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A Stepwise Nicolson-Ross-Weir-Based Material Parameter Extraction Method

Olli Luukkonen, Stanislav I. Maslovski, and Sergei A. Tretyakov *Fellow, IEEE*

***Abstract*—Approaches of automated evaluation of electromagnetic material parameters has received a lot of attention in the literature. Among others, one method is to retrieve the material parameters from the reflection and transmission measurements of the sample material. Compared to other methods, this is a rather wide-band method, but suffers from an intrinsic limitation related to the electrical thickness of the measured material. In this paper we propose a novel way to overcome this limitation. Although being based on the classical Nicolson-Ross-Weir (NRW) technique, the proposed extraction technique does not involve any branch seeking and is therefore capable of extracting material parameters from samples thicker than $\lambda/2$, a measure that would otherwise cause problems in the NRW extraction technique. The proposed derivative of the NRW extraction technique is then used to study the effect of thermal noise on the extracted material parameters.**

I. Introduction

In the most simple cases the electromagnetic properties of materials are described plainly with permittivity and permeability. The former of these two is used to describe the electric response of a given material to an electric stimulus whereas the latter is used to describe the magnetic response to a magnetic stimulus, provided that these are physically meaningful. The material parameter extraction methods have recently attracted attention in the literature due to the grown interest towards metamaterials and the need to characterize the electromagnetic properties of these complex man-made materials.

Among the different extraction techniques, there exists a class of methods that are based on measurements (or numerical simulations) of the reflection and transmission coefficients of a planar material sample. The classical Nicolson-Ross-Weir (NRW) technique [1]–[3] belongs to this class, as well as many other more recent methods [4]–[9]. The intrinsic problem of the NRW technique relates to the electrical thickness of the material sample. Namely, the phase of the electromagnetic wave is periodic with a period of 2π, which causes ambiguity in the extracted results. Assuming that one has some knowledge, whether it is *a priori* or *a posteriori*, about the material, one can resolve some of the ambiguities. This encumbers the use of this method for characterization of new complicated structured media. Other possible techniques to overcome the ambiguity of the extraction results will be discussed later on in this paper.

In what follows we will propose a novel way to overcome the equivocation related to the extracted material parameters and study the performance of the proposed technique with examples.

II. Theory

In [1], [2] material parameter extraction technique based on the reflection and transmission measurements was proposed. It was shown that the complex permittivity and permeability (here we use the time-dependency convention $e^{j\omega t}$)

$$\varepsilon = \varepsilon_{\rm r}\varepsilon_0 = (\varepsilon_{\rm r}' - j\varepsilon_{\rm r}'')\,\varepsilon_0, \quad \mu = \mu_{\rm r}\mu_0 = (\mu_{\rm r}' - j\mu_{\rm r}'')\,\mu_0 \quad (1)$$

could be derived from the measured $S-$parameters after some mathematical manipulation. Indeed, using the reflection coefficient from the interface of a semi-infinite material slab

$$\Gamma = \chi \pm \sqrt{\chi^2 - 1}, \quad \chi = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}, \quad (2)$$

and the phase factor for the wave propagating through the measured material slab with thickness d

$$e^{-\gamma d} = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\,\Gamma}, \quad (3)$$

with $\gamma = j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{n^2 - (\omega_{\rm c}/\omega)^2}$, one can calculate the relative material parameters as follows [1], [2]:

$$n^2 = \varepsilon_{\rm r}\mu_{\rm r} = -\left[\frac{c}{\omega d}\ln\left(e^{\gamma d}\right)\right]^2 + \left(\frac{\omega_{\rm c}}{\omega}\right)^2, \quad (4)$$

$$\mu_{\rm r} = \frac{1+\Gamma}{1-\Gamma}\sqrt{\frac{n^2 - (\omega_{\rm c}/\omega)^2}{1 - (\omega_{\rm c}/\omega)^2}}. \quad (5)$$

In the above equations, $c = 1/\sqrt{\varepsilon_0\mu_0}$, ω is the angular frequency and $\omega_{\rm c}$ is the angular cut-off frequency of the transmission line section in which the material sample is positioned. For quasi-TEM transmission lines (and obviously for free-space measurements) $\omega_{\rm c} \to 0$. The parameters S_{11} at the air-material interface and S_{21} through the material can be calculated from the scattering parameters taking into account the phase correction due to the separation of the planes 1 and 2, and ports 1 and 2 (see Fig. 1 where a schematic representation of the setup is shown).

