Automatic Measurement of Complex Dielectric Constant and Permeability at Microwave Frequencies

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***Abstract*—With the advent of the computer and automatic test equipment, new techniques for measuring complex dielectric constant (ϵ) and permeability (μ) can be considered. Such a technique is described where a system is employed that automatically measures the complex reflection and transmission coefficients that result when a sample of material is inserted in waveguide or a TEM transmission line. Measurement results of ϵ and μ for two common materials are presented.**

Introduction

THE MEASUREMENT of complex dielectric constant and complex permeability is required not only for scientific but also for industrial applications. For example, areas in which knowledge of the properties of materials at microwave frequencies (as described by ϵ and μ) are applications of microwave heating, biological effects of microwaves, and nondestructive testing.[1]

Numerous measurement methods suitable for different ranges of the numerical values of ϵ and μ have been given in the books edited by Von Hippel [2], [3] and in publications of the American Society for Testing and Materials. It is possible, however, to rapidly make measurements over the frequency range from 100 MHz to 18 GHz with a computer-controlled network analyzer such as the Hewlett-Packard Model 8540 series, and by means of appropriate data processing, to determine the complex values of ϵ and μ for materials.

Using the method described in this paper, the complex values of ϵ and μ are determined from measurements made directly in the frequency domain. A somewhat analogous method has been developed [7] where measurements are made in the time domain of the transient response to subnanosecond pulses from a dielectric material. With the time-domain measurement approach, a Fourier transformation is required to determine ϵ and μ from the measured transient response. Furthermore, with this approach, the frequencies at which ϵ and μ values are obtained are band-limited, depending on the time response of the pulse and its repetition frequency. Using the system described in this paper, discrete frequencies in less than 20-kHz steps may be selected anywhere within the entire 100-MHz to 18-GHz band.

Automatic Measurement System

A computer-controlled network analyzer is used to measure the parameters of a network consisting of a section of transmission line containing the sample of material. The transmission line section may either be waveguide or a TEM transmission line. The network is shown schematically in Fig. 1. If coaxial-to-waveguide adapters are required, additional lengths of waveguide are inserted between them and the sample holder. This is to insure that the higher order evanescent modes due to the coaxial-to-waveguide adapters are significantly attenuated prior to reaching the sample under test.

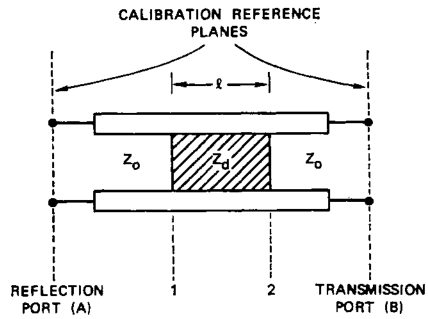

Fig. 1. Transmission line section containing dielectric material.

Under computer control, network analyzer system calibration and measurement are obtained at the reference planes indicated in Fig. 1. This is done over a number of predetermined frequencies at which the complex values of ϵ and μ for the material are to be determined. The normalized scattering parameters (S_{ij}) of the transmission line section containing the material are measured at the calibration reference planes at ports A and B and corrected for system errors included in the calibration data. The measured scattering parameters are normalized to the characteristic impedance (Z_0) of the transmission line section. The reflection coefficient (S_{11}) at the air-to-dielectric interface, and the transmission coefficient (S_{21}) through the material, are found at reference planes 1 and 2. These coefficients are found directly from the measured scattering parameters after the appropriate phase corrections have been applied to account for the shift in the reference planes from ports A and B to the material interfaces (i.e., planes 1 and 2, Fig. 1).

From the complex reflection and transmission coefficients, the computer associated with the network analyzer determines the real and imaginary parts of the dielectric constant and permeability, the loss tangent, and the attenuation per unit length of material. In addition, data taken at several frequencies are used to find the average group delay through the sample. Average group delay, in turn, is used to automatically resolve phase ambiguities that result when the sample length of material is greater than a wavelength in the dielectric. The complex ϵ and μ, loss tangent, and attenuation data are automatically listed by a teleprinter, or rapidly plotted on an $X-Y$ recorder, or both.

Data Processing Techniques

The equations used to compute complex ϵ and μ from the measured reflection (S_{11}) and transmission (S_{21}) coefficients are presented below together with the equations that relate ϵ

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[1] For an example of a system that can be used for nondestructive testing see [1].

and μ to attenuation and loss tangent. As will be seen, an infinite number of roots exist in the solution for the equations ϵ and μ, but the correct root is related to the length of the sample in terms of wavelength within the material. The means of determining the correct solution is shown.

