Question-1

Quality assurance checks on the previous batches of medications found that it is four times more likely that a medicine is able to produce a satisfactory result than not.

Given a small sample of 10 medicines, you are required to find the theoretical probability that, at most, 3 medicines are unable to do a satisfactory job:

1.Propose the type of probability distribution that would accurately portray the above-mentioned scenario, and list out the three conditions that this distribution follows.

Answer:-

Two Possible Outcomes: Each medicine can either produce a satisfactory result (success) or not (failure).
Independent Trials: The quality assurance checks on each medicine are assumed to be independent of each other.
Constant Probability: The probability of a medicine producing a satisfactory result remains the same for each medicine.
Fixed Number of Trials: The sample size is fixed at 10 medicines.
Now, to calculate the probability that at most 3 medicines are unable to produce a satisfactory result, we need to find the probabilities for each possible outcome (0, 1, 2, and 3 medicines unable to do a satisfactory job) and then add them up.

Calculate the required probability.

Let X be the number of drugs that produce an unsatisfactory result after testing of 10 drugs. Therefore, X follows a binomial distribution with n = 10. As it is specified that it is 4times more likely that a drug is able to produce a satisfactory result than not.

Let p be the probability for an unsatisfactory result.

So, $p+4p = 1$ { The sum of the probabilities in a probability distribution is always 1. }

$\Rightarrow 5p = 1$ Therefore, $p = 1/5 = 0.2$

Probability (drug produces unsatisfactory result) i.e., $p=0.2$Probability (drug produces satisfactory result) = $1-p =0.8$

Therefore, the theoretical probability that at most, 3 drugs are not able to do a satisfactory job can be defined as cumulative probability of X, denoted by F(x), which is the probability that the random variableX takes a value less than or equal to x. Therefore, $F(x) = P(X \le x)$

$\Rightarrow F(3) = P(X \le 3)$

$\Rightarrow F(3) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

Formula for Binomial distribution

$P(x = r) = {}^{n}C_{r}(P)^{r}(1-P)^{n-r}$

n = no. of Trials

P = Probability of unsatisfactory results = 0.2

r = no. of unsatisfactory results after n number of trials. i.e. r = 0, 1, 2, 3

$P(x=0) = {}^{10}C_{0}(0.2)^{0}(1-0.2)^{10-0} = 0.107$

$P(x=1) = {}^{10}C_{1}(0.2)^{1}(1-0.2)^{10-1} = 0.268$

$P(x=2) = {}^{10}C_{2}(0.2)^{2}(1-0.2)^{10-2} = 0.302$

$P(x=3) = {}^{10}C_{3}(0.2)^{3}(1-0.2)^{10-3} = 0.201$

$P(x \le 3) = P(x=0)+P(x=1)+P(x=2)+P(x=3)$

$= 0.107+0.268+0.302+0.201$

$= 0.878$

Hence,

$F(3) = P(X \le 3) = 0.878 = 87.8\ \%$

Therefore, the probability that at most 3 drugs are unable to do satisfactory job is 87.8%

Question 2:

For the effectiveness test, a sample of 100 medicines was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the interval in which the population mean mightlie – with a 95% confidence level:

1. Discuss the main methodology using which you will approach this problem. State allthe properties of the required method. Limit your answer to 150 words.

Answer:-

The main methodology for approaching the problem would be to use Central Limit Theorem in order to estimate the population mean in the form of an interval.

Central limit theorem states that no matter how the original population is distributed, the sampling distribution will follow these three properties:

1. Sampling distribution's mean ($\boldsymbol{\mu\bar{x}}$)= Population mean ($\boldsymbol{\mu}$)
2. Sampling distribution's standard deviation (Standard error) = σ √n , where σ is the population's standarddeviation and n is the sample size.
3. For n > 30, the sampling distribution becomes a normal distribution.

CLT lets us assume that the sample mean would be normally distributed with mean (μ) and therefore sampling distribution's standard deviation σ √n can be taken as (approx. S √n) where (S) is standard deviation of the sample and (σ) is population's standard deviation. Moreover, we only know the standarddeviation of the sample in the given case.