The difficulty with the aforementioned extraction technique relates to the periodicity of the phase factor for the wave propagating through the measured material slab that causes the equation (3) to have an infinite number of roots, that is, $e^{-\gamma d} = e^{-\alpha d}e^{-j(\beta d + 2\pi m)}$, where α and β are the attenuation

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O. Luukkonen is with the Nokia Corporation FI, NRC Helsinki, P.O.Box 407 FI-00045, Nokia Group, Finland (email: olli.j.luukkonen@nokia.com)

S. I. Maslovski is with the Departmento de Engenharia Electrotécnica, Instituto de Telecomunicações - Universidade de Coimbra, Pólo II, 3030 Coimbra, Portugal. (email:stas@co.it.pt)

S. A. Tretyakov is with the Department of Radio Science and Engineering/SMARAD CoE, Aalto University, P.O. 13000, 00076 AALTO, Finland. (email: sergei.tretyakov@aalto.fi)

and propagation constants in the measured sample, respectively, and m is some integer. There are, however, ways to determine the correct root, and, respectively, the branch of the natural logarithm in (4). These include, just to name a few, matching of the measured group delay with the calculated one [2], performing the measurements only below the first Fabry-Perot resonance of the sample [4], [6], [8], evaluating the correct branch by expanding the exponent function in the preceding measurement point in a Taylor series and matching this approximation to the result of the extraction method for the next measurement point [7], using other iterative methods [5], or determining the correct branch based on the extraction results [3]. The latest requires, however, knowledge about the properties of the material, *e.g.*, about passivity of the material. In [9] this was done by comparing the retrieved values to the ones calculated through Kramers-Kronig relations. Here we propose another simple technique for removing this ambiguity related to the determination of the correct branch.

In the previously proposed extraction methods it is needed to verify the correct branch of the solution at each measurement point separately. If this is not done, for a material sample with sufficiently large electrical thickness the automated extraction procedure does not perceive the change of the branch but remains on the branch corresponding to the choice $m = 0$. In the proposed method we measure the phase difference between the preceding measurement points (rather than the phase delay at a given frequency point) and this way use the information about the phase delay at the previous measurement point. Since the phase difference between the preceding measurement points from the automated extraction procedure point of view is rather independent from the choice of the branch, we are able to remain in the correct branch throughout the extraction procedure. This way one does not need to verify the branch of each individual measurement point separately, but it is sufficient to verify only the correct branch choice at the first measurement point. Let us see the mathematical representation of this approach in the following.

Starting from the phase factor appearing in (3) and (4), let us write its inverse in the following form:

$$e^{\gamma d} = e^{j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{n^2-(\omega_\mathrm{c}/\omega)^2}} = e^{\alpha d}e^{j(\beta d+2\pi m)} = Ae^{j\phi}, \quad (6)$$

We can now rewrite the natural logarithm of the exponential appearing in (4) in a slightly different way:

$$\ln\left(e^{\gamma d}\right) = \ln\left(\left|e^{\gamma d}\right|\right) + j\arg\left(e^{\gamma d}\right). \quad (7)$$

Let us assume that we have a set of frequency points $\omega_0, \omega_1, \omega_2, \ldots \omega_N$ (corresponding to $\gamma_0, \gamma_1, \gamma_2, \ldots \gamma_N$) at which we have measured both the reflection and transmission from the sample. For all these frequency points we have a corresponding argument of (6): $\phi_0, \phi_1, \phi_2, \ldots \phi_N$. The argument term can now be written at a specific frequency point N as

$$\phi_N = \phi_0 + \sum_{i=1}^{N} \arg\left(\frac{e^{\gamma_i d}}{e^{\gamma_{i-1} d}}\right), \quad (8)$$

where ϕ_0 is the value of the phase at the first measurement point.

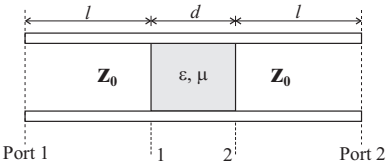

Fig. 1. A schematic figure of the setup used for extraction of the material parameters.