Referring to Fig. 1, the propagation factor for a wave propagating through the material is defined as [4]

$$P = e^{-\gamma l} = e^{-(\alpha + j\beta) l} \tag{1}$$

where

- γ propagation constant,
- α attenuation constant,
- β phase constant.

A time factor of $e^{j\omega t}$ is not explicitly shown in (1). The phase constant is equal to

$$\beta = \frac{2\pi}{\lambda_g} \tag{2}$$

where λ_g is the transmission line guide wavelength. Consistent with the definition of the propagation factor [(1)], ϵ and μ are defined in terms of their real and imaginary parts as follows:

$$\epsilon = \epsilon_r \epsilon_0 = (\epsilon_r' - j\epsilon_r'')\epsilon_0 \tag{3}$$

$$\mu = \mu_r \mu_0 = (\mu_r' - j\mu_r'')\mu_0. \tag{4}$$

The reflection coefficient (Γ) at the interface between the air-filled transmission line and dielectric-filled line when the material sample is infinite in length may be found from the measured reflection (S_{11}) and transmission (S_{21}) coefficients for a sample of finite length (l):[2]

$$\Gamma = \chi \pm \sqrt{\chi^2 - 1} \tag{5}$$

where

$$\chi = \frac{S_{11}^2 - S_{21}^2 + 1}{2 S_{11}}. \tag{6}$$

The propagation factor P can, in turn, be found from S_{11}, S_{21}, and Γ:

$$P = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma}. \tag{7}$$

The complex dielectric constant and permeability can be determined from P and Γ:

$$\frac{1}{\Lambda^2} = \left(\frac{\epsilon_r \mu_r}{\lambda_0^2} - \frac{1}{\lambda_c^2}\right) = -\left[\frac{1}{2\pi l} \ln\left(\frac{1}{P}\right)\right]^2 \tag{8}$$

$$\mu_r = \frac{1 + \Gamma}{\Lambda(1 - \Gamma)\sqrt{\dfrac{1}{\lambda_0^2} - \dfrac{1}{\lambda_c^2}}} \tag{9}$$

where

- λ_0 free space wavelength,
- λ_c cutoff wavelength of the transmission line section ($\lambda_c = \infty$ for a TEM line),

and

$$\operatorname{Re}\left(\frac{1}{\Lambda}\right) = \frac{1}{\lambda_g}.$$

[2] In terms of the air-filled line characteristic impedance Z_0 and the dielectric-filled line characteristic impedance Z_d, $\Gamma = (Z_d - Z_0)/(Z_d + Z_0)$. Note that $S_{11} = \Gamma$ when l is infinite.

Equation (8) has an infinite number of roots since the imaginary part of the logarithm of a complex quantity [P in (8)] is equal to the angle of the complex value plus $2\pi n$, where n is equal to the integer of (l/λ_g). Equation (8) is ambiguous because the phase of the propagation factor P does not change when the length of the material is increased by a multiple of wavelength. However, the delay through the material is strictly a function of the total length of the material and can be used to resolve the ambiguity.

The phase ambiguity is resolved by finding a solution for ϵ and μ from which a value of group delay is computed that corresponds to the value determined from measured data at two or more frequencies. For this method to work, the discrete frequency steps at which measurements are obtained must be small enough so that the phase of the propagation factor (P) changes less than 360° from one measurement frequency to the next. With the use of an automatic measurement system as described in this paper, discrete frequency steps, small enough to meet this requirement, can easily be selected. The group delay at each frequency may be computed for each solution of ϵ and μ assuming that the changes in ϵ and μ are negligible over very small increments of frequency:

$$\tau_{gn} = l \cdot \frac{d}{df}\left[\left(\frac{\epsilon_r \mu_r}{\lambda_0^2} - \frac{1}{\lambda_c^2}\right)_n^{1/2}\right] \tag{10}$$

where f is the frequency in hertz and τ_{gn} is the group delay in seconds for the nth solution of (8) and (9). The measured group delay is determined from the slope of the phase of the propagation factor versus frequency:

$$\tau_g = \frac{1}{2\pi} \frac{d(-\phi)}{df} \tag{11}$$

where ϕ is the phase in radians of P. Accuracy in the determination of τ_g may be increased by applying numerical differentiation techniques [5] where the slope is computed using data for three or more frequencies. The correct root, $n = k$, is found when

$$\tau_{gk} - \tau_g \simeq 0.$$

Once the correct values of ϵ and μ have been found at each of the measurement frequencies, the loss tangent and attenuation per unit length may be determined. In general, for both an electrically and magnetically lossy material, a loss tangent is defined by[3]

$$\tan \delta = \frac{\delta_r''}{\delta_r'} \tag{12}$$

where

$$\delta_r' = \mu_r' \epsilon_r' - \mu_r'' \epsilon_r''$$

[3] Note that for the case when $\mu_r'' = 0$, $\tan \delta = \epsilon_r''/\epsilon_r'$.