2. Find the required interval.

Answer:-

- Sample size (n) = 100
- Sample mean ($\mu\bar{x}$) = 207
- Sample standard deviation (S) = 65

Given the sample size, mean and standard deviation, we can say that the confidence interval for μ lies in the range of $\left(\mu \bar{x} - \frac{z \times s}{\sqrt{n}}, \mu \bar{x} + \frac{z \times s}{\sqrt{n}}\right)$ i.e., the population mean and sample mean differ by a margin of error given by $\frac{z \times s}{\sqrt{n}}$. Here z^* is the z-score associated with a y% confidence level i.e. 95% in the above case.

Z* Values for Commonly Used Confidence Levels are:

Confidence Level	Z*
90%	± 0.65
95%	± 1.96
99%	± 2.58

Therefore, we can say that the population mean (μ) lies between 194.26 seconds and 219.74 seconds.

Question 3:

1. The painkiller needs to have a time of effect of at most 200 seconds to be considered as having donea satisfactory job. Given the same sample data (size, mean and standard deviation) as that in the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilise two hypothesis testing methods to take a decision. Take the significance level at 5%. Clearly specify the hypotheses, the calculated test statistics and the final decision that should be made for each method.

Answer:-

To test the claim that the newer batch of painkillers produces a satisfactory result and passes the quality assurance test, we will use two hypothesis testing methods: Z-test and T-test. Given the sample conditions and significance level, let's perform both tests.

Hypotheses:

Null Hypothesis (H0): The newer batch of painkillers has a time of effect greater than 200 seconds (not satisfactory).

Alternative Hypothesis (H1): The newer batch of painkillers has a time of effect of at most 200 seconds (satisfactory).

Null(H0) and Alternate Hypothesis(H1) are:

H0 : $\mu \leq 200$ seconds, the time of effect to be considered as having done a satisfactory job.

H1 : $\mu > 200$ seconds, the time of effect to be considered as not having done a satisfactory job.

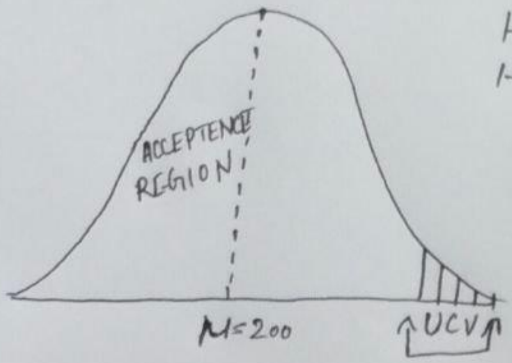
<u>Test Type:</u>

> sign in alternate hypothesis tells us that it would be a One-tailed test (i.e. upper-tailed test). This is a directional hypothesis and therefore the rejection region will lie on the right side of the distribution.

- Sample size n = 100
- Assumed Sample mean (μ) = 200
- Sample mean ($\mu\bar{x}$)= 207
- Sample standard deviation $\sigma\bar{x}$= 65.
- Significance level α = 5% i.e. 0.05

Critical Value Test:

$H_0 :- \mu \leq 200$
$H_1 :- \mu > 200$



1) $n = 100$
2) Assumed sample mean $(\mu) = 200$
3) sample mean $(\mu_{\bar{x}}) = 207$
4) sample standard deviation $(S) = 65$
5) significance level $(\alpha) = 5\% = 0.05$

Cumulative probability till critical area $= 1 - 0.05$
$= 0.95$

Z score for $0.95 = 1.645$ } from Z - table

$\therefore$ upper critical value $(UCV) = \left(\frac{\mu + Z \times \sigma}{\sqrt{n}} \right)$

$$= \frac{200 + 1.645 \times 65}{\sqrt{100}}$$

$$= 200 + 10.69$$

$$= 210.69$$

Since the sample mean ($\boldsymbol{\mu\bar{x}}$) i.e., 207 seconds is less than the Upper Critical Value of 210.69 seconds, it lies in the acceptance region. Therefore, we fail to reject the null hypothesis which states that the drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job.

p-value Test:

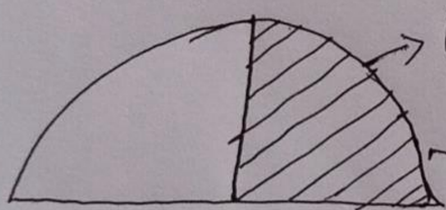
Z score for sample mean is given by:-

$$Z = \frac{\overline{u}x - u}{(\sigma/\sqrt{n})}$$

$$= \frac{207-200}{(65/\sqrt{100})}$$

$$= 1.077$$

The P-value for $Z = 1.77 = 0.8577$] from Z-table



P-value $= 1 - 0.08577$

$= 0.1423$

$= \alpha = 5\%$

$Z = 1.076$

Since the sample mean is on the right side of the distribution and this is a one-tailed test, the p value would be (1 - 0.8577) = 0.1423 (14.23%) Since the p-value is greater than the significance level (0.1423 > 0.05), we fail to reject the null hypothesis which states that the drug needs a time of effect <= 200 seconds to do a satisfactory job.