Next we write for the refractive index n at N-th sample

$$\sqrt{(n_N)^2 - (\omega_\mathrm{c}/\omega_N)^2} = \frac{1}{k_0 d}\Bigg[- j\ln\left(\left|e^{\gamma_N d}\right|\right) + \phi_0 + \sum_{i=1}^{N} \arg\left(\frac{e^{\gamma_i d}}{e^{\gamma_{i-1} d}}\right)\Bigg]. \quad (9)$$

Instead of using (5), it is more convenient to express the permeability through the relative wave impedance z [6], [8]:

$$z = \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \quad \mu_\mathrm{r} = z\sqrt{\frac{n^2 - (\omega_\mathrm{c}/\omega)^2}{1-(\omega_\mathrm{c}/\omega)^2}}, \quad (10)$$

because the passivity condition used to remove the ambiguity in (2) is simply equivalent to the choice $\Re(z) \geq 0$.[1]

Similarly, for the phase factor the ambiguity can be overcome by expressing the exponential in terms of the $S-$parameters and the impedance directly, without the use of the intermediate quantity Γ:

$$e^{\gamma d} = \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} + \frac{2S_{11}}{\left(z - \frac{1}{z}\right) S_{21}}. \quad (11)$$

It can be verified that (11) is equivalent to Eq. (6) of Ref. [7].

The proposed technique can be used for material parameter extraction without worrying about the correct branch of the solution in the measured frequency band provided that $\Delta\phi = \phi_i - \phi_{i-1} < \pi$. Also, in order to unambiguously determine the values of the material parameters the thickness d of the sample should remain small at the lowest measurement frequency: $d < \lambda/2$, in which case m can be set to 0 at this frequency point. Otherwise one needs to set the integer m corresponding to the electrical thickness of the sample at ω_0.

Examining (11) we see that this equation is sensitive to the inaccuracies of the S-parameters at the frequencies where either $S_{11} \to 0$ or $S_{21} \to 0$ while evaluating ϕ_N. For a noisy measurement this will induce uncertainty in the extracted material parameters and may cause problems in finding the correct branch in the vicinity of these frequencies. In the following we will show examples how the improved extraction technique performs for different materials also under noisy conditions.

III. Examples of numerical extraction of ε_r and μ_r for different materials

The proposed extraction technique was used to extract relative material parameters for different analytical examples. We tested the extraction method for dispersive$-\mu$ or dispersive$-\varepsilon$ materials. The reflection and transmission parameters for the

[1]This is a trivial case in many software packages since the standard procedure $w = \mathrm{sqrt}(z)$ maps the complex plane z onto the positive real half of the complex plane w ($\Re(w) \geq 0$).

samples were calculated analytically assuming an incident plane wave at the normal angle. First, the dispersive material parameters followed the Lorentzian dispersion

$$\left.\begin{matrix}\varepsilon_r(\omega)\\ \mu_r(\omega)\end{matrix}\right\} = 1 - \frac{A^2}{\omega^2 - \omega_{0,\varepsilon,\mu}^2 - j\Gamma\omega}, \tag{12}$$

where $A = 2\pi \times 4.3\,\mathrm{GHz}$, $\Gamma = 2\,\mathrm{GHz}$, $\omega_{0,\mu} = 2\pi \times 7\,\mathrm{GHz}$, and $\omega_{0,\varepsilon} = 2\pi \times 12\,\mathrm{GHz}$. The relative material parameters of the non-dispersive materials equaled to unity. The thickness of the samples $d = 12.5\,\mathrm{mm}$ was chosen to be sufficiently large so that the sample thickness exceeds the wavelength inside the material ($d > \lambda$) within the operational band.

Several cases of a material extraction with dispersive, double-negative and plasmonic materials were considered. The simulation model is illustrated in Fig. 1. The samples were positioned in the middle of the waveguide and the reflection and transmission parameters were calculated at the interfaces of the sample. For each material case we studied the performance of the extraction method in the absence and presence of noise. In order to simulate thermal and network analyzer noise, we added a random uncorrelated component to the calculated real and imaginary part of the $S-$parameters. The randomness was normally distributed with the mean value $\mu_\mathrm{d} = 0$ (we use suffix d here to distinguish the mean value from the permeability) and with the total standard deviation $\sigma = 0.005$.[2]

In Fig. 2(a) the extracted complex material parameters with noise for the dispersive$-\varepsilon$ are shown. For the dispersive$-\varepsilon$ case we see the electrical resonance occurring at the frequency 12 GHz in the extracted relative permittivity. In the case when we have no noise in the system (results not shown here for brevity), the extracted relative permeability in the dispersive$-\varepsilon$ case (or the relative permittivity in the dispersive$-\mu$ case) remains constant and we have no problems in finding the correct branch of the solution at the material thicknesses $d = m\lambda/2$, where m is some integer other than zero. As we include noise into the system, we see that at the thickness resonances (when $|S_{11}|$ approaches zero)) and in the stop band (when $|S_{21}|$ approaches zero) the accuracy of the extracted material parameters is deteriorated due to the noise. Nevertheless, we remain on the correct branch after every thickness resonance and above the stop band. For this particular example the thickness of the sample was chosen to be sufficiently small so that the frequency band for which $|S_{21}| \approx 0$ would not be too wide.

