}$

Note: the rectangular form is the one that works best in Python, the others have rounding err.

2°) Apply  $\log$  to the complex plane  $\rightarrow \mathcal{L}(z)$

called  $\mathcal{L}(w)$  in paper

This is the term 'lnr' in Python  
(natural log of the radius =  $\mathcal{L}nr$ )

Depending on the complex representation,  
some tricks apply:



↳ On the exponential repres. of complex:

$$z = |r| e^{i\theta}$$

$$\ln(z) = \ln(|r|) + i\theta \Rightarrow \text{this is the one used by the shader code}$$

But Python has rounding errors with this one!

### 3:) Escher transform

From the paper:  $h/w = w^\alpha$   
where 'w' is the complex plane

$$\alpha = \frac{2\pi i + \ln(256)}{2\pi i} = 1 - \frac{\ln(256)}{2\pi i}$$

$$\left( \frac{(2\pi i + \ln(256))(-i)}{2\pi i} \right) \xrightarrow{(-i)(i)=1} \frac{2\pi \cdot 1 - \ln(256)i}{2\pi \cdot 1} = 1 - \frac{\ln(256)}{2\pi}$$



4:) Combine all steps  $\rightarrow$  Complex + Log +  
transform + Exp

$$h(w) = w^\alpha \quad / \text{trick: } e^{L(x)} = x$$

$$L(z^b) = b L(z)$$

$$h(w) = e^{L(w^\alpha)}$$

$$h(w) = e^{\alpha L(w)}$$

$L(w)$  is the log of the  
complex plane we had before

Note that if we do  $e^{L(w)} = w \rightarrow$  what we had  
started with

$\hookrightarrow$  so the term  $\alpha$  is the rotation