2. You know that two types of errors can occur during hypothesis testing – Type I and Type II errors – whose probabilities are denoted by α and β, respectively. For the current sample conditions (sample size, mean and standard deviation), the value of α and β come out to be 0.05 and 0.45, respectively.

In hypothesis testing:

Type-1 (α) error refers to the scenario where we decided to reject the NULL hypothesis even thoughit was TRUE

Type-2 (β) error refers to the scenario where we failed to reject the NULL hypothesis when it wasFALSE

There are two possible scenarios:

Scenario-1: The value of α and β come out to be 0.05 and 0.45 respectively.

Scenario-2: The value of α and β are controlled at 0.15 each.

We try to keep the α error limited to a very low value when there are side-effects related to acceptanceof alternate hypothesis. From the scenario stated in question, if there is any side effect related to the overdose of the painkiller drug, we need to keep the α error to a minimum i.e., we are conservative inrejecting the null hypothesis.

So, if the painkiller drug has significant side effects, we would like to keep the value of α and β comeout to be 0.05 and 0.45 respectively.

On the contrary, we tend to be relaxed with the α error if there were no major side effects in the painkiller drug. This would mean that we can be aggressive about changing the status quo and establishthe alternate hypothesis. In such cases, our β error limit would be very restrictive.

Question 4:

Once one batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:-

A/B testing, also known as split testing, is a method used to compare two different versions of a marketing element, such as a webpage, ad, or email, to determine which one performs better in terms of a desired outcome. In this case, the marketing team wants to decide which tagline to use in their online ad campaign to attract new subscribers. A/B testing can help them make an informed decision by providing empirical evidence of which tagline is more effective in driving the desired actions (e.g., clicking on the ad, subscribing).

Stepwise Procedure for A/B Testing:

Define the Objective: Clearly outline the goal of the A/B test. In this case, the objective is to determine which tagline is more effective in attracting new subscribers.

Select the Variants: Choose the two taglines (Variant A and Variant B) that the marketing team wants to test. These should be distinct and reflective of the different approaches the team is considering.
Random Sampling: Randomly divide your target audience into two groups: Group A and Group B. It's important to ensure that these groups are as similar as possible in terms of characteristics that might affect the outcome (e.g., demographics, interests).

Implement the Variants: Display Variant A to Group A and Variant B to Group B. This can be done on a website, through an email campaign, or any other relevant platform.

Data Collection: During the testing period, gather data on user interactions with both taglines. This could include metrics such as click-through rates, conversion rates, subscription rates, and other relevant user actions.

Statistical Analysis: Analyze the collected data to determine the performance of each variant. Common statistical tests used for A/B testing include chi-squared tests for categorical data (e.g., conversion rates) and t-tests for continuous data (e.g., time spent on page).

Calculate P-Values: Calculate the p-values associated with the statistical tests. The p-value indicates the probability of observing the data if there is no real difference between the variants. A lower p-value suggests a more significant difference.

Compare Results: If the p-value is below the pre-defined significance level (e.g., 0.05), it suggests that there is a statistically significant difference between the two taglines. The variant with the higher performance (e.g., higher click-through rate or conversion rate) is the more effective option.

Consider Practical Significance: While statistical significance is important, also consider the practical significance of the difference. Even if a difference is statistically significant, it might not be practically significant enough to warrant a change in strategy.

Make a Decision: Based on the statistical analysis and practical considerations, decide which tagline to use for the online ad campaign. Choose the variant that aligns best with the marketing team's goals and objectives.

Monitor and Optimize: After implementing the chosen tagline, continue to monitor its performance and make further optimizations if necessary. A/B testing is an iterative process that can be used to refine marketing strategies over time.

A/B testing allows the marketing team to make data-driven decisions and avoid making choices based solely on intuition or assumptions. It provides a structured and objective way to evaluate the effectiveness of different options and select the one that maximizes the desired outcome.