We applied the extraction procedure also for the simultaneously dispersive$-\varepsilon$ and $-\mu$ test material. These results are reported in Fig. 2(b). Similarly to the dispersive$-\varepsilon$ or dispersive$-\mu$ material case, we can see that noise (with $\mu_\mathrm{d} = 0$ and $\sigma = 0.005$) increases the uncertainty of the extracted material parameters around the thickness resonances and band gaps.

The proposed extraction technique is capable of maintaining the correct branch of the solution in the vicinity of the thickness resonances although the level of the noise would increase. Further, in very noisy situations the level of the noise can be reduced through different averaging methods and in this way improve the applicability of the proposed extraction technique. However, the pole related to the band gap cannot necessarily be overcome this way, but instead we can adjust the electrical thickness of the sample to be sufficiently small. As discussed above with a couple of examples, noise brings uncertainty to the extracted material parameter results and can, in the worst case, lead to losing the correct branch of the solution and to erroneous extracted material parameters.

As an example of this, we have added uncorrelated noise with the normal distribution ($\mu_\mathrm{d} = 0$, $\sigma = 0.05$) to the real and imaginary parts of the S-parameters calculated for simultaneously dispersive$-\varepsilon$ and $-\mu$ material. The $S-$parameters were sampled at 5000 points per 1 GHz and then averaged using a 100-point window. From the resulting $S-$parameters we then extracted the material parameters plotted in Fig. 2(c) for the sample thickness 20 mm.[3] We extracted also the material parameters from a sample with the thickness of 12.5 mm. The material parameter extraction for the thinner sample gave the same results as shown in Fig. 2(b) for a less noisy situation without averaging of the S-parameters. However, in the case of the thicker sample in Fig. 2(c) we see that the branch of the correct solution is lost in the first band gap at 7 GHz and so-called anti-resonances appear in the extracted material parameters. At these anti-resonances (at 7 GHz for the extracted permittivity and at 12 GHz for the extracted permeability) the imaginary part of either one of the material parameters becomes positive, despite the fact that passive materials are used, and therefore the results are non-physical (time dependency $e^{j\omega t}$ is used in this paper).

Keeping in mind the restrictions on the sample thickness in the proposed extraction technique we studied also double negative material parameter (DNG) cases. First, using the material parameters following the Lorentz dispersion as above, but having the material resonance frequencies at $\omega_{0,\mu} = 2\pi \times 7.4\,\mathrm{GHz}$ and $\omega_{0,\varepsilon} = 2\pi \times 7.2\,\mathrm{GHz}$ we have plotted the extracted material parameters from a material sample with the sample thickness 5 mm with normally distributed noise ($\mu_\mathrm{d} = 0$, $\sigma = 0.005$) in Fig. 2(d). In Fig. 2(e) the shown material parameters are extracted from a 12.5-mm thick material sample whose relative permeability followed the same Lorentzian dispersion as in (12) with $\omega_{0,\mu} = 2\pi \times 7\,\mathrm{GHz}$ and the relative permittivity followed the Drude dispersion

$$\varepsilon_\mathrm{r}(\omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega\,(\omega - j\gamma_\mathrm{p})}, \tag{13}$$

where $\omega_\mathrm{p} = 2\pi \times 9\,\mathrm{GHz}$ and $\gamma_\mathrm{p} = 5\,\mathrm{GHz}$. For both cases the proposed technique was capable of extracting the material parameters correctly despite the exotic characteristics of the

[2]The values of the noise here and in the following in the paper may not represent the actual situation in a measurement setup (they rather exaggerate it compared to a standard waveguide measurement). The noise has been generated in the paper plainly to show the possible pitfalls of the proposed material parameter extraction method.

[3]Since the generated noise is random, the preceding extracted material parameter results are never exactly the same. In this paper the material parameters extracted from the noisy reflection and transmission parameters are typical results one gets from the used mathematical sample using the proposed extraction technique.