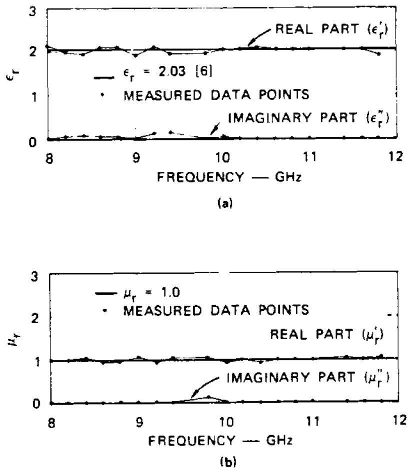

Fig. 2. Measured dielectric constant and permeability of Teflon in the X-band region. (a) Relative dielectric constant (ϵ_r). (b) Relative permeability (μ_r).

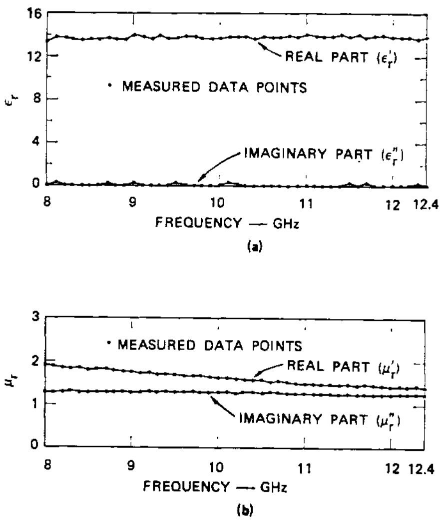

Fig. 3. Measured dielectric constant and permeability of Eccosorb SF-5.5 in the X-band region. (a) Relative dielectric constant (ϵ_r). (b) Relative permeability (μ_r).

and

$$\delta_r'' = \mu_r'\epsilon_r'' + \mu_r''\epsilon_r'.$$

The attenuation in nepers per length is found from the equation

$$\alpha = \frac{\pi\sqrt{2\delta_r'}}{\lambda_0}\left[\sqrt{1+\tan^2\delta} - 1\right]^{1/2}. \tag{13}$$

Measured Results

The results of measuring the complex ϵ and μ of Teflon and Emerson and Cuming Eccosorb SF-5.5 are shown in Figs. 2 through 4. Measurements of both materials were obtained with a sample inserted in X-band (RG-52) waveguide and at an ambient temperature of 22°C. The Teflon sample length was 1 in; the Eccosorb sample length was 0.090 in.

Teflon has a relatively low ϵ_r' (2.03 [6]) in the microwave region and has a very low loss at X band ($\tan\delta < 0.0004$ [2]). It is a nonmagnetic material and hence its relative permeability (μ_r) is 1. The measured data shown in Fig. 2 correspond closely to these known properties of Teflon.

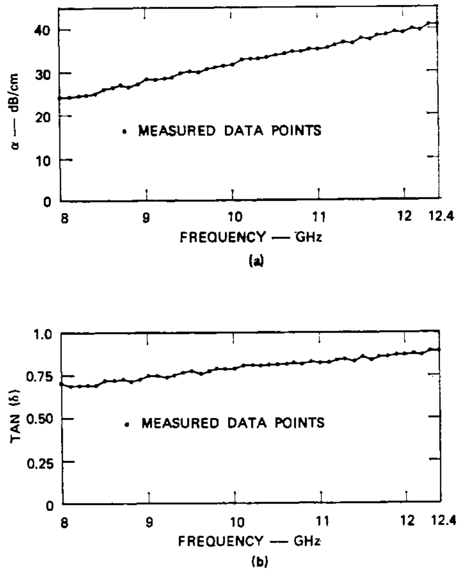

Fig. 4. Measured attenuation and loss tangent of Eccosorb SF-5.5 in the X-band region. (a) Attenuation (α). (b) Loss tangent [tan (α)].

