What is Chi-Square Distribution?

$$\frac{X-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$\Rightarrow \left(\frac{X_1-\mu}{\sigma/\sqrt{n}}\right)^2 + \left(\frac{X_2-\mu}{\sigma/\sqrt{n}}\right)^2 + \cdots \sim \chi^2_n$$

It can be shown that

$$\left(\frac{X_1-\bar{X}}{\sigma/\sqrt{n}}\right)^2 + \left(\frac{X_2-\bar{X}}{\sigma/\sqrt{n}}\right)^2 + \left(\frac{X_3-\bar{X}}{\sigma/\sqrt{n}}\right)^2 + \cdots$$

follows chi-square (χ^2_{n-1})

$$\Rightarrow \frac{(n-1)S^2}{\sigma^2} = \frac{\Sigma(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

[Assume that we found S_1^2, S_2^2, S_3^2, etc as your observations]

$\downarrow$

what shd be your value of σ^2

Proof that it follows a chi-square distribution:

$$\Sigma\left(\frac{X-\mu}{\sigma/\sqrt{n}}\right)^2 = \Sigma\frac{X^2+\mu^2-2\mu X}{\sigma/\sqrt{n}}$$

$$= \frac{(\Sigma X^2)+\mu^2(n)-2(\Sigma\mu X)}{\sigma/\sqrt{n}} = \frac{(\Sigma X^2)+n\mu^2-2\mu n\bar{X}+n\bar{X}^2-n\bar{X}^2}{\sigma/\sqrt{n}}$$

$$= \underbrace{(\Sigma X^2)+n\bar{X}^2-2(\Sigma X)\times\bar{X}}_{\Sigma(X-\bar{X})^2} + \underbrace{2(\Sigma X)(\bar{X})}_{2n\bar{X}^2} - 2\mu n\bar{X} - n\bar{X}^2 + n\mu^2$$

$$+ \left(n\bar{X}^2 - 2\mu n\bar{X} + n\mu^2\right)$$

$$\downarrow$$

$$\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right)^2 \rightarrow N(0,1)$$

From CLT for $n>30$

$\therefore$ This follows χ^2 with $(n-1)$ dof.

T-Distribution Definition:

Definition of T-distribⁿ.

$$\frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}}{\sqrt{s^2/\sigma^2 \times \frac{(n-1)}{(n-1)}}} = \underbrace{\frac{N(0,1)}{\sqrt{\dfrac{\chi^2_{df=n-1}}{n-1}}}}$$

This is the t- distribution

So, if the no. of the estimators in the regression equation is P, then the t-stat for individual coeffficients follow a t-dist with n-(P+1) dof

number of dof for Regression coeffiecents

→ If $Z_1, Z_2, Z_3 \cdots Z_K$ are Standardized normal distributions, then $Z = \sum_{i=1}^{K} Z_i^2$ follows a Chi-Square distribution with K dof.

→ If Z_l is a Standard normal distribution, and Z_m is a χ^2_K, then $t = \dfrac{Z_l}{\sqrt{\frac{Z_m}{K}}}$ follows a t-distribution dof K (Also Z_l & Z_m have to be independent)