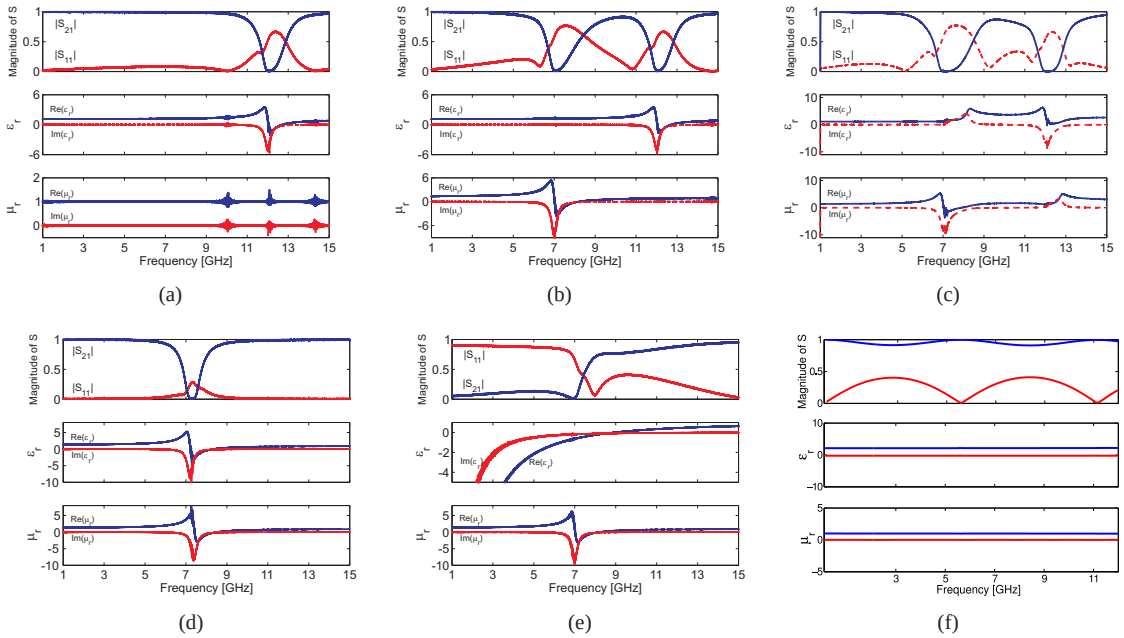

Fig. 2. The reflectance and transmittance, complex relative permeability, and complex relative permittivity extracted from the $S-$parameters for a slab of (a) dispersive$-\varepsilon$ material, simultaneously dispersive$-\varepsilon$ and $-\mu$ material with thicknesses of (b) 12.5 mm and (c) 20 mm. Similarly, in (d) the results are for simultaneously dispersive$-\varepsilon$ and $-\mu$ DNG material slab following Lorentzian dispersion, and in (e) the relative permittivity is of plasmonic nature. Fig. (f) presents the results for a 3D array of dielectric spheres $\epsilon_r = 10$ in the martrix with $\epsilon_r = 2$. The volume fraction of spheres is 0.1. All the analytical results are calculated with noise and in (c) the results are erroneous due to a larger thickness of the material sample.

samples given that the thickness of the material samples were chosen sufficiently small according to the above discussion.

Finally, we have tested the method for a 3D array of dielectric spheres ($\epsilon_r = 10$) in a dielectric host ($\epsilon_r = 2$), see Fig. 2(f). For the inclusion volume fraction of 10% the Maxwell Garnett formula gives $\epsilon_{\text{eff}} = 2.35$. The value extracted from the numerically simulated S-parameters of a composite slab agrees with this estimate very well, and the correct branch is maintained despite two thickness resonances of the samples within the simulated frequency band.

IV. Discussion and Conclusions

Our results suggest that the proposed technique is efficient when applied to extracting material parameters for realistic material samples from the $S-$parameters. The problems at the frequency points where more simplistic extraction procedures based on the NRW technique fail, that is, at thickness resonances and in stop bands, can be overcome using the proposed technique. In the latter case the thickness of the material sample needs to be chosen small enough so that the transmission inside the stop band would not vanish over a wide frequency band: In the presence of noise having a normal distribution ($\mu_{\text{d}} = 0$, $\sigma = 0.05$) we succeeded in extracting the material parameters correctly when the thickness of the sample was small enough. This was not so for a thicker sample, in which case the correct branch of the solution was lost. Interestingly, in this case for the thicker sample the extracted material parameters showed similar type of anti-resonances that can be often seen in the extracted material parameter due to the finite electrical size of the unit cells which is not properly accounted for in the NRW extraction procedure.

In conclusion, we have derived a material parameter extraction method based on the NRW technique that, unlike its predecessor, is capable of extracting the material parameters without any additional branch-seeking algorithms or methods. Nevertheless, the proposed technique has certain limitations: the electrical thickness of the sample at the first sample point should be smaller than $\beta d < \pi$ and the frequency interval between any two subsequent sample points needs to be sufficiently small. In our mathematical analysis we were able to extract material parameters correctly from the samples by making sure that in the presence of a band gap, the thickness of the sample remained small enough.

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