Some noticeable variations in the 9.2- to 9.8-GHz frequency range are indicated in Fig. 2, however, from the average values of ϵ and μ. Furthermore, values (missing in Fig. 2) obtained at three frequencies were erroneous. These errors are attributed to the fact that the network analyzer signal source was not frequency stabilized at the time measurements were obtained on the Teflon material.[4] Without a frequency-stabilized signal source, the frequencies at the time of measurement do not exactly repeat those at the time of calibration, which produces errors in the phase of the measured scattering parameters. The accuracy of the values of complex ϵ and μ depends significantly on the accuracy of the measured scattering parameter phase data.

The Eccosorb SF-5.5 is an absorber type material which resonates at 5.5 GHz. The basic composition of the material is silicone rubber presumably loaded with lossy magnetic particles. When backed with a metallic surface, the material is reported to be effective in absorbing a normally incident wave at 5.5 GHz, although some absorption occurs at other frequencies and other angles of incidence as well. The requirement of a metallic surface backing suggests that the absorber is a magnetically lossy material which absorbs the high currents existing along the metallic surface excited by the incident wave. That the material is magnetically lossy is confirmed by Fig. 3 which indicates a value of μ_r'' greater than zero over X band. Fig. 3 also indicates that the material is not electrically lossy ($\epsilon_r'' \simeq 0$ over the band) but does have a relatively high dielectric constant ($\epsilon_r' \simeq 13.7$ over the band). Furthermore, μ_r' is greater than 1.0, varying from about 1.9 to 1.4 over the band. Attenuation and loss tangent[5] are shown in Fig. 4 for the Eccosorb material. Attenuation varies from about 24 to 40 dB/cm and loss tangent from about 0.7 to 0.9 over X band.

[4] A frequency-stabilized signal source was used, however, for the Eccosorb SF-5.5 measurements.

[5] Loss tangent ($\tan\delta$) $= \mu_r''/\mu_r'$, since $\epsilon_r'' = 0$.

Complex ϵ and μ data have been presented for two commercially available dielectric materials as examples of results obtained using the automatic measurement system described in this paper. This system, however, has also been found useful in determining the dielectric properties of rock, soil, and other nonstandard materials.

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Location and Recognition of Discontinuities in Dielectric Media Using Synthetic RF Pulses

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***Abstract*—RF pulses are used in radar and sonar to detect and locate targets in extended media. Short RF pulses (and impulses) can be used to find buried objects or voids by echo sounding, or can be used to probe snow fields or the depths of the earth. Although similar to radar in principle, there are important differences in these applications that can lead to significant variations in the design approach.**

A novel system is described, with a computer-programmed tunable RF source, that tunes rapidly through all the frequencies of a nanosecond pulse spectrum, makes individual "CW" measurements, stores them, and finally computes a synthetic echo. Target signatures can be "recognized" by calculating correlation functions. Experimental results are presented.

I. INTRODUCTION

PULSE SYSTEMS based on related technologies have been used for widely different applications. The two main types of echo sounding systems are 1) *radar and sonar*, where in general long RF pulses ("long" in terms of RF cycles) are used to locate moving objects in space hundreds or thousands of pulse lengths away, and 2) *time-domain reflectometry* (TDR), where a step function or impulse is used in a closed system to locate and diagnose a discontinuity.

It is possible to replace the pulsed transmitter by a swept frequency generator as in FM radar or in pulse compression systems.

From time to time unconventional applications come along. Pulse radar concepts have been applied to or considered for nondestructive testing, such as finding faults in dielectric bodies [1], finding old sewers or buried cables or abandoned coal mines in the ground, or finding burial chambers in ancient pyramids [2]. Radioglaciology is concerned with making RF soundings in glaciers [3]. RF waves have been used to determine the water content of snow fields, and considered for probing lunar and planetary surfaces [4]. Marine applications are more likely to use sound than electromagnetic waves, but the principles of echo sounding remain the same. In medicine, ultrasonic waves again have better penetration combined with resolution in biological tissue than electromagnetic waves.

What many of these applications have in common over the conventional radar scenario is, first of all, that the target does not move (or moves slowly), and second, a resolution of the order of one wavelength is often required or desirable. The system to be described in this paper makes CW measurements at many selected frequencies covering a wide spectrum. It uses a computer to control the sequence of measurements, to store the measured parameters, and to process them. Finally, a synthetic pulse echo is displayed on an oscilloscope or plotted on a chart recorder. Earlier work on this system has been reported briefly elsewhere [5].

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II. PRINCIPLE OF OPERATION

A. *Reasons for Choice of Synthetic-Spectrum Method*

To cope with the problems of generating a wide-band RF pulse, separate but coherent CW oscillators can be used to generate the frequencies of the pulse spectrum. After appropriate amplitude and phase adjustments, these signals can be combined to form a pulse train. This approach is referred to as the synthetic-spectrum technique.